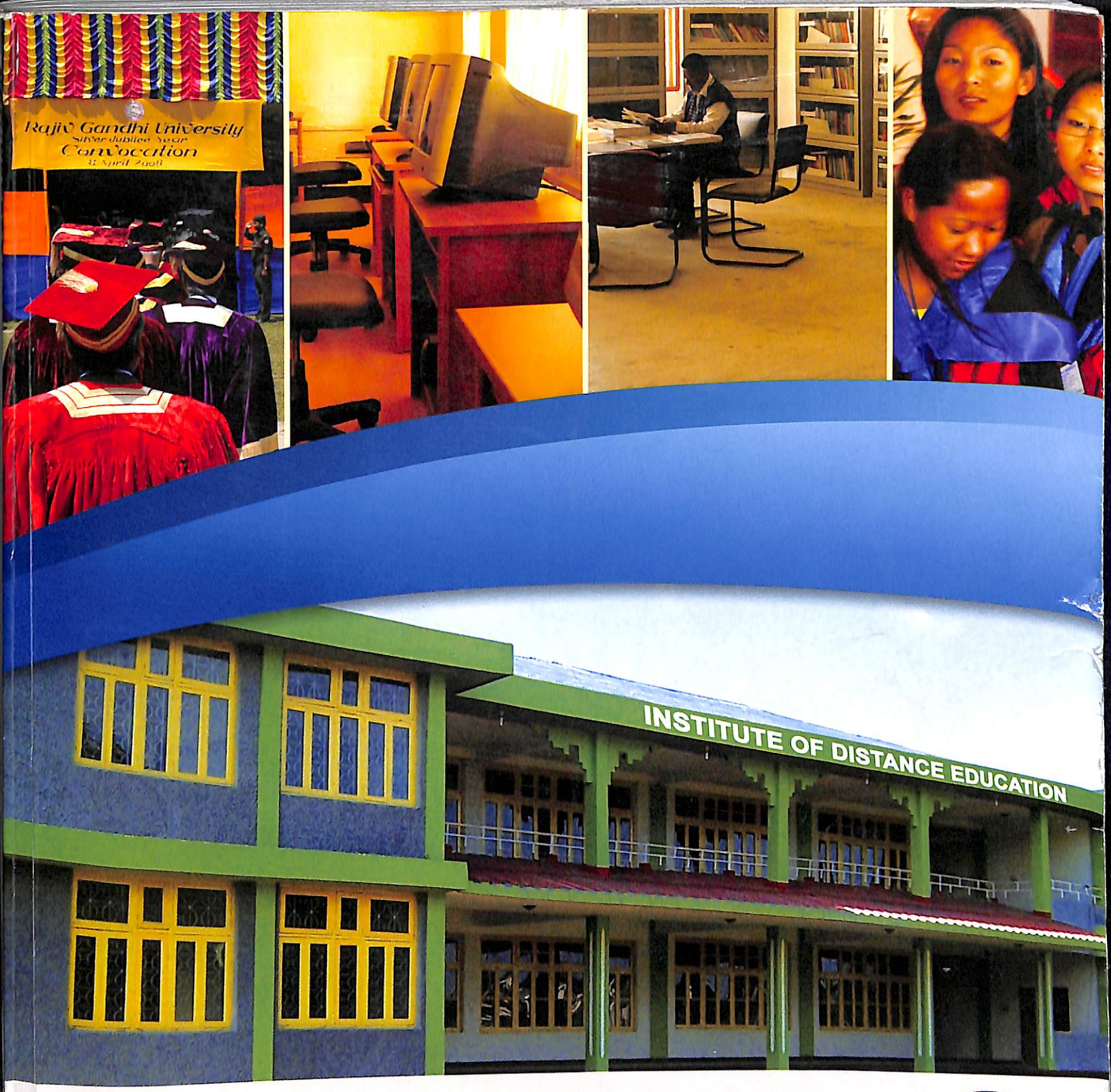

M.A. (Education)

FIRST YEAR

MAEDN-403

INSTITUTE OF DISTANCE EDUCATION
Rajiv Gandhi University

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METHODOLOGY OF RESEARCH IN EDUCATION

METHODOLOGY OF RESEARCH IN EDUCATION

MA [Education]

First Year

MAEDN - 403

RAJIV GANDHI UNIVERSITY
Arunachal Pradesh, INDIA - 791 112

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About the University

Rajiv Gandhi University (formerly Arunachal University) is a premier institution for higher education in the state of Arunachal Pradesh and has completed twenty-five years of its existence. Late Smt. Indira Gandhi, the then Prime Minister of India, laid the foundation stone of the university on 4th February, 1984 at Rono Hills, where the present campus is located.

Ever since its inception, the university has been trying to achieve excellence and fulfill the objectives as envisaged in the University Act. The university received academic recognition under Section 2(f) from the University Grants Commission on 28th March, 1985 and started functioning from 1st April, 1985. It got financial recognition under section 12-B of the UGC on 25th March, 1994. Since then Rajiv Gandhi University, (then Arunachal University) has carved a niche for itself in the educational scenario of the country following its selection as a University with potential for excellence by a high-level expert committee of the University Grants Commission from among universities in India.

The University was converted into a Central University with effect from 9th April, 2007 as per notification of the Ministry of Human Resource Development, Government of India.

The University is located atop Rono Hills on a picturesque tableland of 302 acres overlooking the river Dikrong. It is 6.5 km from the National Highway 52-A and 25 km from Itanagar, the State capital. The campus is linked with the National Highway by the Dikrong bridge.

The teaching and research programmes of the University are designed with a view to play a positive role in the socio-economic and cultural development of the State. The University offers Undergraduate, Post-graduate, M.Phil and Ph.D. programmes. The Department of Education also offers the B.Ed. programme.

There are fifteen colleges affiliated to the University. The University has been extending educational facilities to students from the neighbouring states, particularly Assam. The strength of students in different departments of the University and in affiliated colleges has been steadily increasing.

The faculty members have been actively engaged in research activities with financial support from UGC and other funding agencies. Since inception, a number of proposals on research projects have been sanctioned by various funding agencies to the University. Various departments have organized numerous seminars, workshops and conferences. Many faculty members have participated in national and international conferences and seminars held within the country and abroad. Eminent scholars and distinguished personalities have visited the University and delivered lectures on various disciplines.

The academic year 2000-2001 was a year of consolidation for the University. The switch over from the annual to the semester system took off smoothly and the performance of the students registered a marked improvement. Various syllabi designed by Boards of Post-Graduate Studies (BPGS) have been implemented. VSAT facility installed by the ERNET India, New Delhi under the UGC-Infonet program, provides Internet access.

In spite of infrastructural constraints, the University has been maintaining its academic excellence. The University has strictly adhered to the academic calendar, conducted the examinations and declared the results on time. The students from the University have found placements not only in State and Central Government Services, but also in various institutions, industries and organizations. Many students have emerged successful in the National Eligibility Test (NET).

Since inception, the University has made significant progress in teaching, research, innovations in curriculum development and developing infrastructure.

About IDE

The formal system of higher education in our country is facing the problems of access, limitation of seats, lack of facilities and infrastructure. Academicians from various disciplines opine that it is learning which is more important and not the channel of education. The education through distance mode is an alternative mode of imparting instruction to overcome the problems of access, infrastructure and socio-economic barriers. This will meet the demand for qualitative higher education of millions of people who cannot get admission in the regular system and wish to pursue their education. It also helps interested employed and unemployed men and women to continue with their higher education. Distance education is a distinct approach to impart education to learners who remained away in the space and/or time from the teachers and teaching institutions on account of economic, social and other considerations. Our main aim is to provide higher education opportunities to those who are unable to join regular academic and vocational education programmes in the affiliated colleges of the University and make higher education reach to the doorsteps in rural and geographically remote areas of Arunachal Pradesh in particular and North-eastern part of India in general. In 2008, the Centre for Distance Education has been renamed as "Institute of Distance Education (IDE)."

Continuing the endeavor to expand the learning opportunities for distant learners, IDE has introduced Post Graduate Courses in 5 subjects (Education, English, Hindi, History and Political Science) from the Academic Session 2013-14.

The Institute of Distance Education is housed in the Physical Sciences Faculty Building (first floor) next to the University Library. The University campus is 6 kms from NERIST point on National Highway 52A. The University buses ply to NERIST point regularly.

Outstanding Features of Institute of Distance Education:

(i) At Par with Regular Mode

Eligibility requirements, curricular content, mode of examination and the award of degrees are on par with the colleges affiliated to the Rajiv Gandhi University and the Department(s) of the University.

(ii) Self-Instructional Study Material (SISM)

The students are provided SISM prepared by the Institute and approved by Distance Education Council (DEC), New Delhi. This will be provided at the time of admission at the IDE or its Study Centres. SISM is provided only in English except Hindi subject.

(iii) Contact and Counselling Programme (CCP)

The course curriculum of every programme involves counselling in the form of personal contact programme of duration of approximately 7-15 days. The CCP shall not be compulsory for BA. However for professional courses and MA the attendance in CCP will be mandatory.

(iv) Field Training and Project

For professional course(s) there shall be provision of field training and project writing in the concerned subject.

(v) Medium of Instruction and Examination

The medium of instruction and examination will be English for all the subjects except for those subjects where the learners will need to write in the respective languages.

(vi) Subject/Counselling Coordinators

For developing study material, the IDE appoints subject coordinators from within and outside the University. In order to run the PCCP effectively Counselling Coordinators are engaged from the Departments of the University, The Counselling-Coordinators do necessary coordination for involving resource persons in contact and counselling programme and assignment evaluation. The learners can also contact them for clarifying their difficulties in then respective subjects.

SYLLABI-BOOK MAPPING TABLE

Methodology of Research in Education

Syllabi	Mapping in Book
UNIT-I: Educational Research - Scientific Enquiry and Theory Development. - Meaning, Nature, Scope and Principles of Educational Research, Need and Purpose. - Types of Educational Research: Fundamental, Applied and Action Research.	**Unit 1:** Educational Research **(Pages 3-34)**
UNIT-II: Research Process in Education - General Steps, Formulation of Problem and Its Objectives, Review of Related Literature and Variables in Research Problems. - Hypothesis: Meaning, Sources, Types and Testing.	**Unit 2:** Research Process in Education **(Pages 35-96)**
UNIT-III: Methods of Educational Research - Historical Research, Descriptive Research, Survey Research. - Experimental Research: Designs of Experimental Research, Characteristics. - Internal and External Validity in Experimental Research. - Qualitative Research: Phenomenological, Ethnographical.	**Unit 3:** Methods of Educational Research **(Pages 97-155)**
UNIT-IV: Tools and Techniques in Educational Research - Inquiry Forms, Observation, Interview, Sociometry, Rating Scale and Questionnaire. - Sampling Techniques in Educational Research. - Data Collection Procedure. - Analysis of Data and Reporting. - Organization and Statistical Analysis of Data. - Interpretation of Data. - Writing of Research Proposal and Report.	**Unit 4:** Tools and Techniques in Educational Research **(Pages 157-238)**
UNIT-V: Measures of Central Tendency and Variability - Measures of Central Tendency and Variability and Their Computation and Uses.	**Unit 5:** Measures of Central Tendency and Variability **(Pages 239-291)**
UNIT-VI: Correlation - Correlation: Concept and Its Applications. - Methods of Computing - Rank Difference and Pearson's Coefficient of Correlation.	**Unit 6:** Correlation **(Pages 293-322)**
UNIT-VII: Normal Probability Curve and Tests of Significance - Properties and Applications. - The Concept of Statistical Significance. - Testing the Significance of Mean, Proportion and Correlation.	**Unit 7:** Normal Probability Curve and Statistical Significance **(Pages 323-345)**
UNIT-VIII: Hypothesis Testing - Testing the Significance of Difference between Means, Proportion and Correlation. - Chi-Square (χ^2), Types of Errors, One-Tailed and Two-Tailed Tests (ANOVA-One Way).	**Unit 8:** Hypothesis Testing **(Pages 347-388)**

CONTENTS

INTRODUCTION

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Research is the search for knowledge or a systematic investigation in order to establish facts. The basic aim of research is to discover, interpret and develop methods and systems to advance human knowledge on diverse scientific matters. Methodology of research refers to the way research can be conducted. It is also known as the process of collecting data for various research projects. Research methodology is thus the science of studying how research is conducted scientifically. It helps to understand both the products as well as the process of scientific enquiry. A research process involves selection and formulation of a research problem, research design, sample strategy or sample design, as well as the interpretation and preparation of research report. Research can be undertaken in the form of descriptive/survey research, applied or fundamental research, quantitative or qualitative research, conceptual or empirical research, and also some other types of specific research.

One can also define research as a scientific and systematic pursuit of information on a specific topic. Scientifically, research can also be termed as scientific investigation. Thus, research and scientific enquiry can be considered synonymous. Consequently, research is a more specialized form of scientific enquiry which in turn is the result of gathering of data, information and facts for the specific purpose. There are several types of research designs depending on the type of research study being conducted. The quality of a good research design depends on the extent of its flexibility, efficiency and economy. Research design is also determined by adhering to certain basic principles of research design, such as the randomization principle and the local control principle.

Educational research is a systematic attempt to gain a better understanding of educational process, which is generally meant for improving its efficiency. Research in education is a systematic process for developing a theory by applying scientific methods. It is an impartial, objective, empirical and logical analysis and recording of controlled observation that will finally lead to the development of a theory, principles, laws, etc., and will help us to predict about the phenomenon in future. A research is said to begin with a question or a problem. The purpose of a research is to find out solutions through the application of systematic and scientific methods. Thus, research is a systematic approach to a purposeful investigation. A few important factors in research methodology include the validity and reliability of research data and the level of ethics. A job is considered half done if the data analysis is not conducted properly.

Researchers working in the field of education have a huge task in front of them. The state of education system and educational policies in India has thrown up many fundamental questions. These questions can only be answered through comprehensive and thorough research that follows a scientific and data-centric methodology. Since the issues in education are complex and manifold, the research required is also multi-layered and time-consuming. This book covers the multiple aspects related to educational research and educational statistics, including the scope of educational research, review of related literature, the significance of hypotheses in any kind of research, sampling, tools of educational research, and preparation of research report and its presentation.

This book, *Methodology of Research in Education*, has been designed keeping in mind the self-instruction mode format and follows a SIM pattern, wherein each unit begins with an 'Introduction' to the topic followed by the 'Unit Objectives'. The content is then presented in a simple and easy-to-understand manner, and is interspersed with 'Check Your Progress' questions to test the reader's understanding of the topic. 'Key Terms' and 'Summary' are useful tools for effective recapitulation of the text. A list of 'Questions and Exercises' is also provided at the end of each unit, which includes short-answer as well as long-answer questions.

UNIT 1 EDUCATIONAL RESEARCH

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Structure

1.0 INTRODUCTION

Simply defined, research is a search for knowledge. One can also define research as a scientific and systematic pursuit of information on a specific topic. Scientifically, research can also be termed as scientific investigation. Thus, research and scientific enquiry can be considered synonymous. The only difference between the two is that while it is possible to employ scientific method without research, it is not possible to conduct any research without employing scientific methods. Thus, research is a more specialized form of scientific enquiry which in turn is the result of gathering of data, information and facts for the specific purpose.

There are several types of research designs depending on the type of research study being conducted. The quality of a good research design depends on the extent of its flexibility, efficiency and economy. Research design is also determined by adhering to certain basic principles of research design, such as the randomization principle and the local control principle. This unit will discuss the research process and the research proposal. A research proposal is like a foreword to the exhaustive research paper or dissertation and thus acts as a presentation paper for the researcher. Once the research proposal is approved, the researcher can go ahead and perform his/her research process.

You will also learn how a research design functions as a proactive tool for the researcher. It involves the technique of determining the methods of data collection and

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the ways in which the research work should be performed, giving due consideration to time and cost constraints.

Educational research is a systematic attempt to gain a better understanding of educational process, which is generally meant for improving its efficiency. A research design is a predefined part of a research study. It is a statement that clearly defines the problem for which the research is being done.

In this unit, you will be also familiarized with the concept of scientific method, research design, need and significance of research in education in detail.

1.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Describe the significance of scientific enquiry and the theory developed using scientific method
- Explain how scientific theory is used in research
- Discuss the nature and scope of educational research
- Elaborate the objectives, the characteristics and types of educational research
- Identify the purpose, process and scope of educational research
- Recognize problems faced by researchers in India

1.2 SCIENTIFIC ENQUIRY AND THEORY DEVELOPMENT

Science refers to organized knowledge, but this knowledge and these facts are seldom conclusive. New experiences and additional information constantly change previous findings and replace them with generalizations that confirm the latest bodies of findings.

A scientific enquiry is an investigation or experiment carried out to dispel or confirm various scientific theories. Most scientific enquiries are done practically in labouratories with specialized equipment.

The scientific method is based on techniques used to investigate phenomena, acquire new knowledge or correct and integrate previous knowledge. Any method is termed scientific when the inquiry is based on experiential and computable evidences subject to specific principles of reasoning. As per the *Oxford English Dictionary*, ***"The scientific method is a method or procedure that has characterized natural science since the 17th century, consisting in systematic observation, measurement, and experiment, and the formulation, testing, and modification of hypotheses."***

The key characteristic of the scientific method is that researchers support a theory when the predictions given for any specific theory are confirmed and challenge a theory when its predictions prove false, even though procedures differ from one field of inquiry to another. Theories that include extensive domains of inquiry may combine many independently derived hypotheses together in a logical and supportive structure. Theories are developed on the basis of scientific inquiry and are normally intended to be objective so as to reduce biased interpretations of results. The overall process of theory development involves making assumptions by defining hypotheses and deriving predictions as logical consequences. The experiments are then carried out based on those defined predictions

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to establish whether the original assumption was correct. The scientific method steps are used to establish a theory.

Objectives of Scientific Inquiry

The objective of a scientific inquiry is to acquire knowledge in the form of testable explanations that can predict the results of future experiments. The more enhanced an explanation is at making predictions, the more beneficial it is in proving the predictions that it is correct. The most successful explanations that elucidate and formulate accurate predictions for broad range of conditions are termed as scientific theories. The power of a theory is related to how long it has persisted without distortion of its core principles.

Scientific Enquiry Skills

There are many scientific enquiry skills that must be observed in order to develop scientific theory. Some of which are as follows:

- Raising/asking questions
- Ways of enquiry
- Predicting and hypothesising
- Making careful observations
- Using tools accurately and safely
- Making a record of evidence to present their findings
- Considering significant evidences
- Evaluating reliable evidences and findings accurate results
- Developing ideas from evidence

The same is the case with social sciences. The scientific method can also be applied to subjects in social sciences.

Steps in Scientific Method

The steps involved in scientific method are as follows:

(i) Collection of data as per the problem at hand, according to some adequate plan and their systematic observation.

(ii) Observations are made with a well defined purpose and they are recorded in definite terms.

(iii) Classification and organization of data on the basis of similarities, variations, activities, causes and results.

(iv) Generalization of data for the purpose of formulating principles and theories. The principles and theories must be specifically defined so that it can solve the problems in the related field.

(v) Verification of generalizations through controlled experiments by tested prediction of results and by repetition of experiments. Correlation coefficient of original and verification of results is also calculated and probable errors are estimated. It is also determined whether the error lies in procedure or apparatus.

(vi) Assumptions and limitations are noted down on the basis of verification of results.

(vii) Reporting the research in detail.

(viii) Announcement of the results before the general public for practical use.

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Steps in Scientific Process

The steps involved in a scientific process are as follows:

(i) ***Purposeful Observation:*** Observation should be accurate and extensive, and it must be done under various controlled conditions.

(ii) ***Analysis-Synthesis:*** This include the following:

- The essential elements in a problematic situation must be picked out by analysis.
- Similarities as well as dissimilarities must be isolated.
- Exceptions are to be given special attention.

(iii) ***Selective Recall:*** A wide range of experiences is essential.

(iv) ***Hypothesis:*** It is nothing but a tentative solution to the problem. There may be more than one solution depending on the nature of the problem.

(v) ***Verification by Inference and Experiment:*** Here only one variable is manipulated and judgment is made on the adequacy and accuracy of data.

Redman and Mory define research as a '*Systematized effort to gain new knowledge*'. According to Clifford Woody, *research includes defining and redefining problems, formulating hypothesis or suggested solutions; collecting, organizing and evaluating data; and making deductions'*.

Scientific Theory

Theories are systematic statements that explain a particular segment of phenomenon by specifying certain relationship among variables.

Kerlinger has defined a theory as: '*...a set of interrelated constructs (concepts), definitions and propositions that present a systematic view of phenomena by specifying relationship among variables with the purpose of explaining and predicting the phenomena*'.

A theory can be explained on the following concepts:

(i) Theory is a set of interrelated concepts, definitions and propositions.

(ii) The interrelated concepts and definitions in a theory help us to understand the phenomena in a systematic manner.

(iii) Theory establishes a relationship among various variables in a systematic manner. With the help of this relationship, we can predict the future nature of the phenomena.

(iv) A theory helps us to formulate a hypothesis on the basis of which future research can be based.

Social Science and Research

Research plays a very significant role in the field of social science. In order to study the importance and relationship between social science and research, social research is conducted or undertaken. The research that attempts to measure, describe, explain and predict the social and economic phenomena or social behaviour of human beings is known as 'social research'.

One of the main objectives of conducting social research is to find out information about the behaviour of an individual and solutions to the problems related to human relations. The outcome of social research provides the following benefits:

- It helps professionals in earning their livelihood.
- It helps students in knowing how to write a report for various findings.
- It helps philosophers to think on wider new perspectives.
- It helps in developing new styles for creative work.

In order to conduct social research and examine the social life of human beings, social scientists use different methods. Quantitative and qualitative are the two methods of research that are generally used by social scientists to conduct a research. In quantitative method, numerical data is collected and then analysed in order to measure the social phenomena. Qualitative method is basically the study of data, such as words, pictures and objects. However, the data collected with the help of this method is not very effective and cannot be generalized very easily.

Social research is very helpful for a country as it helps the government to explore the following things:

- Social and economic structures
- Social attitudes
- Social values and behaviours
- Factors motivating individuals and groups of a society

Researchers share a close relationship with government analysts, such as economists, statisticians and operational researchers. The relationship between researchers and government analysts is very much essential in order to find out high quality research data. Social research also informs about development, implementation and evaluation of a wide range of government policies.

Social research also helps to examine the consequences of government policies and economic changes in an organization, and the effects of globalization and its impact on small-scale and cottage industries.

1.3 RESEARCH PLANNING

An organization, in order to conduct a research in its work environment, has to first prepare a research plan. Research planning helps design a research plan, which consists of information related to the process of implementing a research. Organizations design research plan to gain knowledge about the market value of their products, services and programmes. The more effective and efficient the research plan, the shorter is the time to complete the research. An efficient research plan must focus on the main objective of the research.

Types of Research Planning

There are several types of planning that helps in implementing a research. The two major types of research planning are as follows:

(i) **Primary:** It involves collecting the data about a given subject through various research methods, such as surveys, interviews and observations, and analysing the data to use its findings and results for planning. Primary research can be used for business, personal and academic purposes.

(ii) **Secondary:** It involves evaluating the results of primary research planning. It provides a broader perspective and contains reference to the relevant documents related to primary research planning.

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Research Planning Considerations

The following considerations are to be kept in mind while planning a research:

- The purpose for which the research is being done.
- The audiences, such as bankers, employees and customers for whom the research is being done.
- The information that is needed to make the decision for planning a research.
- The sources, such as employees and customers from which information should be collected.
- The way in which information is collected. For example, questionnaires, interviews and observing staff helps in collecting the information.
- The accurate time in which information is to be collected and analysed.
- The requirement and availability of resources for collecting the information.

Designing a Research Plan

A research plan is designed to perform the research efficiently. To design a research plan, you need to perform the following steps:

(i) **Identifying the Need of the Research:** The first step in planning a research is to identify the need of the research. The objective of identifying the problem must be clearly stated; otherwise, the objective of the research cannot be achieved. Before implementing the research, a researcher must have an adequate knowledge of the area in which the research is to be done. By acquiring adequate knowledge in a specific area, a researcher can easily identify the problem efficiently.

(ii) **Selecting the Research Method:** A researcher uses all research methods to collect data and determine the most appropriate method. Selecting the right method enables the researcher to collect data in the right manner and plan the research without any difficulty. A researcher must be very confident about the methods used and the findings.

(iii) **Collecting Data:** Data collection is a process of systematic gathering of data for a particular purpose. The various sources that can be used for collecting data are interviews, questionnaires and existing records. An interview is a data collection technique that involves oral questioning one by one or as a group. A questionnaire is another data collection technique in which written questions are presented to the people which are to be answered by them. Existing records technique involves collecting data by using already available data collected by other researchers. This saves time and reduces the chances of errors.

(iv) **Analysing the Collected Data:** It is a process of applying some systematic techniques to evaluate the data. A good researcher starts thinking how he/she will analyse the data, long before the data is actually analysed. The analysed data is then used for implementing the research.

(v) **Documenting the Analysed Data:** The last and the most important step in planning a research is the documentation of the findings from the analysed data. The money and the time incurred in the research project are wasted if the findings are not documented or communicated effectively. It is very important to provide the collected information to other researchers so that they can check if there is any error in the findings.

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1.4 RESEARCH DESIGN

Research design is a structure that gives an outline of the overall research work. It is the result of better planning and implementation of a good strategy. Different authors have given different definitions of research design. According to a researcher, Kerlinger, research design is the plan, structure and strategy of investigation conceived so as to obtain answers to research questions and to control variance. Bernard Phillips defines research design as the blueprint for collection, measurement and analysis of data.

The decisions that you need to take to formulate a research design should be based on the following questions:

- What is the research all about?
- Why is the research being made?
- What kind of data is required for the research?
- From where can the data be obtained?
- How much time will the research take?
- What is a sample research design?
- What is the style of report preparation?

A research design helps a researcher to organize ideas and check for flaws and inadequacies in the collected data. Research design involves the following elements:

- A statement that clearly defines the problem for which research is being done.
- Procedures and techniques for gathering the information required for research design.
- Methods that need to be implemented for processing and analysing the data required for research design.

The overall research design can be divided into the following four parts:

- **Sampling Part:** It includes the method of selecting samples to be observed for the research study.
- **Observational Part:** It includes the conditions under which you need to make observations.
- **Statistical Part:** It is based on the questions of how many samples need to be observed and how the analysis of gathered data be performed.
- **Operational Part:** It involves the techniques that help to implement the strategies specified in the sampling, statistical and observational designs.

Need for Research Design

Before starting the research process, formulation of an efficient and appropriate research design is important. A research design is significant as it has the following advantages:

- It helps in the smooth functioning of various research operations.
- It requires less effort, time and money.
- It helps to decide the methods and techniques to be used for collecting and analysing data.

The researcher needs to consider the following factors before creating a research design:

- The source of information source.

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- Skills of the researcher and his/her coordinating staff.
- Problem objectives.
- Nature of the problem.
- Availability of time and money for the research work.

Features of a Good Research Design

A good research design can be characterized by flexibility, efficiency and low cost, but it has many other features. On the basis of the description of design, a research design has the following features:

- It states the sources and types of information required for solving the problem for which research is being carried out.
- It is a strategy for indicating the approach to be adopted for gathering and analysing data.
- It includes performing research work according to time and budget constraints.
- It minimizes preconception and maximizes the reliability of collected and analysed data.
- It minimizes experimental errors in an investigation.
- It provides various aspects for dealing with a problem.

Research design fully depends on the type of research study that you are conducting. If the research study is exploratory, then major emphasis is on the discovery of ideas. So, a research design should be flexible to implement the different aspects of a phenomenon. Whereas, when the purpose is to obtain accurate description of a research study, the design that maximizes reliability of the collected data is considered a good design. The availability of time, money, skills of the research staff and the method of obtaining information must be considered while creating experimental design, survey design and sample design.

Steps in Research Design

The steps in a research design primarily depend on the type of research being conducted. The general steps involved in a research process are as follows:

Step 1: Preparing the research question or problem

Step 2: Assessing the available literature

Step 3: Creating hypotheses

Step 4: Constructing the research design

Step 5: Collecting data

Step 6: Analysing the data

Step 7: Interpreting the results

Step 8: Writing the research report

Step 4, i.e., constructing the research design, involves three subordinate steps, which include the process of creating a research design. The three subordinate steps can further be explained as follows:

1. **Identifying Variables:** This involves identifying the variables to be studied and determining their types. The most common types of variables are dependent, independent, controlled and other variables. Dependent variables include points, such as responses of subjects and outcomes of survey or criterion variables. Independent variables, on the other hand, are those which are explanatory or predictor variables.
2. **Formulating Functional Definitions:** Here, the researcher explores the possibilities and the ways in which the variables can be operationalized.
3. **Selecting Design for Data Analysis:** This is the preliminary step of data collection, and hence involves determination of what design option to choose for analysing the data being collected.

Types of Research Design

The several research designs are classified on the basis of the study performed in the research. These research designs are listed below.

1. Research Design in Exploratory Research Studies

Exploratory research design is also known as formulative research design. In this research design, a specific subject is investigated. It helps to generate a set of hypotheses or research-based questions that can be used in a later stage. The three methods that are applied for explorative research studies are as follows:

(i) ***Surveying the Literature:*** It is the simplest method for formulating the research problem in which along with new literature, previous hypothesis are reviewed and evaluated for future research.

(ii) ***Experience Survey:*** It is a type of research that involves practically experienced persons in the research work. For such survey, people with more innovative ideas are carefully selected as respondents and then, the investigators interview the respondents. Thus, experience survey enables the researcher to concisely define the problem. This survey also provides information about the practical possibilities for different research works.

(iii) ***Analysis of Insight-Stimulating Examples:*** It includes an intensive study of selected instances of a phenomenon. In this method, attitude of the investigator and intensity of study and ability of the researcher are required to unify the diverse information of the problem.

Thus, in exploratory research study, the applied method needs to be flexible regardless of the type of the method, so that the different aspects of the problem can be considered. In exploratory research design, the following considerations are kept in mind:

- Small sample size must be used.
- Data requirements real scope are unclear.
- General objectives are considered rather than specific objectives.
- No definite suggestions are made after research analysis.

2. Research Design in Descriptive Studies

Descriptive research study describes the characteristics of a particular problem or individual or group. Descriptive studies include specific predictions concerned with study, facts and characteristics concerning individual, group or situations. Most of the social research is based on descriptive research studies. In descriptive studies, the questions related to 'what', 'why', 'where' and 'who' need to be answered.

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The following steps must be followed while designing the descriptive study:

(i) **Formulating the Objectives of a Study:** This step specifies the objectives to ensure that the collected data is related to the study, otherwise research will not provide the desired result.

(ii) **Designing the Data Collection Methods:** This step helps to select the methods, such as observation, questionnaires, interview and examination of records for collecting the data.

(iii) **Processing and Analysing the Data:** The data collected for research study must be processed and analysed. Processing and analysing of data includes analysing the data collected through interviews and observations, tabulating the data and statistical computations.

(iv) **Reporting the Searched Data:** For representing the report, its layout should be well planned, which helps in presenting it in a simple and an effective style.

In descriptive studies, the following considerations are kept in mind:

- The phenomenon under study must be described.
- The data may be related to the behavioural variables of the respondent.
- The recommendations are definite.
- The objectives are specific, data requirements are clear and large samples are used.

3. Research Design in Quantitative Studies

A quantitative research design is based on the assumption that things can be measured and calculated using objective units and measurements. Here, the researcher does a scrutiny of the data collected from the subjects. This involves questioning or surveying a large number of respondents. However, the researcher may not interact with the concerned subjects. Similarly, individual opinions and choices of the subjects are not considered in such a study and a random classification is carried out. A research design in quantitative study uses predefined concepts and variables. Such a design thus focuses on generalizations that further help in predictions, explanations and understanding.

4. Research Design in Qualitative Studies

A qualitative research design can be explained by considering the features that makes it different from the quantitative research design. A qualitative research design assumes that things can be interpreted and subjective judgments can be passed. Research design, therefore, involves creating research questions that help the researcher to infer conclusions based on the subjects' responses. He/She personally interacts with the concerned subjects. Opinions and choices of the respondents or participants are considered and scrutinized. Research design, thus, involves developing a theory unlike testing a predefined theory. It uses definitions and concepts that evolve or emerge while the study is being conducted. Thus, it is more concerned with questions like 'what' and 'why'. A qualitative research design focuses more on patterns and theories that aim at facilitating a comprehensive understanding of the concerned field of research.

5. Research Design in Experimental Research Studies

Experimental research design is usually applicable when we are determining the cause and effect relationship or deriving the cause and effect inferences in any experimental research study. This research design is instrumental in answering some of the important psychological questions that are based on the concept of 'what causes what'.

The objective of experimental research design is to establish the cause and effect relationships between variables. The four types of variables related to the experimental research design are as follows:

(i) **Independent Variables:** These signify conditions or measures in the experimental design that can be changed.

(ii) **Dependent Variables:** These are those which can be measured and signify the effect or result in the experimental design.

(iii) **Control Variables:** Control variables are those which remain constant in the experimental design.

(iv) **Random Variables:** Random variables are those which can vary their values in different conditions in the experimental design.

There are many variations in experimental designs, which are created to achieve different results and resolve different problems. We can define the simplest form of experimental design by creating two similar groups, which are equivalent to each other in all respects, except for the fact that one group will receive the treatment and another group will not receive the treatment. The group that receives the treatment can be termed as 'treatment group' and the group that does not receive the treatment is termed as 'comparison' or 'control group'.

The formation of two similar groups that are equivalent to each other is ensured by randomly assigning people or participants into two groups from a common pool of people or participants. The success of the experiment is based on the concept of random assignment of people into two groups.

However, as two people cannot be exactly similar, in the experimental design, we refer to the idea of probability and say that two groups are probabilistically equivalent or equivalent in the probabilistic ranges.

Basic Principles of Research Design

Research design is always based on some principle. RA Fisher has classified three basic principles of research design, which are discussed below.

1. Replication Principle

In this principle, an experiment is repeated more than once thereby applying each action in more than one experimental unit. For example, to examine two varieties of flowers, you need to divide the varieties of flowers in two parts. You sow the first variety in one part of the field and the second in the other part, and then you draw a conclusion. This application can be applied repeatedly in several parts of the field. The result obtained after applying this principle is more reliable than the result obtained without applying the replication principle. This principle increases the accuracy with which its main effects and interactions can be estimated. The main disadvantage of this replication principle is the computational problem.

2. Randomization Principle

The randomization principle protects the research design from extraneous factors while conducting an experiment. It indicates that you must design or plan the experiment in such a way that unrelated factors could be included as a possibility. For example, if you sow one variety of flowers in the first part of a field and the other variety of flowers in the second part of a field, then there is a possibility that soil fertility of both the parts of

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the field may be different. In such a situation, the result may not be realistic; therefore, you need to apply randomization principle by giving the results, which are based on some random sampling technique. This principle estimates experimental errors in the research work in a better way.

3. **Local Control Principle**

This is an important principle in which extraneous factors are measured and then eliminated from the experimental error. The experiment is planned in such a manner that two-way analysis of variance can be performed. For example, to apply the control principle, you need to divide the field into several blocks. Each block is first divided into parts equal to the number of treatments and then the treatments are randomly assigned to these parts. In each block, the extraneous factors are fixed to measure the variability, which caused the two-way analysis of variance. You can then eliminate the variability caused due to extraneous factors from the experimental error using the local control principle.

The researcher should be well acquainted with the various principles and aspects of research design. This enables him/her to explore a variety of options of presenting the outline of his/her proposed research. This can also enable you to submit one or more research designs, pertaining to your field of endeavour and thus keep room for flexibility in submitting your research proposal. Thus, the next step to perform after preparing the research design and getting it approved is of writing the research proposal. Once the research proposal gains approval, the researcher can start conducting the research process.

1.5 EDUCATIONAL RESEARCH

Educational research refers to a systematic attempt to gain a better understanding of educational process, generally for the purpose of improving its efficiency. It is actually an application of scientific method to study educational problems. The main concerns of educational researches are to understand, explain, predict and control human behaviour in individuals and social situations so that events or situations can be further improved.

1.5.1 Purpose of Educational Research

Researches in education are conducted for fulfilling the following purposes:

(i) To solve the immediate local problems in education.

(ii) To ascertain principles and develop procedures for use in the field of education.

(iii) To determine the extent we should go to for educating children and adults.

(iv) To answer the questions related to education through reflective thinking, to determine what should be done in the future on the basis of what is as present, and what was in the past.

(v) To discover new applications of principles and laws in the field of education.

1.5.2 Need and Importance of Research in Education

Research in education is needed because of the following reasons:

(i) Education has direct links with history, philosophy, sociology, psychology and economics. It is through an intensive process of scientific enquiry about the philosophical, sociological psychological, economic impact on various aspects of education that scientific theories can be established.

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(ii) Education is considered to be both science and arts. So, there is a need to add scientific knowledge to it for enrichment and improvement. This helps in adjusting educational programmes according to the situation. Research aids in giving suggestions on what to do and what not to do.

(iii) Introduction of democratic principles into the field of education has given rise to numerous problems like the problem of individual differences, expansion of relations, discipline, etc. The solution to these problems needs systematic research and experimentation so that defects of traditional methods may be avoided.

(iv) Meaning and application of many concepts of education have changed today. So the limits of educational research have to be extended from the formal and conventional modes of education to the non-formal and innovative system based on ecological and cybernetic models.

(v) The 21st century has seen many new changes taking place in the field of knowledge due to scientific and technological development. Education has to play a convincing role here too so that we can accept these changes with pleasure and adapt to them. Educational research will help us in this regard to construct curriculums, to prepare new textbooks and to adopt new methods of teaching.

1.5.3 Steps in Educational Research

The following steps are followed for all educational research:

(i) **Identification and Definition of the Problem:** The problem to be researched is selected either by surveying material available on the topic or through experience when the researcher comes across a problem in an actual situation. This problem is actually a question that can be answered using scientific enquiry and procedure. After studying various concepts, the problem is defined in definite and clear terms so that it becomes clears what data or evidence will be required to solve this issue.

(ii) **Formulating the Research Problem:** The two types of research problems are those that relate to states of nature, and those that study relationships between variables. At the start, the researcher must identify the problem he/she will focus on, i.e., he/she must decide the area of interest, or specific characteristic of a subject that he/she would be probing. At the outset, the problem can be generalized and then the ambiguities, if any, can be resolved. Thereafter, the workability of an identified solution needs to be taken into account before a working formulation of the problem can be finalized. As is apparent from this, the starting point of any scientific enquiry process is to formulate the general topic into a specific research problem.

There are two logical steps involved in the process of formulating the research problem, viz., understanding the issue comprehensively, and rephrasing it into relevant terms to prepare for the subsequent analysis. One of the effective methods that may be used to understand the problem is to discuss it with colleagues, associates or experts on the matter. In an academic environment, the researcher can approach a guide or colleague who has successfully conducted several research studies. Often, the guide will summarize the issue generally and then leave it to the researcher to fine tune and rephrase the problem into operational terms. On the other hand, in private businesses or public/government institutions, the problem may be put forward by administrative agencies with whom the researcher can discuss the origins of the problem and also take on board the issues that need to be considered while determining possible solutions.

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The researcher must simultaneously study relevant literature so that he/she gets acquainted with the problem. In this matter, he/she can review two broad types of literature—conceptual literature covering the relevant concepts and theories, and empirical literature where the subject matter comprises similar studies undertaken earlier. The purpose of this review is to obtain information regarding the data and other materials that are available and will enable the researcher to set out his/her research problem meaningfully. Subsequently, the researcher will rephrase the problem in as specific terms as possible. This activity—formulating or defining a research problem—is one of the most important steps in the entire research process. By defining the problem clearly, it will be possible to distinguish relevant data from irrelevant data. There, however, needs to be focus on determining the objectivity and validity of the background data.

Professor W.A. Neiswanger correctly states that the statement of the objective is of basic importance because it determines the data which are to be collected, the characteristics of the data which are relevant, relations which are to be explored, the choice of techniques to be used in these explorations and the form of the final report. If the researcher identifies certain pertinent terms, he/she should clearly define these while formulating the problem. We note that the final formulation of a problem is often done in several steps; wherein a number of interim formulations are arrived such that each one is more specific, analytical and realistic than the preceding one.

(iii) **Development of Working Hypotheses:** After concluding a comprehensive study of the available literature, the researcher should state the working hypothesis or hypotheses clearly. A working hypothesis is an assumption which is made in order to establish and test its logical or empirical consequences. The development of appropriate research hypotheses is particularly important as these hypotheses provide the focal point and a direction for the research activity. They also have a bearing on which tests are to be conducted to analyse the data and, indirectly, on the quality of data which is to be used for the analysis. The hypotheses must be specific to the piece of research which is to be tested. A hypothesis provides guidance to the researcher by ring-fencing, or clearly identifying, the area of research and thereby ensuring that he/she remains on the right track. It sharpens his/her focus onto the key aspects problem. The hypothesis also determines the type of data required and the methods which are to be followed for the data analysis. The following approach can be useful for developing working hypotheses:

- Discussing the problem with colleagues, associates and experts in order to understand its origin as well as the key objectives in seeking a solution.
- Examining the data and other available information, in order to identify trends, peculiarities and other nuances.
- Reviewing studies that have been conducted on similar problems.
- Personally conducting exploratory investigation, such as field surveys, or sample surveys with interested parties so as to obtain insight into the practical side of the problem.

Hence, good working hypotheses can be developed through forward thinking, discussions with colleagues or experts, and a study of the available information (including data as well as similar studies). Working hypotheses are effective when they are precisely and clearly defined. It should be noted that there are occasions where the nature of the problem is such that working hypotheses are not required. This could be in the case of exploratory or formative researches, i.e., which do not seek to test the hypothesis. As a general practice, however, specification of working hypotheses is a basic step in the research process.

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(iv) **Clarification of Research Procedure:** The procedure and method of research are clarified here. This refers to the general strategy to be followed for collecting and analysing data. The research method depends on the nature of the problem under study and the type of data required.

(v) **Collection of Data:** The method or technique to be used for collecting data is specified at this stage. First, the sample is chosen and then research tools, such as questionnaires, tests, interviews, etc., are used on the sample.

(vi) **Analysis and Interpretation of Data:** In this step, data are first arranged in systematic form, and then they are analysed and interpreted in the context of the hypothesis. Appropriate quantitative as well as qualitative techniques are used for processing the data. This step helps in testing the hypothesis.

(vii) **Drawing Conclusions and Development of Theory and Principles:** This is the final step where conclusions are drawn on the basis of results and laws or principles (generalizations) are developed for general use.

(viii) **Preparation of the Report or the Thesis:** Finally, the researcher prepares a report of his/her work. This report must be written very diligently, keeping certain guidelines in mind. The layout should have the following format:

(a) The preliminary pages or introduction
(b) The main body of text
(c) The conclusion

The preliminary pages of the report should comprise its title and the date, followed by acknowledgements and the foreword. This should be followed by the contents and then the lists of tables, graphs and charts that are present in the report. The main text of the report should carry the following:

(a) ***Introduction:*** The introduction should clearly mention the objective of the research and explain the methodology that has been employed in the report. It should also touch upon the scope of the study and its limitations, if any.

(b) ***Summary of Findings:*** Following the introduction, the report should state the findings and recommendations in simple language. If there are many findings, they should just be summarized here.

(c) ***Main Report:*** The main body of the report should be arranged logically and should comprise clearly distinguished sections.

(d) ***Conclusion:*** At the end of the main text of the report, the researcher should again summarize the key findings. This should be done clearly and precisely in the manner of final summing up.

The appendices must be provided at the end of the report and should enlist all technical data. Also, the bibliography listing all the books, journals, reports, etc., that have been consulted, should be provided at the end of the report. Finally, the report must provide an index, particularly in the case of a published research report.

1.5.4 Scope of Educational Research

The field of educational research can be classified into the following categories:

(i) **Educational Psychology:** Researchers in this field help the teacher to understand the child in the classroom in order to improve the teaching–learning process. This research provides the following information:

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- Relative effectiveness of socio-cultural forces on the development of children.
- Usefulness of learning theories in various educational settings.
- Relative effectiveness of various learning theories via field experiments.
- Identification of factors conducive to learning.
- Role of physical/intellectual inefficiencies and defects in learning.
- Understanding the personality of children in the class.
- Effects of parental and teacher's attitude toward children on learning.
- Understanding the problems of physically and socially handicapped children in the school system.
- Role of teachers and textbooks in removing delinquency in adults.

(ii) **Philosophy of Education:** Research in this field can provide us the following information:

- Role of logic in various areas of education from concept information to theory development.
- Role of knowledge, beliefs and values in developing educational theories.
- Role of ideologies and religion for improving educational practices.
- Development of a practical philosophy in the Indian context.
- Discovering new implications of ancient Indian philosophies in the present scenario.
- Determining the contributions of various Indian philosophers and their implications at present.
- Reorganization of the social structure and educational system in India.

(iii) **Sociology of Education:** Various dimensions of research in this field are given below:

- Effects of changes in the demographic structure on education.
- Effects of the New Education Policy (1986, 1992) on expansion of education and employment.
- Role of educational institutions in bringing about social change and vice versa.
- Role of social and cultural factors in bringing about social and educational equity.
- Role of teachers as agents of social change, modernization and social equity.
- Education in disadvantaged sections of the society.
- Minorities and their problems.
- Reservation policy and its impact on the social system.

(iv) **Educational Management and Administration:** Research in this field can help us understand the following aspects:

- Problems of educational administration in India and its impact on performance of students.
- Impact of educational planning and legislations on performance of students.
- Development of management theories and their implications on educational institutions.
- Role of teachers and principal in enhancing the performance of students.
- Impact of recruitment policies on output.
- Supervision and performance.

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- Contribution of NGOs to education.
- Effects of liberalization and privatization of higher education in India.

(v) **Comparative Education:** Research in this field helps us understand the following aspects of education:

- Administrative and educational policies of different countries and their impact on society as a whole.
- Impact of various systems of education in the world on each other.
- Comparison of educational progress in various countries of the world.
- Impact of economic progress on education.
- Allocation of budget on education in different countries and its impact on educational progress.

(vi) **Curriculum Development:** Nature and scope of research in this field can be understood by the following topics:

- Structure of the curriculum in India from the primary to higher level.
- Analysis of psychological demands of learners at different stages of education.
- Analysis and organization of curriculum in various subjects.
- Curriculum in relation to needs of the learner and the society.
- Analysis of textbooks at different stages of learning.
- Modernization of curriculum in relation to changing needs.
- Inculcation of national values through curriculum development.

(vii) **Guidance and Counselling:** Research in this field helps us to understand the following aspects of education:

- Role of the family and neighbourhood in teaching children to adjust to society.
- Construction of tools for diagnosing adjustment problems of students.
- Methodology of vocational guidance for children belonging to different strata of society.
- Identification of factors contributive to success in the life of students.
- Adaptation of foreign tests and inventories to Indian situations.

(viii) **Educational Technology:** Research in this field contributes in the:

- Development of new teaching strategies by action researches.
- Role of technology in teaching-learning process.
- Application of psychology in solving teaching problems.
- Application of technological equipment and laws in education.
- Development of new audio-visual aids.

(ix) **Problems of Indian Education:** This research covers:

- Pre-primary education
- Primary education
- Secondary education
- Higher education
- Vocational and technical education
- Non-formal education
- Distance education
- Recommendations of commissions and committees on education
- Continuous and comprehensive evaluation

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- Value education
- Women's education
- Inclusive education
- Teacher education

(x) **Inclusive Education:** Physical handicaps can be genetic or acquired. From diagnosis to their rehabilitation, we come across a number of problems that are to be investigated scientifically to arrive at a definite solution.

1.5.5 Principles of Educational Research

Research is a systematic process for developing a theory by applying scientific methods. It is an impartial, objective, empirical and logical analysis and recording of controlled observation that will finally lead to the development of a theory, principles, laws, etc., and will help us to predict about the phenomenon in future.

A research is said to begin with a question or a problem. The purpose of a research is to find out solutions through the application of systematic and scientific methods. Thus, research is a systematic approach to a purposeful investigation. Several authors and management gurus have defined research in different ways. Some of the proposed definitions of research are as follows:

- Redman and Mory have defined research as a systematized effort to gain new knowledge.
- In the words of the renowned researcher Clifford Woody, research comprises defining and redefining problems, formulating hypothesis or suggested solutions, collecting, organizing and evaluating data, making deductions and reaching conclusions, and carefully testing the conclusions to determine whether they fit the formulating hypothesis or not.
- D. Slesinger and M. Stephenson in the *Encyclopaedia of Social Sciences* define research as manipulation of things, concepts or symbols for the purpose of generalizing to extend, correct or verify knowledge, whether that knowledge aids in construction of theory or in practice of an art.

The main aim of research is to uncover answers to questions by applying scientific procedures. Research aims to discover hidden truths. While each research initiative has a particular purpose, the objectives of research can be broadly characterized as follows:

(i) **Exploratory/Formulative Research:** It attempts to get familiar with a concept or to develop new insights into it.

(ii) **Descriptive Research**: It seeks to accurately portray the key characteristics of an individual, a situation or a group.

(iii) **Diagnostic Research:** It establishes the frequency with which an event occurs, or the frequency with which it is associated with something else.

(iv) **Hypothesis-Testing Research:** This type of research tests the hypotheses of a causal relationship between variables.

1.5.6 Characteristics of Good Research

The process of research helps increase the creative ability of a decision-maker. The various characteristics of research are as follows:

- **Interdisciplinary Team Approach:** This approach is based on the principle of using expertise and experience of different personnel working in different disciplines within an organization. An individual cannot be an expert in all the areas of operation. So, researchers take help from other experts, who are specialists in their respective fields. Under interdisciplinary team approach, an expert may use old solutions, which were used in the past as research material for finding the most appropriate solution to a problem.
- **Methodological Process:** The researcher uses scientific methods and techniques to provide optimum solution to problems. The scientific methods include observing and defining a problem and formulating hypothesis related to the results of the scientific methods and techniques. If the hypothesis is accepted, its results should be executed in an organization; but if the hypothesis is not accepted, another hypothesis is formulated.
- **Objectivistic Approach:** The aim of an organization is to have optimal solutions to various problems. It is essential to measure the desirability of a solution for achieving the organizational objective. This measured desirability helps in comparing the alternative courses of action with respect to their outcomes.
- **Economical in Nature:** In an uncertain and complex situation, research helps in reducing the costs of inventory, thereby improving profits. For example, in inventory control, research can provide scientific rules for reducing acquisition costs and inventory-carrying costs.

The qualities of good research are as follows:

(i) **Good Research is Systematic:** This means that the research lays out clear steps in a specified sequence in compliance with well defined rules. Being systematic does not mean that the research cannot be based on creative thinking. On the other hand, it dramatically reduces guesswork-based and intuitive conclusions.

(ii) **Good Research is Logical:** This implies that the use of sound logic provides a foundation for reasoning, induction and deduction, which are of great significance for carrying out high quality research. Induction entails reasoning from a part to the whole, while deduction is the process of reasoning, wherein a premise is driven to a conclusion which is based on that very premise. In fact, logical reasoning leads to more meaningful research and better eventual decision-making.

(iii) **Good Research is Empirical:** This means that research is related to one or several aspects of a real situation and uses concrete data which provides a basis for external validity to the research results.

(iv) **Good Research is Replicable:** Good research allows for research results to be verified by replicating the study, thereby building a sound basis for decisions.

Further, Best and Kahn (1992) have summarized the main characteristics of research as follows:

- Research seeks to find the solution to a problem. In this objective, it could answer a question or even determine the relationship between several variables.
- Research creates generalizations, principles and theories that enable the prediction or anticipation of future occurrences. Research studies specific objects, groups or situations and then applies these characteristics observed to a larger population than the sample observed. Research goes far beyond simply retrieving or gathering information. There are many schools where the research teams gather and tabulate statistical information. This information can be used for decision-making, but it is not necessary to do so.

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- Research is based on observations or empirical evidence. There are many questions which are interesting or relevant but, since they cannot be observed, do not become research procedures. Research does not accept revelation or dogma as a basis for establishing knowledge. Research only accepts that which can be verified by observation.
- Research requires accuracy of observation and description. A researcher relies on quantitative or numerical measuring devices which are accepted as precise means of description. He/She identifies or creates appropriate data gathering instruments or procedures and employs effective mechanical, electronic, or psychometric techniques to improve human observation, recording, computation and analysis of data.
- Research entails obtaining new data from first-hand sources, or uses existing data towards a new purpose. Teachers often make their students undertake a project which requires them to write a paper detailing the life of a prominent person. The students go about this project by reading encyclopedias, books, or periodicals and summarize the information in writing. This is not research, the information is not new. Simply rewriting or representing what is already known may be a valuable learning experience, but it is not research. It does not provide any new information.
- Research may sometimes appear to be random or unsystematic, however it is actually always based on carefully designed procedures, and by applying rigorous analysis. Although researchers may sometimes employ trial and error methodologies, research is not a blind, random investigation—where the researcher is just experimenting to see what happens.
- A good researcher requires significant expertise. He/She is already aware of what is known about the problem including the investigations carried out by others. The researcher has familiarized himself/herself with the related literature and also fully grasps the terminology, concepts and technical skills necessary to thoroughly assess the data that he/she has gathered.
- The researcher must apply objectivity and logic and must also remove all his/her personal biases. He/She must employ all possible tests in order to comprehensively validate the procedure followed, the data sourced, and the results or conclusions that have been arrived at. The researcher should not make any effort to be additionally persuasive in order to prove an emotionally held conviction of his/her. His/Her focus must be on testing, and not on proving the hypothesis. Total objectivity is just as rare as absolute righteousness, and therefore the researcher must not allow bias or emotion to affect his/her analysis.
- Research involves the quest for answers to unsolved problems. Pushing back the frontiers of ignorance is its goal, and originality is frequently the quality of a good research project. However, previous important studies are deliberately repeated, using identical or similar procedures, with different subjects, different settings, and at different times. This process is replication, a fusion of the words repetition and duplication. Replication is always desirable to confirm or to raise questions about the conclusions of a previous study.
- Research must be carried out patiently and not in a rushed manner. Its outcomes and results are mundane rather than spectacular, and the research team must be prepared to face disappointment in the pursuit of answers to hitherto unanswered questions.
- The process and outcomes of research are meticulously recorded. Every key term is defined, restrictive factors are acknowledged, procedures are carefully described, all references are recorded, results are objectively documented, and the final outcomes are presented with caution and restraint. The final research reports and supporting data are made available for associates and other scholars to study, analyse, evaluate and even replicate.

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1.5.7 Research Approaches

Almost all types of research primarily follow the two basic approaches depending on the need of quality and the amount of data available. These two approaches are: (a) Quantitative approach and (b) Qualitative approach.

(a) Quantitative Approach

This approach involves creation of data in quantitative terms. It can be further classified into the following approaches:

- **Inferential Approach:** It is used to form a database to infer features or relationships of population. This usually means survey research in which a model of population is studied.
- **Experimental Approach:** This approach attempts to establish a cause–effect relationship among the groups of subjects that make up the research study. It is characterized by a greater control over the research environment where some variables are operated to scrutinize their effect on other variables.
- **Simulation Approach:** This entails the creation of an artificial environment within which relevant information and data can be produced. It is useful in building models for understanding future conditions.

(b) Qualitative Approach

This approach is concerned with the subjective assessment of human attitude, opinions and behaviour. It generates results either in non-quantitative form or in non-numerical data. This technique focuses on group interviews and in-depth interviews in its approach. It can be further classified into the following approaches:

- **Ethnographic Approach:** This is concerned with studying an entire culture. The researcher studies an integral cultural group in a natural setting over a specific period of time. By a cultural group, we mean any group of individuals, who share a common social identity, location or any other characteristics of interest. For example, an ethnographic study of hurricane victims in crisis, a group of children in kindergarten, or a cultural group in tribal India.
- **Phenomenological Approach:** This focuses on the subjective experiences of people over a long period of time. It aims at understanding the 'lived experience' of the individuals being studied.
- **Field Research:** This is a broad approach to qualitative research that facilitates collection of data. The basic idea involved in this research is that the researcher goes into the field to observe the phenomenon in its natural state. He/She takes extensive field notes, which are then coded and analysed in various ways.

Check Your Progress

1. What are the steps involved in a scientific process?
2. Give an advantage of social research.
3. What is the main aim of any research?
4. Give one characteristic of a good research.
5. What is the objective of experimental research design?
6. What is the main disadvantage of replication principle of research design?
7. Give an advantage of randomization principle.

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1.6 TYPES OF EDUCATIONAL RESEARCH

The types of research depend on the field in which the specific research study is performed. The different types of research are as follows:

1. Experimental Research

Experimental research involves conducting tests in a simulated or real space and time. The emphasis of most experimental research is on establishing cause and effect relationships, acquiring sensitivity of the dependent factor on an independent factor, keeping other independent factors under control and finding the conditions under which reactions take place.

The main features of experimental research are: (i) Isolation of factors into dependent, independent and catalystic, (ii) Replication of the experiment to ensure the reliability of the results and (iii) Measurement of the result, i.e., inputs, conditioning environment and output. The hallmark of experimental research is precision and accuracy. Everything is structured, controlled, monitored, measured and reported.

There are different types of experimental research. These are: (i) Natural or uncontrolled as in the case of natural phenomena where only observation of the phenomena is done, results analysed and conclusions drawn; (ii) The laboratory-situation research where a simulated environment is created with the input and conditioning variables manipulated to find the output behaviour and (iii) Field experiment where research is conducted in a social setting, with the researcher having a low manipulative power as far as the input variables and the conditioning factors are concerned.

The first type of research is used in the case of totally uncontrollable phenomena like gravitational force, weather, astronomical or celestial events. The second type is used in physical, biological and psychological sciences. The third one is based on management, business and social sciences.

There are several experimental designs like: completely randomized design, completely randomized block design, Latin square design, cross-sectional design, longitudinal design, etc.

Experimental research is considered as most scientifically valid. The purpose of experimental research is to establish 'cause and effect' relationships from observed findings. The effects of specific variables in a process can be understood by keeping other variables constant or using controlled experimental research. This is practised till the design and execution of the experimental hypothesis provides results. Research and marketing managers need to be confident about the conclusions drawn from the research.

Experiments call for selecting matched groups of subjects, subjecting them to different treatments by controlling extraneous variables and checking whether observed response differences are statistically significant. To the extent that extraneous factors are eliminated or controlled, the observed effects can be related to variations in the treatment. For instance, Indian airlines might introduce in-flight Internet service on one of its regular flights from Delhi to New York. It might charge 1,200 rupees one week and only 700 rupees the next week. If the plane carried approximately the same number of first class passengers each week and the particular weeks made no difference, any significant difference in the number of calls made could be related to the difference in price charged. Trying other prices and including other air routes could elaborate the experimental design.

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Experimentation is not easy to define. In most circumstances, experiments must create artificial situations so that they can obtain the particular data needed and can measure it accurately. Artificiality, in general, is the essence of an experimental research. Experimental research is commonly used in sciences, such as sociology, psychology, physics, chemistry, biology, medicine, etc.

2. Ex-Post Facto Research

Ex-post facto is a term used to define an action taken to change the effect given to a set of circumstances. This action relates to a past endeavour and bases the new effect on the same set of circumstances existing at that time. Ex-post facto research is similar to experimental research, which is conducted to deal with situations that occur in or around an organization. Studies that investigate possible cause and effect relationships by observing an existing condition or state of affairs and searching back in time for probable causal factors are collectively known as 'ex-post facto research'.

The characteristics of an ex-post facto research are as follows:

(i) Exploration of possible causes and effects.

(ii) No manipulation of independent variables as it has already been applied.

(iii) A control or comparison group.

(iv) Intact groups are used.

(v) Researcher takes the effect-dependent variable and examines it retrospectively.

(vi) Flexible by nature.

This research is used in one or more of the following cases:

- Where more powerful experimental designs are not possible to apply.
- When one is unable to select, control and manipulate the factors necessary to study the cause and effect relationships directly.
- When control variables except a single independent variable may be unrealistic and artificial.

Advantages of ex-post facto research are as follows:

- Shows a correlation where more rigorous experimentation is not possible.
- It is an exploratory tool.
- Useful to avoid artificiality in the research.
- Shows cause and effect relationships.

Disadvantages of ex-post facto research are as follows:

- Lack of control for independent variable and randomizing subjects.
- Never certain if causative factor has been included or identified.
- Relationship between two factors does not estimate cause and effect.
- May be regarded as too flexible.

3. Survey Research

Survey is a fact-finding study. It is a method of research involving the collection of data directly from a population or a sample at a particular point of time. The purpose of survey is to provide information, explain phenomena, make comparisons, etc. It is concerned with cause and effect relationships that can be useful for making predictions,

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knowing about customers knowledge, beliefs, preferences and satisfaction, and measuring all these magnitudes for general population.

A company, such as Air India might prepare its own survey to collect the information it needs or it might add questions to an anthology survey that carries the questions that are common to several companies. It can then provide services at a much lower cost. It can also put questions across to an ongoing consumer panel run by itself or another company. A mall intercept study may also be carried out by having the researcher approach people in a shopping mall and asking them questions. The survey methodology is popular among students for two reasons. Firstly, it seems familiar and easy to do. Most students have taken part in either an interview or questionnaire survey and many have conducted a survey in their secondary school days. Secondly, people are often interested and the survey is a useful tool for gathering a wide range of information.

A survey collects information from a sample of the population or sometimes, the organizations that are interested in participating in it. This may involve gathering information either at one point in time, that is, cross-sectional studies or following a group of people over a period of time, that is, longitudinal studies. Most of the non-academic surveys, such as surveys in market research, are of the first type. The type of information that can be gathered from people includes factual information, their level of knowledge, attitude, personalities, beliefs and preferences.

Steps in Conducting a Survey

(i) Clarify the purpose

(ii) Define the study population

(iii) Sample selection and estimating the sample size

(iv) Decide what information to collect

(v) Decide how to measure the information

(vi) Collect the data

(vii) Record, analyse and interpret the data

Clarifying the Purposes

It is important to be absolutely clear and explicit about the purposes right at the beginning. Surveys can be used for following two purposes:

(i) To know how common a characteristic is, that is, a descriptive survey.

(ii) To learn something about the causes for these characteristics, that is, analytic survey.

Defining the Study of Population

The next step is to define the exact subject of the study. It is vital to ensure that the subject of study relates to the purpose of the survey. This usually includes specific personal criteria, time and place.

4. **Historical Research**

Historical research turns history or the past to study the patterns there, their impact on the present, evolutionary process and so on. In a sense most of the researches are historical in nature because it uses data and information pertaining to the past. Hence research depends heavily on the past. How deep should research go into the past? This is a relevant question right now. Perhaps historical research concentrates on the deep distant past.

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Enquiry into the trade, commerce, business and economy of Ancient India (5000–3500 BC), the administrative system during the reign of Asoka, etc., are truly historical research. Historical research requires extraordinary skills on the part of the researcher to live the distant past, to visualize the environment, then to analyse and synthesize the undercurrents and overtones of the past. Everything must be brought before the mind's eye.

We can also say that historical research studies the bygone social effects that may have given rise to current situations. The study of the current state of Indian labour based on the past labour union movements to formulate the Indian labour policy is an example of historical research.

5. **Formulative/Exploratory Research**

It helps examine a problem with suitable hypothesis. This research on social science is mainly significant for clarifying concepts and innovations for further research. The researchers are mainly concerned with the principles of developing hypothesis and testing the hypothesis with statistical tools.

6. **Case Study**

This research undertakes intensive research that requires a thorough study of a particular unit, such as industrial or banking for data collection.

Besides these, there are several other types of research, such as evaluation research, assessment research and comparative research.

It is difficult to categorize a particular research under any major head. Irrespective of the nature and method of research, the research problem is essentially treated in an interdisciplinary manner. Interdisciplinary treatment means borrowing of an idea from related disciplines connected with the research topic for more authenticity. For example, management is not an individual discipline in its own right; rather, it requires an integral approach of various disciplines like finance and human resources.

1.6.1 Pure/Fundamental Research

This research is mainly concerned with identifying certain important principles in a specific field. It intends to find out information that has a broad base of application. The purpose of this kind of research is to develop theories, laws or principles by observing broad generalizations or phenomena. These theories or laws were previously unknown to anybody, e.g., discovery of the Law of Gravity by Newton, Law of Operant Conditioning by Skinner, and so on. Other examples of fundamental research are John Robinson's imperfect competition theory in economics and Maslow's hierarchy of needs theory in motivation, etc.

The researcher selects the problem from any source, which generally is not traditional. When the researcher finds the solution to the problem through his/her research, he/she comes to know that he/she has done something useful economically or socially.

1.6.2 Applied Research

This research aims at finding a solution to an immediate problem faced by a society or an industrial organization. It is supposed to discover a solution to some basic practical problems. It suggests corrective methods to minimize a social or business problem. Applied research is an application of pure research and its new interpretation in a different

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situation. Sometimes two pure laws are compared and third new law emerges. This is called 'applied research'.

Both the above researches use the scientific method of reflective thinking but their objectives are different. Problems involved in applied research have definite relevance to human aspirations; but pure research is not directly related to specific human needs, at least in the beginning.

Steps involved in applied research are as follows:

(a) A growing concern is studied and points of weaknesses in the system are isolated.

(b) Some of these weaknesses are selected for investigation.

(c) Investigations followed by solution either in the laboratory or in the field.

(d) Solution is modified and installed so that it may work in practice.

(e) Solution must be maintained by planning it in the organization so that it may become a permanent part of the system.

1.6.3 Action Research

It is 'research during action', such as observation of students in a classroom. Its findings are to be related in terms of local applicability. It means action research is generally used to solve local problems. As a result, its findings cannot be generalized beyond a particular setting. Its purpose is to improve school practices and educational outputs.

Action research, thus, is a research initiated to resolve an immediate problem or a reflective process of progressive problem. There are two types of action research: participatory action research and practical action research.

As per Reason & Bradbury (2002), "***action research is an interactive inquiry process that balances problem solving actions implemented in a collaborative context with data driven collaborative analysis or research to understand underlying causes enabling future predictions about personal and organizational change.***"

The action research development includes various scientific methods that regulate the actions taken on the research those results from the reflective perceptive of the actions. It exists between the following:

- Those who are more driven by the researcher's agenda and those more driven by participants.
- Those who are motivated primarily by influential goal attainment and those motivated primarily by the aim of personal, organizational or societal transformation.

Action research consequently has two aspects. The first aspect is to sort out a problem or issue in practice by an action researcher to obtain a solution. The second approach refers to the traditional research approach that includes hypothesis formulation and prediction, experimentation and analysis of collected evidences to prove the theory. What is most significant in both approaches is that the action researcher must be unbiased, honest and rigorous.

1.7 PROBLEMS ENCOUNTERED BY RESEARCHERS IN INDIA

Researchers in India face many problems—particularly those who are carrying out empirical research.

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Some of the more significant problems are as follows:

(i) The lack of scientific training in the methodology of research is a major stumbling block for researchers in India. There is a severe shortage of high quality researchers. Many researchers undertake research activities without any in-depth knowledge of research methods. Most of the work, which is carried out in the name of research, is not based on any sound method. For many researchers and also to their guides, research means a cut and paste activity without adding any thought or insight on the collated materials. The consequences of this are obvious—the results often do not reflect reality. Hence, a systematic study of research methodology is a key prerequisite. The researcher must have a strong understanding of the methodologies before taking on a research assignment. This can be achieved by providing intensive and short-duration courses.

(ii) Research departments at universities have very little interaction with business establishments, government departments and research institutions. A large body of non-confidential primary data remains unused by the researchers due to lack of proper contacts. Efforts need to be made to achieve interaction between all concerned parties for better and more realistic research. There is a need to develop a university-industry interaction platform so that the academics can get insight from practitioners on what aspects need to be researched, and practitioners find that the research done by the academics is relevant and can be applied.

(iii) Most of the businesses in India are of the opinion that the material supplied by them to researchers will be misused, and they are therefore reluctant to provide any information to researchers. The resultant need for secrecy seems to be embedded in business units, so much so that it results in a very strong barrier for the researchers. Hence, there is a requirement to create the confidence that any information provided for research will not be misused.

(iv) Research studies that overlap with each other are undertaken due to lack of information. The result of this is a duplication of effort and a waste of resources. This issue can be resolved through regular and updated compilation of a summary of issues and the places where research is being done. Additionally, specific attention must be given to identifying research topics across those disciplines of applied science that are of immediate concern to business and industry.

(v) There is no code of conduct for researchers. Also, inter-university and inter-departmental rivalries are common. Hence, there is a requirement to develop a code of conduct for researchers which can resolve this issue.

(vi) Quite often, researchers in India face the issue of a lack of adequate and timely secretarial assistance—including computer support. This results in delays in the completion of research studies. A serious effort must be made to ensure that researchers have access to efficient and timely secretarial assistance. The University Grants Commission (UGC) must play a key role in resolving this issue.

(vii) The library management process is usually unsatisfactory. As a result, researchers spend time unproductively to trace books, journals or reports, rather than accessing the relevant content from them.

(viii) A number of libraries are unable to access copies of old or new Acts/Rules, reports or government publications in time. This issue is more acute in libraries which are not situated in large cities. Therefore, efforts must be made to ensure that libraries receive regular supply of all relevant governmental publications.

Check Your Progress

8. What is meant by educational research?
9. What are the two logical steps involved in the process of formulating a research problem?
10. What is meant by 'working hypothesis'?

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(ix) There is a lack of availability of published data from various government and non-government agencies. Researchers also face the problem that the published data varies significantly due to differences in coverage by the concerned agencies.

(x) At times, there is the problem of conceptualization as well as problems relating to the process of data collection and other related issues.

ACTIVITY

1. 'Meaning and application of many concepts of education have changed today.' Give an example to support the statement above, whereby a present-day law in the field of education has been altered from its original form due to research recommending the change.
2. Give a detailed account of a situation where educational psychology has benefitted from research.

DID YOU KNOW

Educational researchers have come to the consensus that educational research must be conducted in a rigorous and systematic way. The findings of educational research also need to be interpreted within the context in which they were discovered as they may not be applicable in every time or place.

1.8 SUMMARY

- Research is defined as an activity involving technical and organized search for relevant information on a particular topic. It primarily emphasizes on finding solutions of problems in a methodical way.
- A research plan plays an important role in achieving an organization's objectives.
- The main aim of research is to uncover answers to questions by applying scientific procedures. Research aims to discover hidden truths. Other objectives of research are exploratory, descriptive, diagnostic and hypothesis testing.
- One of the main objectives of conducting social research is to find out information about the behaviour of an individual and solutions to the problems related to human relations.
- The researcher uses scientific methods and techniques to provide optimum solution to problems. The scientific methods include observing and defining a problem and formulating hypothesis related to the results of the scientific methods and techniques. If the hypothesis is accepted, its results should be executed in an organization; but if the hypothesis is not accepted, another hypothesis is formulated.
- Almost all types of research primarily follow the two basic approaches depending on the need of quality and the amount of data available. These two approaches are: (a) Quantitative approach and (b) Qualitative approach.
- Action research is 'research during action', such as observation of students in a classroom. Its findings are to be related in terms of local applicability. It means action research is generally used to solve local problems. As a result, its findings cannot be generalized beyond a particular setting. Its purpose is to improve school practices and educational outputs.

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- Research design involves determining the overall structure of a research study, identifying the variables pertaining to the research, formulating the operational definitions of these variables, and finally selecting the design for analysing the data or information that is collected by the researcher.
- A good research design needs to be flexible, reliable, and efficient and aims at minimizing errors and preconceptions.
- Educational research refers to a systematic attempt to gain a better understanding of educational process, generally for the purpose of improving its efficiency. Its purpose is to solve the immediate local problems in education, to ascertain principles and develop procedures for use in the field of education and to discover new applications of principles and laws in the field of education among others.
- The field of educational research can be classified into educational psychology philosophy of education, sociology of education, educational management and administration, comparative education, curriculum development, counselling, educational technology, problems of Indian education, and inclusive education.

1.9 KEY TERMS

- **Applied research:** This research aims at finding solution to an immediate problem faced by a society or an industrial organization
- **Control variables:** These are kept constant in the experimental design
- **Dependent variables:** These are measured and signify the effect or result in the experimental design
- **Ethnographic approach:** This is concerned with studying an entire culture and the researcher studies an integral cultural group in a natural setting over a specific period of time
- **Experimental research:** This research enables one to calculate the findings, employ statistical and mathematical devices and measure the results thus quantified
- **Fundamental research:** This type of research is mainly concerned with identifying certain important principles in a specific field
- **Independent variables:** These signify the conditions or measures in the experimental design that can be changed
- **Random variables:** These can vary their values in different conditions in the experimental design
- **Research design:** It is the plan, structure and strategy of investigation conceived so as to obtain answers to research questions and to control variance
- **Research planning:** It helps in designing a research plan, consisting of information related to the process of implementing a research
- **Research process:** It is a series of actions or steps that are essential to be performed in a specific order
- **Research proposal:** It is an application that proposes to pursue or conduct a research study and aims at presenting the idea around which the research study revolves

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1.10 ANSWERS TO 'CHECK YOUR PROGRESS'

1. The steps involved in a scientific process are as follows:
   - Purposeful observation
   - Analysis-Synthesis
   - Selective recall
   - Hypothesis
   - Verification by inference and experiment
2. Social research helps to examine the consequences of government policies and economic changes in an organization, and the effects of globalization and its impact on small-scale and cottage industries.
3. The main aim of any research is to uncover answers to questions by applying scientific procedures. Research aims to discover hidden truths which are thus far undiscovered.
4. Good research allows for research results to be verified by replicating the study, thereby building a sound basis for decisions.
5. The objective of experimental research design is to establish the cause and effect relationships between variables.
6. The main disadvantage of replication principle of research design is the computational problem.
7. The randomization principle protects the research design from extraneous factors while conducting an experiment. It indicates that you must design or plan the experiment in such a way that unrelated factors could be included as a possibility.
8. Educational research refers to a systematic attempt to gain a better understanding of educational process, generally for the purpose of improving its efficiency. It is actually an application of scientific method to study the educational problems.
9. There are two logical steps involved in the process of formulating the research problem, viz., understanding the issue comprehensively, and rephrasing it into relevant terms to prepare for the subsequent analysis.
10. A 'working hypothesis' is an assumption which is made in order to establish and test its logical or empirical consequences. The development of appropriate research hypotheses is particularly important as these hypotheses provide the focal point and a direction for the research activity. They also have a bearing on which tests are to be conducted to analyse the data and, indirectly, on the quality of data which is to be used for the analysis.

1.11 QUESTIONS AND EXERCISES

Short-Answer Questions

1. What are the different categories of the objectives of research?
2. What are the different types of research?
3. Define the steps involved in applied research.
4. Why are researches in education conducted?
5. What are the features of a good research?
6. What is randomization principle?
7. List the steps involved in a research process.
8. What is the significance of educational research?

Long-Answer Questions

1. Elaborate on the steps of the scientific method.
2. Discuss the characteristics of research.
3. Explain the steps involved in educational research.
4. Elaborate on the scope of educational research under the categories of educational psychology and philosophy of education.
5. Explain the significance of a research design.
6. Write a short note on the process of exploratory research design.
7. Describe the important concepts related to research design.
8. Elaborate the distinguishing features of research design with regard to quantitative and qualitative research studies.
9. Discuss the need and significance of educational research in India. Also elaborate on the problems encountered by the researchers in India.

1.12 FURTHER READING

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UNIT 2 RESEARCH PROCESS IN EDUCATION

Structure

2.0 INTRODUCTION

Research takes advantage of the knowledge which has accumulated in the past as a result of constant human endeavour. It can never be undertaken in isolation of the work that has already been done on the problems which are directly or indirectly related to a study proposed by a researcher. A careful review of the research journal, books, dissertations, theses and other sources of informations on the problem to be investigated is one of the important steps in the planning of any research study. A review of the related literature must precede any well planned research study.

Hypothesis is an assumption or proposition whose testability is to be tested on the basis of the compatibility of its implications with empirical evidence with previous knowledge (Mouly, 1963). It is also a declarative statement in which the investigator makes a prediction or a conjecture about the outcome of the relationship. The conjecture or the prediction is not simply an 'educated guess'; rather it is typically based on past researches, which investigators gather as evidence to advance the hypothesized relationship between variables.

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The first section of this unit describes the specific purposes which are served by the review of related literature. The unit provides a study guide to the researcher in identifying related literature, and in locating, selecting and utilizing the primary and secondary sources of information available in the library. The unit deals with procedure which the researcher should adopt for organizing the related literature in a systematic manner.

In this unit, you will also learn about the concept of hypothesis testing. For this, a hypothesis needs to be appropriate. Testing a hypothesis means verification of the hypothesis. This unit will describe the application of hypothesis testing in a variety of cases, such as comparing two related terms and testing equality of variance of two normal populations. A number of hypothesis tests, such as *t*-test and *Z*-test, facilitate the process of hypothesis testing.

2.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Explain the significance of research process in education
- Formulate a research problem
- Know about the variables used in a research process
- Discuss the specific purposes served by the review of related literature
- Describe the procedure the researcher should adopt in organizing the related literature in a systematic manner
- Explain the concept of hypothesis
- Describe the procedure of hypothesis testing
- Describe the various types of hypothesis testing
- Explain the statistical techniques involved in hypothesis testing

2.2 RESEARCH PROCESS IN EDUCATION: GENERAL STEPS, OBJECTIVES AND FORMULATION OF PROBLEM

Educational research or research in education refers to a range of methods which helps individuals to evaluate different aspects of education. Educational researchers have accredited that the educational research must be performed in a rigorous and systematic way.

As per Creswell (2002), "*Educational research is a cyclical process of steps that typically begins with identifying a research problem or issue of study. It then involves reviewing the literature, specifying a purpose for the study, collecting and analysing data, and forming an interpretation of information. This process culminates in a report, disseminated to audiences such that it is evaluated and used in the educational community*"

2.2.1 General Steps

Accomplishing or conducting any educational research is based on exploration, description, explanation or prediction of educational phenomenon using systematic data collection and analysis procedures. All the educational research methodology follows a planning, data collection and analysis, and report writing process and consists of the following nine significant steps:

Planning: The planning process includes the following five steps, Steps 1 to 5.

Step 1: Identify the Problem or Topic: The review literature facilitates to achieve an understanding of the present state of knowledge pertaining to the research design. It helps in recognizing if the research problem or topic has already been investigated and if a revision or replication is required, how to design the research process and what data collection methods to be used.

Step 2: Review Prior Research: Researchers use the literature review to identify the underlying principle required for the research process. Specifically, literature review can be utilized at the beginning of the research process to elucidate about the topic of research and to provide a rationale for the research process that one has planned. In addition, the literature review can also be used to facilitate the research design by providing assistance regarding appropriate sample size or identifying proper data collection methods or mechanisms that can be used in the research process.

Step 3: Determine the Purpose, Research Questions or Hypotheses: Determining the purpose of research helps in recognizing and determining how the research should be accomplished or conducted, what research design should be used, and what the research question(s) or hypothesis(es) should be formulated for the research study. Four significant and universally used purposes to conduct any educational research are to explore, describe, predict or explain.

Step 4: Consider Research Inferences: Inferences are the realistic methods that may affect the conducted research for education field. These include the underlying goals and principles formulated for research process and the significance of research study. Inferences can be further linked to the research problem or topic, research purpose and research question(s) or hypothesis(es).

Step 5: Construct a Research Proposal: Typically, the research proposal refers to a detailed description as how the research study will be conducted. It includes the title of research and the researchers employed in the research study, statement of the research problem and research purpose, review of relevant literature, research question(s) or hypothesis(es), what information or variables are to be collected, the participants of the research study and possible benefits or risks of research under study, the design and procedure for collecting data, what data collection method(s) will be used, and how the collected data will be analysed.

Data Collection: The data collection process includes the following step, Step 6.

Step 6: Collect Data: Data collection process focuses on information accomplishment which in turn will answer the research questions and will also support the research hypotheses. Data collection process illustrates what variables will be investigated during the research process, the unit of analysis or participants of the research study (such as, population and sample), participant or subject safeguards, procedures used to select participants, the methods and procedures used for data collection, and any reliability or validity of collection methods.

Report Writing Process: The report writing process includes the following three steps, Steps 7 to 9.

Step 7: Analyse Data: Data or statistical analysis depends on the type of data collected, i.e., whether the collected is quantitative data, qualitative data, or both. For quantitative

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data analysis, there are a range of statistical analysis tools that can be used to categorize statistical relationships between variables. For qualitative data analysis, the data analysis methods usually involve holistically recognizing patterns, categories and themes.

Step 8: Determine Research Findings: The research findings can be determined by analysing the data that is collected for the research. From quantitative data, the statistical information and general findings can be determined. From qualitative data, the primary details and specific findings can be determined.

Step 9: Report Conclusions, Implications and Limitations: Conclusions are statements that can be used to interpret and evaluate the results established from the research study. Ensure that the result interpretation must relate to the hypotheses or research questions of the study.

2.2.2 Objectives of Research Process

Research objectives can be defined as the systematic method of collecting data from selected sample or on the phenomenon under study and analysing the information in order to achieve the end result and to check the validity of the hypothesis that was formulated before the research was started. The research objectives are defined on the basis of either general or specific goals and by formulating the research hypothesis.

The following are some significant characteristics that provide quality in research process:

- Firmness of research process
- Trustworthiness of research process
- Reliability/Validity of research process
- Usefulness of research process, i.e., implications of research methodology for policy making and practice in educational research
- Originality of research process

Probable Aims and Objectives of Research Process

The following are the probable aims and objectives of a research process:

- Identify research aims/questions of the research problem, i.e., questions to answer - what, when, where and who
- Categorize nature/type of the research problem, i.e., scholarly review, empirical work, new or secondary analysis
- Recognize ontological position, epistemological and methodological assumptions of the research problem
- Explain the research problem, i.e., why did it happen
- Declare prediction of the research problem, i.e., what is to be expected
- Understand the research problem, i.e., how is it clutched with human experience
- Interpret the research problem, i.e., analyse the aim of research
- Identify location of the research area, date of research, sample selection and methods used in the research
- Support the conclusions appropriately by evidence
- Recommendation, i.e., proposal to the research problem to explain how ought it be

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- Change and emancipation, i.e., how can it be transformed for the enhancement
- Critique and disruption, i.e., defining that what are the limitations and hidden assumptions and how can these assumptions be challenged/interrupted

2.2.3 Formulation of Research Problem

A 'Research Problem' is considered as the initial step and the most significant prerequisite in the research process. It provides the groundwork or foundation of a research study because if the research problem is well prepared then the anticipated fine research study will pursue. To solve a research problem, it is required to identify what the problem is. Basically, you have to identify the type of problem and then decide what must be done to achieve desired result.

Typically, identifying a research problem and the methodology adopted to formulate it will determine approximately every step of research that is required in the research study.

Formulation of the research problem is considered as the input into the research study while the output is referred as the quality of the contents of the research report.

The following are some significant steps that formulate a 'Research Problem' in education:

1. Identify a wide area of significance for the academic/professional field of research.
2. Analyse the wide area into sub-areas logically or by having discussion with other research colleagues, if any.
3. Select the sub-area where the research study is to be conducted following the process of elimination of other sub-areas.
4. Reverse the research questions that are to be answered through the research study. This also helps in the formulation of the hypothesis based on the research objectives.
5. Evaluate all the formulated objectives to establish the feasibility of accomplishing them precisely considering the time factor and other problem factors, such as economic factor and human resource expertise.

2.2.4 Variables in Research Problem

An important step in designing all quantitative research projects is defining or identifying the variables that will be manipulated, measured, described, or controlled. A variable is a label or name that represents a concept or characteristic that varies (e.g., gender, weight, achievement, attitudes toward inclusion, etc.).

Conceptual and Operational Definitions of Variables

Conceptual (i.e., constitutive) definition uses words or concepts to define a variable, i.e., Achievement: What one has learned from formal instruction and Aptitude: One's capability for performing a particular task or skill.

Operational definition is an indication of the meaning of a variable through the specification of the manner by which it is measured, categorized, or controlled, for example, Weschler IQ score, Income levels below and above $45,000 per year and Use of holistic or phonetic language instruction.

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Types of Variables

Three variable labels defined by the context within which the variable is discussed are

- Independent and dependent variables
- Extraneous and confounding variables
- Continuous and categorical variables

Independent and Dependent (i.e., Cause and Effect)

Independent variables act as the 'cause' in that they precede influence, and predict the dependent variable whereas Dependent variables act as the 'effect' in that they change as a result of being influenced by an independent variable. For example:

- The effect of two instructional approaches (independent variable) on student achievement (dependent variable).
- The use of SAT scores (independent variable) to predict freshman grade point averages (dependent variable).

Some situations do not lend themselves to the use of the terms *independent* or *dependent* because it is difficult to discuss them in causal terms. For instance, the relationship between attitude and achievement, that is, do positive attitudes cause high achievement or does high achievement cause positive attitudes?

The relationship between creativity and critical thinking, that is, do high levels of creativity cause higher levels of critical thinking or do higher levels of critical thinking cause greater creativity?

Extraneous and Confounding Variables

Extraneous variables are those that affect the dependent variable but are not controlled adequately by the researcher.

- Not controlling for the socio-economic status of students involved in a study of the effects of instructional technologies.
- Not controlling for the key-boarding skills of students in a study of computer-assisted instruction.

Confounding variables are those that vary systematically with the independent variable and exert influence on the dependent variable.

- Not using counselors with similar levels of experience in a study comparing the effectiveness of two counseling approaches.
- Not using the same test to measure the effectiveness of two instructional approaches.

Continuous and Categorical

Continuous variables are measured on a scale that theoretically can take on an infinite number of values. For example:

- Test scores range from a low of 0 to a high of 100.
- Attitude scales that range from very negative at 0 to very positive at 5.
- Students' ages.

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Categorical variables are measured and assigned to groups on the basis of specific characteristics. For example:

- Gender: Gender has two levels - male and female.
- Grade level: K-12.
- Socio-economic status: Socio-economic status has three levels - low, middle, and high.

Here, the term *level* is used to discuss the groups or categories.

Continuous variables can be converted to categorical variables, but categorical variables cannot be converted to continuous variables. For instance, IQ is a continuous variable, but the researcher can choose to group students into three levels based on IQ scores - low is below a score of 84, middle is between 85 and 115 and high is above 116. Test scores are continuous, but teachers typically assign letter grades on a ten point scale (i.e., at or below 59 is an F, 60 to 69 is a D, 70 to 79 is a C, 80-89 is a B, and 90 to 100 is an A.

2.3 REVIEW OF RELATED LITERATURE

Review of the related literature; besides, allowing the researcher to acquaint himself/herself with current knowledge in the field or area in which he/she is going to conduct his/her research, serves the following specific purposes:

1. The review of related literature enables the researcher to define the limits of his/her field. It helps the researcher to delimit and define his/her problem. To use an analogy given by D Ary *et al.*, (1972, p. 56) a researcher might say:

   The work of A, B and C has discovered this much about my question; the investigations of D have added this much to our knowledge. I propose to go beyond D's work in the following manner.

   The knowledge of related literature, brings the researcher up-to-date on the work which others have done, and thus to state the objective clearly and concisely.
2. By reviewing the related literature, the researcher can avoid unfruitful and useless problem areas. He/She can select those areas in which positive findings are very likely to result and his/her endeavours would be likely to add to the knowledge in a meaningful way.
3. Through the review of related literature, the researcher can avoid unintentional duplication of well established findings. It is no use to replicate a study when the stability and validity of its results have been clearly established.
4. The review of related literature gives the researcher an understanding of the research methodology which refers to the way the study is to be conducted. It helps the researcher to know about the tools and instruments which proved to be useful and promising in the previous studies. The advantage of the related literature is also to provide insight into the statistical methods through which validity of results is to be established.
5. The final and important specific reason for reviewing the related literature is to know about the recommendations of previous researchers listed in their studies for further research.

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Identifying the Related Literature

The first step in reviewing the related literature is identifying the material that is to be read and evaluated. The identification can be made through the use of *primary* and *secondary* sources available in the library.

In the primary sources of information, the author reports his/her own work directly in the form of research articles, books, monographs, dissertations or theses. Such sources provide more information about a study than can be found elsewhere. Primary sources give the researcher a basis on which to make his/her own judgment of the study. Though consulting such sources is a time consuming process for a researcher, yet they provide a good source of information on the research methods used.

In secondary sources of information, the author compiles and summarizes the findings of the work done by others and gives interpretation of these findings. In them, the author usually attempts to cover all of the important studies in an area reported in encyclopedia of education, education indexes, abstracts, bibliographies, bibliographical references and quotation sources. Working with secondary sources is not time-consuming because of the amount of reading required. The disadvantage of the secondary sources, however, is that the reader is depending upon someone else's judgments on the importance of the study.

The decision concerning the use of primary or secondary sources depends largely on the nature of the research study proposed by the researcher. If it is a study in an area in which much research has been reported, a review of the primary sources would be a logical first step. On the other hand, if the study is in an area in which little or no research has been conducted, a check of the secondary sources is more logical. Sources of information, whether primary or secondary, are found in a library. The researcher must, therefore, develop the expertise to use resources without much loss of time and energy. To aid the researcher in locating, selecting and utilizing the resources, a study guide is provided in relation to their use in educational research.

A researcher should be familiar with the library, its facilities and services. He/She should also be acquainted with the regulations governing the use and circulation of materials. Many libraries use a printed guide that contains helpful information. The guide uses a diagram to indicate the location of the stacks, the periodicals section, reference section, reading rooms, and special collections of books, microfilm or microcard equipment, manuscripts, or pamphlets. The guide lists the periodicals to which the library subscribes and the names of special indexes, abstracts and other reference materials.

The regulations concerning the use of stacks, the use of reserve books, the procedures for securing reference materials held by the library or those that may be borrowed from another library are also included in the guide.

Research scholars and other readers are usually issued a library card giving them access to the stacks. They may take the help of library staff or may carry on the independent searching for the books and other reference materials. After using the books, it is desirable for the readers to leave them on the tables so that the library staff will return them to their proper position on the shelves.

Sometimes a reference is not available in the library. In such a situation, the reader must consult the 'union' catalog, which lists references found in other libraries. Such references may be obtained in the following ways:

(i) **By Inter-Library Loan System:** The reader requests the librarian to borrow the desired reference from other library where it is available.

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(ii) **By Requesting a Pholostatic Copy:** The reader may request the librarian to obtain the photostat of a page or a number of pages of a desired reference from the source.

(iii) **By Requesting an Abstract or Translation of the Portion of a Desired Reference:** Some large libraries have abstracting and translating service that provides abstracts, or copied or translated portions of needed materials at an established fee.

(iv) **By Requesting Microfilm or Microfiche:** The reader may purchase a microfilm that can be projected on library microfilm equipment. A microfiche is a sheet of film that contains microimages of a printed manuscript or book. Its development has been one of the most significant contributions to library and information services by providing economy and convenience of storing and distribution of long runs of scholarly materials.

An even more significant development is the *ultra-fiche.* It has the capacity of 3,200 pages per fiche (Mittal, 1979, p. 10).

Various types of cameras are used to record microimage on roll film. Some of which are described below.

Planetary Camera is either 35 mm or 16 mm still camera which is mounted on a vertical column that can be moved up and down as per the requirement. At a time, it can be loaded with 100 feet of roll film. It does not cost much.

Step-And-Repeat Camera is a costly camera which is specifically used to automatically record microimages on microfiche one-by-one.

Rotary Camera, like the planetary camera, records microimages on roll film and has the capacity to change the reduction ratio as desired.

Flow Camera costs less than half of a planetary camera. Its reduction ratio is fixed unlike previously mentioned types of camera.

All these cameras make use of *silver*, *diazo* or *vesicular* film for recording microimages.

Generally, six types of *Readers* are used for reading microfilms or microfiche (Mittal, 1979, p. 13).

(i) *Cuddly Microfiche Reader* is a portable reader and can be used by keeping it in one's lap. It is very cheap and can be lent to library members for home use.

(ii) *Microfilm* and *Microfiche Readers* is a reader/printer machine that can make copies from both microfilm and microfiche.

(iii) *Universal Machines* essentially archieve by reading the description, storing it and printing it. The example is Universal Tuning Machine (UTM) or computer.

(iv) *Reader/Printer* is a push-button machine which not only helps in reading a microfilm/microfiche but is capable of producing a full-sized paper copy of the frame on the screen.

(v) *Production Printer/Enlarge Printer* is an automatic machine and can print the requisite number of copies of a microfilm or selected portions of a microfilm. It is used for mass production of full-sized copies of microfilms.

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(vi) *Xerox Copyflow Machine* is a costly machine, and therefore, is beyond the reach of ordinary libraries. In can print out a microfilm into a readable size, and as such, a single copy of any requisite document can be had at low cost and in less time.

The card catalog is the index to the entire library collection. It lists the details of publications found in the library with the exception of serially published periodicals.

Generally, the card catalog contains author, title, and subject cards arranged alphabetically. A great deal of information about a book can be found on the cards. Besides the title of the book and the name of the author, the reader will find the date of birth of the author, the edition, the publication date, the number of pages, and the name and location of the publisher. Other items listed on the cards are bibliographies, maps, portraits, illustrations, tables, series (if any) in which a book appears, a brief description of the book—whether the book is a translation and who did the translation.

Library classification systems provide ingenious ways of systematizing the placement and location of books. Every system is based upon a methodology that is logical and orderly to the smallest detail. The two principal systems of library classification in the US are the 'Dewey Decimal' system and the 'Library of Congress' system.

The 'Dewey Decimal' system is a decimal plan with the numbers running from 001 to 999.99. The 'Library of Congress' system is particularly used in large libraries. It provides for 20 main classes instead of the 10 of the 'Dewey Decimal' system. The system uses letters of alphabet for the principal headings and numerals for further sub-grouping.

In a library, all books have a call number or letter that appears in the upper left-hand corner of the author, subject or title card and on the back of the book. These call numbers or letters are used to arrange the books serially on the library shelves and within each classification, the books are arranged alphabetically by author's last name.

Identifying the best available sources pertaining to a problem and extracting the essential information from them is of much importance to a researcher. For this, he/she must develop some library searching techniques so as to save his/her time and effort. Van Dalen (1973, p. 88) has suggested the following valuable guidelines for a researcher:

1. Before using a library, familiarize yourself with its layout, facilities, services and regulations.
2. Learn how to use the microform (microfilm and microfitche) readers, photocopies and other mechanical aids.
3. Look in the stacks and in the periodical, reference, reserved book and rare book rooms, the materials that you will use frequently are placed.
4. Schedule your work session in a library when you will encounter the least competition for resources and services.
5. Make out call slips for all or most of the books needed in one session.
6. Copy all information that the librarian needs to obtain each reference for you, and before closing the periodical index or card catalog, recheck and rectify any errors or omissions.
7. Arrange to spend a block of time in the library that is sufficient to accomplish a specific task.
8. When little time is available, clear up questions that can be answered quickly through the help of reference books that are readily available.

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9. Before initiating search for materials in a library, write down questions that cover precisely the information you wish to locate and group the questions in accordance with the areas in the library where the answers may be found.
10. Compile a list of the present and any previous names of periodicals, organizations, government agencies, research agencies, collectors of statistics, libraries and museums with special collections, and outstanding authorities in your field.
11. Keep a list of the best reference books, indexes, handbooks, historical studies, and legal references in your area of specialization.
12. Obtain copies of the best bibliographies and reprints of significant research studies for your files.
13. Note which periodicals regularly or occasionally print bibliographies, reviews of literature or such other reference material and the issues in which they appear.

There are a number of references that may be useful to a researcher in the field of education. To facilitate the search for such material, a researcher may consult the following carefully compiled volumes:

Constance M. Winchell, ed., *A Guide to Reference Books,* 8th edn. (Chicago: American Library Association, 1967). This comprehensive work has biennial supplements to bring the up-to-date information in a number of languages. It describes and evaluates about 7,500 references and a section is devoted to education.

Albert J. Walford, *Guide to Reference Material.* This is a two-volume work which covers (1) Science and Technology (1966) and (2) Philosophy and Psychology, Religion, Social Sciences, Geography, and History (1968).

Mary N. Barton and Marion V. Bell (1962), Reference Books: *A Brief Guide for Students and Other Users of the Library.* This guide is helpful but considerably shorter.

International Guide to Educational Documentation (1955-1960), (UNESCO, 1963). This is a one-volume international guide to educational books, pamphlets, periodicals, occasional papers, films and sound recordings.

Arvid Burke and Mary Burke, *Documentation in Education.* This guide provides an excellent introduction to literature in the field of education.

The Standard Periodicals Directory, (New York: Oxbridge Publishing Co., 1964-date). This is a directory of over 30,000 entries and covers every type of periodical, with the exception of local newspapers. It is published every year and covers about 200 classifications which are arranged by subject. An alphabetical index is provided.

Christine L. Wyner, *Guide to Reference Books for School Media Centres,* (Littleton, Colo: Libraries Unlimited, 1973). This guide includes 2575 entries with evaluative comments on reference books and selection tools for use in educational institutions. It is indexed by author, subject and title.

Encylopedias. These serve as a store house of information and usually contain well-rounded discussion and selected bibliographies that are prepared by specialists. Encyclopedias are arranged alphabetically by subject and for each field of research, they present a critical evaluation and summary of the work that has been done. In addition, these suggest the research needed in the field and also provide *a selective bibliography.*

The following list provides a sample of encyclopedias that researchers in the field of education might use:

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A Cyclopedia of Education, Paul Monroe, ed., 5 vol., (New York: Macmillan, 1911-13). It is edited by Paul Monroe with the assistance of departmental editors and more than 1,000 individual contributors. It provides excellent bibliographies and is extremely useful for historical and biographical purposes.

The Encyclopedia of Education, ed., Lee C. Deighton, (New York : The Macmillan Company and The Free Press, 1971). The encyclopedia includes more than 1,000 articles. It offers a view of the institutions and people, of the processes and products, found in educational practice. The articles deal with history, theory, research, philosophy, as well as with the structure and fabric of education.

Encyclopedia of Modern Education, Henry D. Rivlin and H. Schueller, ed., (New York: Philosphical Library, 1943). This comprehensive work of about 200 authorities has been edited by Henry D. Rivlin and H. Schueller. It stresses present day problems, trends, theories, and practices. The articles are accompanied by brief bibliographies and there is a system of cross references.

Encyclopedia of Educational Research, Walter Scott Monroe, ed., rev. edn., (New York: Macmillan, 1950). Monroe's Encyclopedia of Educational Research was prepared under the auspices of the American Educational Research Association. It aims to present a critical evaluation, synthesis and interpretation of research studies in the field of education. All the articles, arranged alphabetically, are provided with bibliographies.

Encyclopedia of Educational Research, Chester Harris, ed., 3rd edn., (New York: Macmillan, 1960). Harri's Encyclopedia of Educational Research is also prepared under the auspices of the American Educational Research Association. It is not merely a revision of earlier editions, but it is completely a rewritten volume that has attempted to put into a new perspective.

Encyclopedia of Educational Research, Robert L. Ebel, ed., 4th edn., (New York: Macmillan, 1969). Ebel's Encyclopedia of Educational Research provides concise summaries of research and many references for further research. The articles deal with persistent educational problems and continual educational concerns.

Encyclopedia of Educational Research, Harold E. Mitzel, ed., 5th edn., (New York: The Free Press: A Division of Macmillan Publishing Co., Inc., 1982). The contents of encyclopedia have been classified under 18 broad headings alphabetically ranging from 'Agencies and Institutions Related to Education, Counselling, Medical, and Psychological Services; Curriculum Areas, etc., to Teachers and Teaching'. The new concepts and topics, viz., 'Computer-Based Education', 'Drug Abuse Education', 'Equity Issues in Education', 'Ethnography' and 'Neurosciences' are also included in this volume. These additions reflect recent events and developments in the world to which education must attend.

The International Encyclopedia of Education, Torsten Husen and T. Neville Postlethwaite, ed., (New York: Pergamon Press, 1985). This publication is the first major attempt to present an up-to-date overview on educational problems, practices and institutions all over the world. The information available in this volume provides answers to three basic questions: What is the state of the art in the various fields of education?, What scientifically sound and valid information is available? and What further research is needed in various aspects of education?

The Encyclopedia of Comparative Education and National Systems of Education, T. Neville Postlethwaite, ed., (New York: Oxford Press, 1988). This encyclopedia is in two parts: the first part presents a series of articles about comparative education; the second part provides description of 159 different systems of education in various countries.

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International Encyclopedia of the Social Sciences, (New York: Macmillan Co., 1968). It was prepared under the direction of 10 learned societies. This reference work covers topics in all of the social sciences.

Encyclopedia of Child Care and Guidance, (Garden City, New York: Doubleday and Co., 1968). It is a comprehensive treatment of the nature of the problems of childhood. It also suggests the methods of dealing with such problems.

Encyclopedia of Social Work, (New York: National Association of Social Workers, 1965). This reference work presents extensive articles on all aspects of social work.

Encyclopedia of Philosophy, (New York: McGraw-Hill Book Co. 1971). This encyclopedia contains more than 7,000 articles written by more than 2,000 contributors in all areas of science and engineering.

Encyclopedia of Philosophy, (New York: Macmillan, Free Press 1967). It is an authoritative and comprehensive reference work covering both Western and Eastern thought—ancient, medieval and modern.

Encyclopedia of Indian Education, (New Delhi: NCERT, 2004). It provides a comprehensive description of various concepts, themes and systems pertaining to Indian education in ancient, medieval, pre-independence and post-independence periods.

Dictionaries. They serve as constant guides to the researcher. A few known dictionaries are detailed below:

Dictionary of Education, (New York: McGraw-Hill Book Co., 1973). This dictionary covers 33,000 technical and professional terms. It also includes educational terms used in various countries.

Comprehensive Dictionary of Psychological and Psycho-Analytical Terms, (New York: David McKay Company). It contains more than 13,000 terms. All these are defined in non-technical terms.

Dictionary of Sociology, Totowa, N.J., (Littlefield, Adams and Co.). In this dictionary, sociological terms are defined in non-technical language.

Roget's International Thesaurus of Words and Phrases, (New York: Crowell, Collier and Macmillan). A Thesaurus is the opposite of a dictionary. One turns to the Thesaurus when one has an idea, but does not yet have appropriate word to convey it. Thesaurus lists together the synonyms and antonyms of words. A researcher should use this reference in conjunction with a good dictionary to ensure precision of expression.

Yearbooks, Almanacs and Handbooks, A large amount of current information on educational problems, thought and practices may be found in yearbooks, almanacs and handbooks. Some yearbooks cover a new topic of current interest each year and some others give more general reviews of events. A list of some yearbooks, almanacs and handbooks is given as under:

The Handbook of Research on Teaching, N. L. Gage (ed.), (Chicago: Rand McNally & Co., 1963). This handbook presents a comprehensive research information on teaching with extensive bibliographies.

The Rand McNally Handbook of Education, Arthur W. Foshay (ed.), (Chicago: Rand McNally & Co, 1963). It is a convenient source compilation of the most important

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facts about education in the United States. This handbook provides a quick-reference comparison of education in England, France and Russia.

Education Yearbook, (New York: Macmillan Co., 1972-date). This is an annual publication. It includes statistical data on major educational issues and movements with a comprehensive bibliography and reference guide.

Mental Measurement Yearbook, (Highland Park, New Jersey: Grayphon Press, 1938-date). It is compiled by Oscar K. Buros and provides a comprehensive summary on psychological measurement and standardized tests and inventories. It is published every four years and includes reviews on all significant books on measurement and excerpts from book reviews appearing in professional journals.

Indian Mental Measurement Hand Book: Intelligence and Aptitude Tests, (New Delhi: National Council of Educational Research and Training (NCERT), 1991). The Handbook is one of major efforts of National Library of Educational and Psychological Tests (NLEPTs) published by NCERT to present before the researchers, a review of the standardized tests, particularly in the areas of 'Intelligence' and 'Aptitude'. It makes available the organized information on tests developed in India and the Indian adaptations or standardizations of foreign tests. The information covers not only tests which are commercially available to test users, and those available for restricted use, but also tests for which only specimen sets are available. Test reviews have been included in this Handbook in order to help the readers to evaluate the tests more critically.

The Student Psychologist's Handbook: A Guide to Sources, (Cambridge, Mass: Schenkman Publishing Co., 1969). This handbook describes the major content areas of psychology with sources of information, methods of data collection, and the use of reference materials.

Data Processing Yearbook, (Detroit: Frank H. Gille, 1952-date). This yearbook is published irregularly and includes articles on equipment, techniques, and developments in data processing. It also provides information about institutions offering data processing and computer courses.

United Nations Statistical Yearbook, (New York: United Nations, 1949-date). This is an annual publication. It presents statistical data on population, trade, finance, communication, health and education.

World Almanac-Book of Facts, (New York: Newspaper Enterprise Association, 1968-date). This reference guide is published annually. It provides up-to-date statistics and data concerning events, progress and conditions in social, educational, political, religious, geographical, commercial, financial and economic fields.

The Standard Education Almanac. It provides a record of facts and statistics on virtually every aspect of education.

Directories and Bibliographies. Directories are used by a researcher to locate the names and addresses of persons, periodicals, publishers or organizations when he/she wants to obtain information, about financial assistance or research material and equipments. Directories may help a researcher to find people or organizations who have similar professional interests or who can answer his/her queries or help to solve his/her problems.

A few important directories in the US and the UK are as follows:

- *Guide to American Educational Directories*. It lists in one volume over 12,000 educational and allied directories. The directories are listed alphabetically and are arranged under subject headings.

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- *The Education Directory*, (Washington: US Office of Education, Superintendent of Documents, 1912-date). This directory is published annually in five parts. It deals with names, educational agencies, officials, institutions and other relevant data.
- *NEA Handbook for Local, State and National Associations*, (Washington, DC: National Education Association, 1945-date). This is an annual publication and contains listings and comprehensive reports of state and national officers of affiliated associations and departments.
- *Educator's World*, (Englewood, Colo.: Fisher Publishing Co., 1972-date). This is an annual guide to more than 1,600 education associations, publications, research and foundations.
- *National Faculty Directory*, (Detroit: Gale Research Co., 1964-date). This annual publication lists alphabetically the names and addresses of more than 300,000 full-time and part-time faculty members and administrative officials of colleges and universities in the US.
- *Encyclopedia of Associations*, (Detroit: Gale Research Co., 1964-date). This directory lists alphabetically more than 14,000 national associations of the US. It includes information on membership, addresses, names of executive secretaries and statement of purpose of these associations.
- *Directory of Exceptional Children*, (Boston: Porter Sargent Publishing Co., 1962-date). This directory provides a description of schools, camps, homes, clinics, hospitals and services for the socially mal-adjusted, mentally retarded or physically handicapped in the US.
- *Mental Health Directory*, (Washington, D.C.: National Institute of Mental Health, Government Printing Office, 1964-date). This annual publication lists national, state and local mental health agencies in the US.
- *American Library Directory*, (New York: R.R. Bowker Co., 1923-date). This directory provides a binnaual guide to private, state, municipal, institutional and collegiate libraries in the US and Canada. It includes information on special collections, number of holdings, staff salaries, budgets and affiliations.

Kelley, Thomas (ed.) *Select Bibliographies of Adult Education in Great Britain*, (London: National Institute of Education, 1952). Blackwell, A.M. *A List of Researches in Educational Psychology Presented for Higher Degrees in the Universities of the United Kingdom and the Irish Republic from 1918*. (London: Newnes Educational Publishing Co., 1950).

In India, a very few bibliographical guides to educational research on a national basis have appeared. *Bibliography of Doctorate Theses in Science and Arts* accepted by the Indian Universities for 1946–48 and 1948–50 was published by the Inter-University Board of India. These are listed under the respective universities with subject sub-headings including education.

The Index. A periodical index serves the same purpose as the index of a book or the card file of a library. It identifies the source of the article or of the book cited by listing the titles alphabetically, under author and the readers should read all such directions before trying to locate the references.

A list of some important educational indexes is given below:

Education Index, (New York: H.W. Wilson Co., 1929-date). One valuable and work saving guide created for educators is Education Index. It is published monthly

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(September through June), cumulated annually and again every three years. It indexes more than 250 educational periodicals, and many yearbooks, bulletins, and monographs published in the US, Canada, and Great Britain. The material on adult education, business education, curriculum, educational administration, educational psychology, educational research, exceptional children, higher education, guidance, health and physical education, international education, religious education, secondary education and teacher education are included in this index.

Canadian Education Index, (Ottawa, Ontario: Canadian Council for Educational Research, 1965-date). This index is issued quarterly and indexes periodicals, books, pamphlets, and reports published in Canada.

Current Index to Journals in Education, (New York: Macmillan Information, 1969-date). This index is published monthly and cumulated six monthly and annually. It indexes about 20,000 articles each year from more than 700 education and education-related journals under author and subject headings.

ERIC Educational Documents Index, (Washington, D.C.: National Institute of Education, Government Printing Office, 1966-date). This index is published annually. It is a guide to all research documents in the 'Educational Resources Information Centre' or ERIC collection.

Index of Doctoral Dissertations International, (Ann Arbor, Mich.: Xerox University Microfilms, 1956-date). Published as the issue 13 of *Dissertation Abstracts International* each year, it consolidates into one list all dissertations accepted by American, Canadian, and some European universities during the academic year, as well as those available in microfilm.

International Guide to Educational Documentation, (Paris: UNESCO). This guide is published every five years. It indexes annotated bibliographies covering major publications, bibliographies and national directories written in English, French and Spanish.

British Education Index. This index is compiled by the Librarians of Institutes of Education, and it includes references to articles of educational interest published during the period of four years. The index covers more than 50 periodicals.

Index to Selected British Educational Periodicals, (Leeds: Librarians of Institutes of Education, 1945-date). This index is issued thrice per year and it covers 41 educational periodicals excluding those on fundamental and adult education.

Information about new ideas and developments often appear in periodicals long before it appears in books. There are many periodicals in education and in other closely-related areas that are the best sources for reports on recent research studies. Such periodicals give much more up-to-date treatment to current questions in education than books possibly can. They also publish articles of temporary, local or limited interest that never appear in book form. The periodicals of proper dates are the best sources for determining contemporary opinion and status, present or past.

It has been estimated that there are about 2,100 journals that are specifically related to the field of education. In all such journals, one may also find articles of interest devoted to psychology, philosophy, sociology, and other subjects.

All those engaged in educational research should become acquainted with certain educational periodicals, and they should also learn to use the indexes to them. Knowledge about the editor of a periodical, the names of its contributors, and the associations or institutions publishing it may serve as clues in judging the merit of the periodicals.

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Ulrich's Periodicals Directory; A Classified Guide to a Selected List of Current Periodicals, Foreign and Domestic, (New York: Bowker), provides a comprehensive list of periodicals relating to education. In this directory, periodicals are grouped in a subject classification and are alphabetically arranged. Each entry includes title, sub-title, date of origin, frequency of publication, annual index, cumulative indexes, and item characteristics of each periodical.

In India, many periodicals are published by some associations or institutions. They provide a medium for dissemination of educational research and exchange of experience among research workers, teachers, scholars and others interested in educational research and related fields and professions.

Abstracts include brief summaries of the contents of the research study or article. They serve as one of the most useful reference guides to the researcher and keep him/her abreast of the work being done in his own field and also in the related fields.

In America, the most useful of these references are the following:

The Review of Educational Research. It gives an excellent overview of the work that has been done in the field and about the recent developments. This publication, between 1931 and 1969, reviewed about every three years each of the given 11 major areas of education: (i) Administration; (ii) Curriculum; (iii) Educational Measurement; (iv) Educational Psychology; (v) Educational Sociology; (vi) Guidance and Counselling; (vii) Language Arts, Fine Arts, Natural Sciences, and Mathematics; (viii) Research Methods; (ix) Special Programmes; (x) Mental and Physical Development; and (xi) Teaching Personnel.

Since June 1970, the *Review of Educational Research* has pursued a policy of publishing unsolicited reviews of research topics of the contributor's choice. The role played by this publication in the past has been assumed by the *Annual Review of Educational Research.*

Research In Education (RIE): This represents the most comprehensive publication of research materials in education today. RIE is published monthly since 1966 by the Educational Resources Information Centre (ERIC) and indexed annually. Each monthly issue of RIE is divided into three sections: (1) Document Section; (2) Project Section; and (3) Accession Numbers Section.

Psychological Abstracts: This useful reference is published by the American Psychological Association since 1927. It is published bimonthly and contains abstracts of articles appearing in over 530 journals, mostly educational periodicals. The biannual issues (January–June, July–December) contain both author and subject index.

Education Abstracts: This is a publication of UNESCO, which began in 1949 and has been published monthly except in July and August. Each introductory essay devoted to a particular aspect of education is followed by abstract of books and documents selected from various countries dealing with the topic under consideration.

In addition to the above periodicals, a researcher may also consult the following publications:

(1) *Annual Review of Psychology* (1950-date)

(2) *Child Development Abstracts and Bibliography* (1927-date)

(3) *Psychological Bulletin* (1904-date)

(4) *Sociological Abstracts* (1952-date)

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(5) *Educational Administration Abstracts* (1966-date)

(6) *Sociology of Education Abstracts* (1965-date)

(7) *Mental Retardation Abstracts* (1964-date)

(8) *Dissertation Abstracts International* (1952-date)

In India, National Council of Educational Research and Training (NCERT) has been publishing *Indian Educational Abstracts* to serve the cause of educational research through disseminating information about educational researches available in public domain. The information contains abstracts of the researches carried out in India and abroad relevant to Indian educational scene with bibliographic information. This biannual periodical also includes abstracts of doctoral theses, research projects, published researches in the form of books and articles in the reputed journals.

Many professional periodicals and year books, in India and abroad, include some reviews of research and technical discussions of educational problems in one or all the issues of their series. A list of some of the publications are as follows:

USA: *Journal of Educational Research, NEA Research Bulletin, Educational and Psychological Measurement, Journal of Experimental Education, Research Quarterly, Journal of Research in Music Education, American Educational Research Journal, Reading Research Quarterly, Journal of Educational Psychology, Journal of Psychology, Journal of Social Psychology, Journal of Applied Psychology, Sociology of Education, American Journal of Sociology, American Sociological Review, Sociology and Social Research, Harvard Educational Review, Journal of Teacher Education, Elementary School Journal, History of Education Quarterly, and Educational Forum.*

UK: *British Journal of Educational Psychology.*

India: *Indian Educational Review, Journal of Psychological Researches, Indian Journal of Applied Psychology, Indian Journal of Experimental Psychology, Journal of Education and Psychology, The Education Quarterly, Perspectives in Education, Journal of Educational Planning and Administration, University News, Journal of Higher Education, Indian Journal of Education.*

Theses and dissertations are usually preserved by the universities that award the authors their doctoral and masters degrees. Sometimes these studies are published in whole or in part in various educational periodicals or journals. Because the reports of many research studies are never published, a check of the annual list of theses and dissertations issued by various agencies is necessary for a thorough coverage of the research literature.

In the US, references of doctoral dissertations in all fields, including education, can be found in sources compiled by various agencies. For the period 1912–1938, the Library of Congress issued the annual *List of American Doctoral Dissertations* for published studies. The Association of Research Libraries published the list of *Doctoral Dissertations Accepted by American Universities* from 1933–1934 to 1954–1955. This service was continued by the Index to *American Doctoral Dissertations* 1956–1963, which became the *American Doctoral Dissertations,* 1963–64 to date. It lists all doctoral dissertations accepted by the American and Canadian universities and other educational institutions.

Dissertation Abstracts International, May 1970, abstracts dissertations in the humanities, social sciences, physical sciences and engineering. It is published monthly.

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For each dissertation, there is a 600 word abstract that provides the researcher enough information to satisfy his/her needs. If a researcher wants to read a complete copy of a dissertation that is presented in *Dissertation Abstracts International,* he/she can purchase a microfilm or xerox copy from the University Microfilms. The reference number for placing an order and price are provided in the abstract.

In India, only a few universities publish abstracts of dissertations and theses that have been completed at the institution.

Kurukshetra University, Kurukshetra (Haryana) published *Abstracts of M.Ed. Dissertations*, Vol. I, 1966; *Abstracts of M.Ed. Dissertations,* Vol. II, 1967; *Abstracts of M.Ed. Dissertations,* Vol. III, 1968; *Abstracts of M.Ed Dissertations*, Vol. IV, 1969; *Abstracts of M.Ed. Dissertations,* Vol. V, 1970; *Abstracts of M.Ed. Dissertations and Ph.D. Theses*, Vol. VI, 1973.

M.B. Buch (ed.) *A Survey of Research in Education*, (Centre of Advanced Study in Education, Baroda: M.S. University, 1973). This publication contains all the research studies in education completed in Indian universities up to 1972. The break up of the studies in the said volume is 462 Ph.D. studies and 269 project research. The abstracts of all the studies have been classified into 17 meaningful areas of education. They are (i) Philosophy of Education, (ii) History of Education, (iii) Sociology of Education, (iv) Economics of Education (v) Comparative Education, (vi) Personality, Learning and Motivation, (vii) Guidance and Counselling, (viii) Tests and Measurement, (ix) Curriculum, Methods, and Textbooks, (x) Educational Technology, (xi) Correlates of Achievement, (xii) Educational Evaluation and Examination, (xiii) Teaching and Teaching Behaviour, (xiv) Teacher Education, (xv) Educational Administration, (xvi) Higher Education and (xvii) Non-Formal Education.

M.B. Buch, ed., *Second Survey of Research in Education (1972–1978)* (Baroda: Society for Educational Research and Development, 1979). This publication incorporates 839 research studies completed during the period 1972–1978 and follows the same pattern of organization of 17 research areas as *A Survey of Research in Education (1973).* The first chapter gives a broad perspective of the place and function of research for educational development including historical account of the development of educational research in India. Each subsequent chapter includes a report based on the abstracts of research studies giving the trend of research in the area, including the gaps and high-lighting the research priorities as perceived by the author. The abstracts are arranged alphabetically for each area and continuously numbered throughout the volume. Each abstract contains the title of the study, the objective and/or hypotheses examined, methodology including the sample, tools of research, the statistical techniques used, and the findings. A special feature of this publication is the incorporation of a large number of studies on educational problems completed in the university departments of social sciences and humanities other than the departments of education. The trend reports are based not on the research completed during the period 1972–1978, but on the total research activities during the period 1940–1978.

M.B. Buch, ed., *Third Survey of Research in Education (1978–1983)*, New Delhi: National Council of Educational Research and Training, 1987. The publication comprises 20 chapters beginning with a comprehensive review for the general trend of research in education in India based on a quantitative and qualitative analysis of the studies. The trend reports in different areas of education have been developed by eminent educationists on the basis of studies conducted during the period of four decades, from 1943 to 1983. In all, 1481 research abstracts have been presented after being classified

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under the 17 areas. Each research abstract reports in brief the problem, objectives of the study, research techniques adopted, and the findings and conclusions of the study. A special feature of the volume is the chapter on 'Research on Indian Education Abroad', which presents a review of 192 doctoral dissertations submitted to American and British universities, covering a period of around two decades. Another significant inclusion in the volume is the chapter on 'Priorities in Educational Research'. The volume also makes available at one place a complete list of all researches in education conducted in India till 1983.

M.B. Buch, ed., *Fourth Survey of Research in Education (1983–1988)*, New Delhi: National Council of Educational Research and Training, 1991.

This publication, available in two volumes, covers researches in education till 1988. It comprises 31 chapters beginning with a comprehensive review of the general trend of research followed by trend reports in different areas of education developed by eminent educationists on the basis of studies conducted during the period of about four-and-a-half decades—1943 to 1988. In all, 1,652 research abstracts have been presented after classification in 29 areas. The volume makes available a complete list of all the 4,703 educational researches conducted in India since 1943. The Fourth Survey has a new dimension. There is a chapter on review of researches at the M.Phil level in Indian Universities.

Fifth Survey of Educational Research (1988–1992), New Delhi: National Council of Educational Research and Training, 1997. This publication is also available in two volumes and covers researches in education conducted during 1989–1992. It has dealt with all the areas of research which were covered in the Fourth Survey with the addition of a chapter on researches in "Distance Education and Open Learning".

Sixth Survey of Educational Research (1993–2000), New Delhi: National Council of Educational Research and Training, 2006. The first volume of this publication was released in 2006 and the second volume is still awaited. The researches in the areas of philosophy of education, teacher education, vocational education, science education, distance education and open learning, women education, guidance and counselling, physical education, health education and sports, language teaching, inclusive education, educational technology and population education conducted in India during the period 1993–2000 have been reported in the first volume.

Many articles of particular interest to a researcher may be located through pamphlets and newspapers. Current newspapers provide up-to-date information on speeches, seminars, conferences, new trends, and a number of other topics. Old newspapers, which preserve a record of past events, movements and ideas are particularly useful in historical inquiries. Some libraries catalog pamphlets and newspapers in their reference sections.

Government documents are a rich source of information. They include statistical data, research studies, official reports, laws and other material that are not always available elsewhere. These are available in national, regional, state as well as local level government offices.

Monographs are also major sources of information on ongoing research. In the US, universities and teachers' colleges publish many research studies in education in the form of monographs. A few examples of these are *Supplementary Educational Monographs, Educational Research Monographs,* and *Lincoln School Monographs.* In England too, various institutes of education publish monographs from time to time. In India, only a limited number of monographs are published by some universities and research organizations.

School Research Information Service (SRIS), Direct Access to Reference Information (DATRIX), and Psychological Abstract Search and Retrieval Service (PASAR) in the United States provide a number of computer-generated reference sources that may save a great deal of time and effort of the researcher. SRIS operated by Phi Delta Kappa (Bloomington, Indiana) provides a computer printout of abstracts for a moderate fee. DATRIX, a development of the University Microfilms (Ann Arbor, Michigan) provides computerized retrieval for *Dissertation Abstracts*, from 1928 to date. The researcher can procure information on Microfiche or Xerographic copy of the complete dissertation which he needs, from *University Microfilms,* on payment. The PASAR furnishes printouts of abstracts of psychological journal articles, monographs, reports, and parts of books for a moderate fee.

Organizing the Related Literature

After making the comprehensive survey of the related literature, the next step for the researcher is to organize the pertinent information in a systematic manner. It should be done in such a way as to justify carrying out the study by showing what is known and what remains to be investigated in the topic of concern. According to Ary *et al.* (1972, p. 67):

The hypotheses provide a framework for organizing the related literature. Like an explorer proposing an expedition, one maps out the known territory and points the way to the unknown territory he proposes to explore. If the study has several aspects, or is investigating more than a single hypothesis, this is done separately for each facet of the study.

One should avoid the temptation to present the literature as a series of abstracts. Rather, it should be presented in such a way as to lay a systematic foundation for the study.

The organization of the related literature involves recording the essential reference material and arranging it according to the proposed outline of the study.

Once pertinent information has been identified, the researcher should record certain essential information for locating the material on 3 × 5 inch index card to serve as a *bibliography card.* To make writing of the final report simpler, it is desirable that the information recorded in the bibliography card should appear, in content and style, exactly as it will appear in the final report.

The basic information in the bibliography card should include name of the author with last name first; title of the book or article; name of the publication (for articles); name of the publisher; date of publication; volume number, page numbers and library call number (for books). If some of this information is not available, the specified space should be left blank so that the missing information can be included immediately upon locating the references.

After recording the essential information on the bibliography cards, it is necessary to arrange the cards according to the location of the material in the library. For example, the researcher may list together all cards pertaining to the material located in the periodical section. Similarly, all the material located in the reserve section may constitute another list, and so on. Then the researcher should make a systematic review of the material located in a specific section of the library and after reviewing each reference on the list, he/she should proceed to another list.

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All the information likely to be used in the final report should be recorded on 4 × 6 inch card to serve as *content card.* The information to be recorded on the content cards will depend on the source from which it is taken. If it is from a primary source, it may include brief bibliographic information comprising author's last name, brief title of the report, specific page numbers on which information is located; sentence statement of the problem; brief description of the study; statements of findings or conclusions, or both; a card code as to the aspect of the research to which the material most closely relates.

The information to be recorded from the secondary source is somewhat different from the primary source. Turney and Robb (1971, p. 55) have given the following suggestions for recording information from a secondary source:

1. Provide brief bibliographic information (as with a primary source).
2. Record on a single card only those statements that are related to the same topic (if all the information cannot be placed on one card, continue statements on another card and staple to the first card).
3. Paraphrase, in complete statements, the most relevant ideas. Record direct quotations only if they are stated concisely and effectively, and if paraphrasing might change the meaning.
4. Place a page number and a paragraph number after each separate statement indicating its location in the reference in case you need to review it again.
5. Code the cards (probably in the upper-right hand corner) according to topic(s) to which it most closely relates.

For the preparation of the report of the related literature, the researcher should arrange the bibliographic and content cards according to the proposed outline of the problem. This can be done with the help of card code.

The report of the related literature should begin with an introductory paragraph describing the organization of the report. After the introduction, the researcher should present the studies most relevant to each aspect of the proposed problem outline. Studies with similar and contradictory results should be reported side-by-side without using excessive space.

Check Your Progress

1. What is the first step in reviewing the related literature?
2. What is a microfiche? What are its significant contributions to library and information services?
3. Why card catalog is used?
4. What are Encyclopedias?
5. What all abstracts include?
6. What does organization of the related literature involve?

Test

1. What is the importance of survey of related literature in educational research? Illustrate by taking a specific research problem as to how the survey of the related literature can be helpful at various stages.
2. Describe the procedure which the researcher should adopt in identifying related literature, and in locating, selecting and utilizing the primary and secondary sources of information available in the library.
3. What library skills are required for a thorough survey of literature related to a research topic in education?
4. Name some important reference books with author's names and some important educational journals you would like to consult in connection with the problem you have selected for research.
5. Describe the procedure which the researcher should adopt in organizing the related literature in a systematic manner.

2.4 OVERVIEW OF HYPOTHESIS

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A hypothesis is an approximate assumption that a researcher wants to test for its logical or empirical consequences. It can contain either a suggested explanation for a phenomenon or a proposal having deductive reasoning to suggest a possible interrelation between multiple phenomena. A deductive reasoning can be defined as a type of reasoning that can be derived from previously known facts.

2.4.1 Characteristics of Valid Hypothesis

There are several characteristics of hypothesis, which are as follows:

- **Conceptually Clear and Accurate:** The hypothesis must be conceptually clear. The concepts and variables should be clearly defined operationally. The definition should use terms which are commonly accepted and communication is not hindered. Hypothesis should be clear and accurate so as to draw a consistent conclusion.
- **Statement of Relationship between Variables:** If a hypothesis is relational, it should state the relationship between the different variables.
- **Testability:** A hypothesis should have empirical referents which mean that it should be testable through the empirical data. Hypothesis involving mystical or supernatural things are impossible to test. For example, the hypothesis 'education brings all-round development' is difficult to test because it is not easy to operationally isolate the other factors that might contribute towards all-round development. Since a hypothesis predicts the outcome of a study, it must relate variables that are capable of being measured. The hypothesis such as '*there is a positive relationship between the learning style and academic achievement of 8th grade students*' can be tested since the variables in the hypothesis are operationally defined, and therefore can be measured.
- **Specific with Limited Scope:** A hypothesis, which is specific with limited scope, is easily testable than a hypothesis with limitless scope. Therefore, a researcher should pay more time to do research on such a kind of hypothesis.
- **Simplicity:** A hypothesis should be stated in the most simple and clear terms to make it understandable.
- **Consistency:** A hypothesis should be reliable and consistent with established and known facts.
- **Time Limit:** A hypothesis should be capable of being tested within a reasonable time. In other words, the excellence of a hypothesis is judged by the time taken to collect the data needed for the test.
- **Empirical Reference:** A hypothesis should explain or support all the sufficient facts needed to understand what the problem is all about.

A few more characteristics of a good hypothesis are as follows:

- It ensures that the sample is readily approachable.
- It maintains a very apparent distinction with what is called theory, law, facts, assumptions and postulates.
- It should have logical simplicity, a large number of consequences and be expressed in quantified form.

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- It should have equal chances of confirmation and rejection.
- It permits the application of deduction reasoning.
- Tools and data should be easily available and effectively used.
- It should be based on study of previous literature and an existing theory, and should be verifiable.

As soon as a research question is formulated, it makes the hypothesis formulation imperative since a *hypothesis* is a tentative solution or an intelligent guess about a research question under study. It is an assumption or proposition whose tenability is to be tested on the basis of its implications with empirical evidence and previous knowledge. Modem investigators agree that, whenever possible, research should proceed from a hypothesis. In the words of Van Dalen (1973), '*a hypothesis serves as a powerful beacon that lights the way for the research worker*'.

2.4.2 Need for Hypotheses Formulation

The reasons for formulating a hypothesis are as follows:

(i) A hypothesis directs, monitors and controls the research efforts. It provides tentative explanations of facts and phenomena and can be tested and validated. Such explanations, if held valid, lead to generalizations, which help significantly in understanding a problem. They thereby extend the existing knowledge in the area to which they pertain and thus help in theory building and facilitate the extension of knowledge in an area.

(ii) The hypothesis not only indicates what to look for in an investigation but also how to select a sample, choose the design of research, how to collect data and how to interpret the results to draw valid conclusions.

(iii) The hypothesis orients the researcher to be more sensitive to certain relevant aspects of the problem so as to focus on specific issues and pertinent facts. It helps the researcher to delimit his/her study in scope so that it does not become broad and unwieldy.

(iv) The hypothesis provides the researcher with rational statements, consisting of elements expressed in a logical order of relationships, which seek to describe or to explain conditions or events that have not yet been confirmed by facts. Some relationships between elements or variables in hypotheses are known facts, and others transcend the known facts to give reasonable explanations for known conditions. The hypothesis helps the researcher relate logically known facts to intelligent guesses about unknown conditions (Ary, *et al.*, 1972, pp. 73–74).

(v) Hypothesis formulation and its testing add a scientific rigour to all type of researches. A well thought set of hypothesis places a clear and specific goal before the researcher and equips him/her with understanding. It provides the basis for reporting the conclusions of the study on the basis of these conclusions. The researcher can make the research report interesting and meaningful to the reader. The importance of a hypothesis is generally recognized more in the studies which aim to make predictions about some outcome. In an experimental study, the researcher is interested in making predictions about the expected outcomes and, hence the hypothesis takes on a critical role. In the case of historical or descriptive studies, however, the researcher investigates the history of an event, or life of a man, or seeks facts in order to determine the *status quo* of a situation and hence may not have a basis for making a prediction of the results. In studies of this nature, where fact finding itself is the objective of the study, a hypothesis may not be required.

Most historical or descriptive studies involve fact finding as well as the interpretation of facts in order to draw generalizations. For all such major studies, a hypothesis is recommended so as to explain observed facts, conditions or behaviour and to serve as a guide in the research process. If a hypothesis is not formulated, a researcher may waste time and energy in gathering extensive empirical data, and then find that he/she cannot state facts clearly and detect relevant relationships between variables as there is no hypothesis to guide him/her.

2.5 HYPOTHESIS TESTING

Hypothesis testing means to determine whether or not the hypothesis is appropriate. This involves either accepting or rejecting a null hypothesis. The researcher has to pursue certain activities contained in the procedure of hypothesis.

In the formulation of hypothesis, the investigator looks for the statements where he/she relates one or more variables to make predictions about the relationships. The hypothesis tells the researcher what to do and why to do it in the context of the problem.

For example, the researcher is interested to study a problem, 'Why does a gifted child become a poor achiever in school'? The researcher then moves towards finding out the causes and factors that have been responsible for his/her poor achievement. He/She makes a conjecture that he/she might be suffering from some disease at the time of the examination. Conjecture is in the form of a hypothesis, and this now determines what the researcher should do to verify whether it is a fact or not. He/She shall go to the student's home, meet his/her parents and enquire about the student's health. All that the investigator is doing is guided by the hypothesis he/she had developed.

Thus, hypothesis refers to a conjecture statement about the solution to a problem, which the researcher goes on to verify on the basis of the relevant information collected by him/her. It is said to be a hunch, shrewd guess or supposition about what the answer to a problem may be. It is a statement which is tested in terms of the relationship or prediction, etc., which after testing is either accepted or rejected.

A hypothesis relates theory to observation and vice-versa. Hypotheses when tested are either rejected or accepted, and help to infer the conclusion, which helps in theory building. Being a specific statement of prediction, a hypothesis describes in concrete (rather than theoretical) terms what you expect will happen in your study. Not all studies have hypotheses. Sometimes a study is designed to be exploratory. In such researches, no formal hypothesis is established, and it may be the case that the actual objective of the study is to explore one or more specific areas more thoroughly in order to develop specific hypotheses or predictions that could be tested through research in the future. A single study could result in one or several hypotheses.

Some definitions of hypothesis are:

- According to Townsend, '*Hypothesis is defined as suggested answer to a problem*'.
- According to McGuigan, '*A hypothesis is a testable statement of a potential relationship between two or more variables*'.

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- According to Uma Sekaran, '*A hypothesis is defined as a logically conjectured relationship between two or more variables in the form of testable statement. These relationships are based on theoretical framework formulated for the research problem. The hypotheses are often statements about population parameters like expected value and variance, for example a hypothesis might be that the expected value of the height of 10-year-old boys in the Scottish population is not different from that of 10-year-old girls.*'
- According to Kerlinger, '*A good hypothesis is one which satisfies the following criteria:*

  (i) *Hypothesis should state the relationship between variables.*

  (ii) *They must carry clear implications for testing the stated relations.*'

This means that (a) Statements contain two or more variables which can be measured, (b) They must state clearly how the two or more variables are related and (c) It is important to note that facts and variables are not tested but relations between variables exist.

2.5.1 Sources of Hypothesis

Since the mind is fed by innumerable streams and sources, it is difficult to pinpoint how a particular good idea came to the researcher. The following are some of the popularly known sources of research hypothesis:

- **Scientific Theories:** A systematic review and analysis of theories developed in the field of psychology, sociology, economics, political science and biological science may provide the researcher with potential clues for constructing a good and testable hypothesis.
- **Expert Opinions:** Discussion with the experts in the field of research may further help the researcher obtain necessary insight and skill into the problem and in formulation of a hypothesis.
- **Method of Related Difference:** When we find that two phenomena differ constantly and the other circumstances remaining the same, we suspect a causal connection. For example, when we find more uncontrolled traffic in a locality, resulting in a greater number of road accidents, we suspect a causal connection between uncontrolled traffic and road accidents. This method also suggested a hypothesis.
- **Intellectual Equipment of Researcher:** Intellectual abilities of a researcher like creative thinking and problem solving techniques are very helpful in the formulation of a good hypothesis.
- **Related Literature:** Related literature is the most important source of hypothesis formulation. A review of this literature may reveal to the researcher the variables that have been considered important in relation to his/her problem, which aspects have already been studied and which still remain to be studied, which theories have supported the relationships and which theories present a contradictory relationship. Familiarity with related literature may give the researcher a tremendous advantage in the construction of hypothesis.
- **Experience:** One's own experience may be a rich source of hypothesis generation. Personal experiences of an individual which has been gained through reading of biographies, autobiographies, newspaper readings or through informal talks among friends, etc., can be a potential source of generation of a hypothesis. For example, a researcher who is working on the effectiveness of guidance in teaching, can think of factors such as the teacher's polite behaviour, techniques of counselling, mastery over the subject, effective use of teaching skills, decision-making capability, perception of his/her competence, perception of student's capacity for better interaction, use of communication skills, etc.
- **Analogies:** Several hypotheses in a branch of knowledge may be made by using analogies from other sciences. Models and theories developed in a discipline may help, through extrapolation, in the formulation of hypothesis in another discipline. By comparing the two situations, analysing their similarities and differences, some rationale may emerge in the mind of the researcher which may take the form of a hypothesis for testing. For example, in a research problem like the studying the factors of unrest among college level students, the researcher insightfully thinks: 'Why was unrest found among school students? and What has changed them: quality of teaching or quality of leadership?'

  Arguing analogically in this way may lead the investigator to some conclusions which may be used for identifying variables and relationships, which form the basis of hypothesis construction. If a researcher knows from previous experience that the old situation is related to other factors Y and Z as well as to X, he/she may reason out that the new situation may also be related to Y and Z.
- **Methods of Residues:** When the greater part of a complex phenomenon is explained by some causes already known, we try to explain the residual part of phenomenon according to the known law of operation. It also provides possible hypothesis.
- **Induction by Simple Enumeration:** Sometimes scientists take common experience as a starting point of their investigation. For example, after observing a large number of scarlet flowers that are devoid of fragrance, we frame a hypothesis that all scarlet flowers are devoid of fragrance. Thus induction by simple enumeration is a source of discovery.
- **Formulation of Hypothesis:** It may also originate from the need and practice of present times.
- **Existing Empirical Uniformities:** In terms of common sense proposition, the existing empirical uniformities may form the basis for scientific examination.
- **A Study of General Culture:** It is also a good source of hypothesis.
- **Suggestions:** When given by other researchers in their reports, suggestions are quite helpful in establishment of hypothesis for future studies.

2.5.2 Procedure of Hypothesis Testing

The procedure for hypothesis testing is as follows:

(i) **Making Formal Statement:** In this step, the nature of a hypothesis is clearly stated, which could be either null hypothesis or alternate hypothesis. Stating a problem in hypothesis testing is of utmost importance, which should be done with proper care, keeping in mind the object and nature of the problem.

(ii) **Choosing a Significance Level:** In this step, a hypothesis is tested on the basis of a present significance level, which has to be adequate in terms of nature and

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purpose of the problem.

(iii) **Sampling Distribution:** In this step, determination of an appropriate sampling distribution and making a choice between normal distribution and *t*-distribution is included.

(iv) **Selection of a Sample Randomly:** In this step, a random sample is selected from the sample data for determining an apt value.

(v) **Probability Calculation:** In this step, the probability regarding viability of the sample result is made dependent on the null hypothesis.

(vi) **Comparison:** In this step, the calculated probability and the value of alpha in case of one-tailed test and alpha in case of two-tailed test is compared.

2.5.3 Types of Hypothesis Testing

Hypothesis is tested to identify the errors occurred in the statements and concepts used in hypothesis. Hypothesis testing can be broadly divided into two types, which are as follows:

- Parametric tests or standard tests of hypothesis
- Non-parametric tests or distribution-free tests of hypothesis

Parametric Tests or Standard Tests of Hypothesis

These kinds of tests assume certain properties of the population sample such as observations from a normal population, large sample size, population parameters like mean and variance. The various parametric tests of hypothesis are based on the assumption of normality. In other words, the source of data for them is normally distributed. They can be listed as follows:

- **Z-Test:** This kind of test is based on normal probability distribution. It is mostly used to judge the significance of mean as a statistical measure. This is the most frequently used test in research studies. It is generally used to compare the mean of a sample with the hypothesized mean of the population. It is also used in case the population variance is known. It is helpful in judging the significance of difference between the means of two independent large samples, to compare the sample proportion to a theoretical value of population proportion and to judge the significance of median, mode and coefficient of correlation.
- ***t*-Test:** This test is based on *t*-distribution and is aptly considered to judge the significance of a sample mean or the difference between the means of two small samples when population variance is not known.
- **χ^2:** This test is based on a chi-square distribution and is used for comparing a sample variance to a theoretical population variance.
- ***F*-Test:** This test is based on *F*-distribution and is also used to compare the variance of two independent samples. It is also used to compare the significance of multiple correlation coefficients.

Non-Parametric Tests or Distribution-Free Tests of Hypothesis

There are situations where assumptions cannot be made. In such situations, different statistical methods are used which are known as 'non-parametric tests'. There are various types of non-parametric tests. The important non-parametric tests are as follows:

- **Sign Test:** This is one of the easiest tests in practice based on the plus/minus sign of an observation in a sample. The sign may be one of the following two types:
  - **One-Sample Sign Test:** This is a very simple distribution-free test and is applied in case of a sample from a continuous symmetrical population, wherein the probability of a sample to be either less or more than mean is half. Here, to test a null hypothesis, all those items which are greater than the alternate hypothesis are replaced by a plus sign and those which are less than the alternate hypothesis are replaced by a minus sign.
  - **Two-Sample Sign Test:** In case of all the problems consisting of paired data, two-sample sign test is used. Here, each pair of values can be replaced with a plus sign in the first value of the first sample with the first value of the second sample. If the first value is less, minus sign is assigned.
- **Fisher-Irwin Test:** This is applied where there is no difference between two sets of data. In other words, it is used where you can assume that two different treatments are supposedly different in terms of the results that they produce. It is applied in all those cases where result for each item in a sample can be divided into one of the two mutually exclusive categories.
- **McNamara Test:** It is applied where the data is nominal in nature, and is related to two interrelated samples. By using this test, you can judge the significance of any observed changes in the same subject.
- **Wilcoxon Matched-Pairs Test:** This test is applied in the case of a matched-pair such as output of two similar machines. Here, you can determine both the direction and the magnitude between the matched values. This test is also called Signed Rank Test.

2.6 HYPOTHESIS TESTING FOR COMPARING TWO RELATED TERMS

Researchers often use hypothesis testing for comparing two population parameters based on the corresponding statistics from each population. For instance, researchers might want to check if the two populations have the same mean, which they can test with the help of hypothesis testing.

In this method two separate scores are to be obtained for each individual sample where the data in each sample set is related in some special way. For example, a group of patient's blood pressure is measured before and after a drug therapy. In this case, the same variable is measured two times for the same set of samples. Hypothesis testing uses *t*-statistic for comparing two related terms, which is described in the following sections:

t-Statistic for Comparing Two Related Terms

The *t*-statistic for comparing two related terms is based on the 'Difference Scores' and not on the 'Raw Scores'.

What is the Difference Scores?

Suppose a sample has $n = 4$ participants. Each individual's blood pressure is measured before and after medication.

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Let,

X_1 = The first score for each person before medication.

X_2 = The second score for each person after medication.

Then, the difference scores are obtained by subtracting the first score from the second score for each participant.

Hence, difference score $(d) = X_2 - X_1$

The following figure illustrates the difference scores:

Subject	*I*	*II*	*d*
A	10	15	5
B	20	25	5
C	15	10	–5
D	25	30	5

The *t*-Statistic Formula for Comparing Two Related Terms

t-statistic formula for comparing two related terms is as follows:

$$t = \frac{M_d - \mu_d}{S_{M_d}}$$

Where,

M_d = The mean for the sample of difference scores or sample mean difference.

μ_d = The mean for the population of difference scores.

S_{M_d} = The standard error for M_d.

Decision

If the *t* value obtained falls in the critical region then reject the null hypothesis, otherwise do not reject it.

- M_d can be calculated as follows:

$$M_d = \frac{\sum d}{n}$$

Where, n = Sample size .

- Sample variance (S^2) of difference score (d) can be calculated as follows:

$$S^2 = \frac{SS}{n-1} = \frac{SS}{df}$$

Where,

SS = It denotes sum of square of deviation.

df = It denotes degree of freedom.

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Also, the formula to calculate *SS* is as follows:

$$SS = \sum d^2 - \frac{(\sum d)^2}{N}$$

Where, N = The size of the population.

- The sample standard deviation is then calculated as follows:

$$S = \sqrt{\frac{SS}{df}}$$

- Next, using sample variance, the estimated standard error is computed as follows:

$$S_{M_d} = \sqrt{\frac{S^2}{n}} \text{ or } \frac{S}{\sqrt{n}}$$

Now, from the above computations, it is clear that all the calculations are done with the *d* scores, which is unique for each subject.

Degrees of Freedom

If there is a sample of n scores, then there will be total of *nd* scores. Degrees of freedom (*df*) describe the number of scores in a sample that are independent and can vary freely. The mean for the sample of difference scores places a restriction on the value of one sample, therefore the degrees of freedom will be $n - 1$ for n scores.

Basic Assumptions for t-Statistic

1. The sample size should not exceed 30.
2. Each observation within each treatment condition should be independent.

Example 1: For the following data set from a study of examining the effect of a treatment on college students by measuring a group of $n = 6$ subjects before and after they receive the treatment, find:

(a) The difference score.

(b) The sample means difference.

(c) Variance for difference scores.

(d) Standard error for the sample means difference.

Subjects	*Before Treatment*	*After Treatment*
I	7	8
II	2	9
III	4	6
IV	5	7
V	5	6
VI	3	8

Solution: (a) Let us denote,

X_2 = The second score for each participant after treatment

X_1 = The first score for each participant before treatment

NOTES

Now, the difference score for each participant is given by: $d = X_2 - X_1$

The difference scores are calculated in the table given below:

Subjects	X_1	X_2	d
I	7	8	1
II	2	9	7
III	4	6	2
IV	5	7	2
V	5	6	1
VI	3	8	5

The required difference scores are: 1, 7, 2, 2, 2, 1 and 5.

(b) The sample mean difference of the score is given by:

$$M_d = \frac{\sum d}{n}$$
$$= \frac{18}{6}$$
$$= 3$$

The required sample means difference is 3.

(c) First, calculate *SS* by calculating d^2, which is given in the table below:

Subjects	X_1	X_2	d	d^2
I	7	8	1	1
II	2	9	7	49
III	4	6	2	4
IV	5	7	2	4
V	5	6	1	1
VI	3	8	5	25

The formula of *SS* is given by:

$$SS = \sum d^2 - \frac{(\sum d)^2}{N}$$
$$= 84 - \frac{18^2}{6}$$
$$= 30$$

The variance for the sample of difference scores is given by:

$$S^2 = \frac{SS}{df}$$
$$= \frac{30}{5}$$
$$= 6$$

Hence, the required answer is 6.

(d) The standard error for the sample mean difference is given by:

$$S_{M_d} = \frac{s}{\sqrt{n}}$$
$$= \frac{\sqrt{6}}{\sqrt{6}}$$
$$= 1$$

Hence, the required answer is 1.

Every hypothesis test contains two opposite statements. One of the statements is null hypothesis and other is alternative hypothesis. These are the two types of hypothesis, which are described as follows:

Null Hypothesis

The null hypothesis states that the population parameter is equal to the claimed value and is denoted by H_0. It is used for comparing statistics with the help of mean, μ. For example, if the average time taken by the student to complete his homework is 5 hours, then, $H_0: \mu = 5$.

Alternative Hypothesis

Before conducting the hypothesis the other possible hypothesis can also be treated vice versa. It is used for comparing statistics assuming that there is a difference between the two.

If the population parameter is *not equal* to the claimed value:

$H_1: \mu \neq 5$

If the population parameter is *greater than* the claimed value:

$H_1: \mu > 5$

If the population parameter is *less than* the claimed value:

$H_1: \mu < 5$

Example 2: Use the dataset of Example 1.1 to find whether there is any significant treatment effect. Use, $\alpha = 0.05$ for two tails.

Solution:

The null hypothesis is: $H_0: \mu_d = 0$

The mean difference is zero as there is no difference between the treatment conditions.

Against the alternative hypothesis 7, which is:

$H_0: \mu_d \neq 0$

There is a significant mean difference. Use, $\alpha = 0.05$ for two tails.

With a sample of $n = 6$, the t statistic has $df = 6 - 1 = 5$.

Hence, for a two tailed test with $\alpha = 0.05$ and $df = 5$, the critical t values are ± 2.571.

NOTES

Now, compute t-value using t-statistic formula:

$$t = \frac{M_d - \mu_d}{S_{M_d}}$$

$$= \frac{3-0}{1}$$

$$= 3$$

The value $t = 3$ is greater than the critical value, so reject the null hypothesis H_0.

Hence, it can be concluded that there is a significant mean difference.

Confidence Intervals (CI) for *t*-Statistics

Confidence interval gives an estimate about a range of values that are centered around the sample statistic. It is calculated by using sample mean so that it can be confidently estimated that the value of the parameter lies in the interval of the known population. The sample mean difference M_d is used to estimate the population mean difference μ_d.

Formula of CI for *t*-Statistics

For the t-test of comparing two related terms, $\mu_d = M_d \pm tS_{M_d}$

Where t stands for t-value and tS_{M_d} stands for standard error of mean differences.

Example 3: Use the dataset of Example 1 to construct a 95% CI.

Solution: Confidence level= 95%

Now, $\alpha = 0.05$ and $df = 5$, the critical t values are ± 2.571.

Hence, the confidence interval is:

$$\mu_d = M_d \pm tS_{M_d}$$

$$= 3 \pm 2.571 \times 1$$

$$= 3 \pm 2.571$$

The required confidence interval to estimate the sample mean difference is: (0.429, 5.571).

Advantages of Hypothesis Testing for Comparing Two Related Terms

The advantage of this study is that it removes individual differences, which lowers sample variability and increases the chances of obtaining significant results.

2.6.1 Hypothesis Testing of Proportions

Many a times, crucial decisions rely on the percentage or proportion of the population that meets certain predefined criteria. For example, a state's Chief Minister might be interested in knowing the percentage of females attending school in that state in order to come up with a policy decision to enhance female literacy. An economist might be interested in the proportion of the firms in an industry that make excessive profits and hence suggest the existence of an oligopolistic market structure, which is a situation where market is controlled by few sellers. A manager of a big enterprise may want to estimate the percentage of employees with an attendance rate of more than 90 per cent. The central point is that we might want to check if the population proportion exceeds or is less than some cut off value. Stated differently, hypothesis testing would allow us to check if the population proportion is significantly different from the hypothesized proportion, which is the one that we ideally desire for our data should possess.

Let us denote the population proportion by θ in which we are interested in testing. Let ρ denote the sample proportion of observations that are considered as successes according to the defined rationale. So, if n is the sample size and X is the number of successes, then,

$$\rho = \frac{X}{n}$$

Let σ_ρ denote the standard error of the sampling proportion. It measures the tendency for the sample proportions, σ to deviate from the unknown population proportion, ρ. So,

$$\sigma_\rho = \sqrt{\frac{\rho(1-\rho)}{n}}$$

Assuming that n is large, we can use the standard Z-test technique for hypothesis testing. Z is defined as,

$$Z = \frac{\rho - \theta}{\sigma_\rho}$$

Note that this variable follows a standard normal distribution since it is in the form,

$$Z = \frac{\rho - E(\rho)}{\text{Standard error of } \rho}$$

Thus, we can use the standard normal tables to determine whether the calculated Z-value exceeds the Z-value at the given level of significance or not. Let us denote this critical Z-value by Z_α where α denotes the level of significance.

The general procedure for testing hypothesis can be thus outlined as follows:

- Set up the null hypothesis, that may be $\theta = \theta_H$, where θ_H is the hypothesized value of θ. This is what we want to test.
- Set up the alternative hypothesis which is complimentary to the null hypothesis. So, if the null is $\theta = \theta_H$, then the alternative is $\theta \neq \theta_H$.
- Choose the appropriate level of significance (α).
- Compute the relevant test statistic Z in this case.
- Find out the critical value (Z_α).
- Use the decision rule to accept or reject the null.

In this case, the decision rule is: If $|Z| > |Z_\alpha|$ then reject the null hypothesis, otherwise accept it.

Example 4: In a random sample of 500 people from a large population in a college, 200 are females. Is it correct to say that the sex ratio in this college is 1:1? Use level of significance as 1%.

Solution: We define success as the number of females in the sample. It is given that,

$n = 500$

$X = 200$

NOTES

NOTES

Since, the given ratio is 1:1, we define the null hypothesis as:

$H_0: \theta = 0.5$

$H_0: \theta = 0.5$, where θ is the proportion of females in the population.

The alterative hypothesis is therefore,

$H_1: \theta \neq 0.5$

The level of significance is given to be 0.01.

We compute,

$$\rho = \frac{X}{n}$$
$$= \frac{200}{500}$$
$$= 0.4$$

$$\sigma_\rho = \sqrt{\frac{\rho(1-\rho)}{n}}$$
$$= \sqrt{\frac{0.4 \times 0.6}{500}}$$
$$= \sqrt{0.00048}$$
$$= 0.02191$$

So, the test statistic in this case is,

$$Z = \frac{\rho - \theta}{\sigma_\rho}$$
$$= \frac{0.4 - 0.5}{0.02191}$$
$$= -4.564$$

Note that it is a two tailed test. So,

$Z_{0.01} = -2.58$

Also,

$|Z| = 4.56$

$|Z_{0.01}| = 2.58$

Thus, $|Z| > |Z_\alpha|$. Hence, we reject the null hypothesis.

Therefore, we cannot claim that the sex ratio is 1:1 in the college is at level of significance 1%.

2.6.2 Hypothesis Testing for Differences between Proportions

In this method, one usually tests a claim made about two population proportions. The two estimated proportions may be different due to a difference in the populations. A hypothesis test helps in determining if there is a difference in the estimated proportions: $p_1 - p_2$ which reflects a difference in the population proportions.

In this section, we shall consider those tests only where the hypothesized difference between proportions is zero, since this is generally the case is.

NOTES

Denoting our two population proportions as $\hat{p}_1$ and $\hat{p}_2$, we can write the null hypothesis as,

$H_0: p_1 - p_2 = 0$ which states that the two population proportions are equal.

Decision

If the *Z*-value obtained falls in the critical region then reject the null hypothesis, otherwise accept it.

Important Notations for the Two Population Proportions

Suppose, there are two populations: Population 1 and Population 2.

p_1 = Population proportion

n_1 = Size of the sample

x_1 = Number of success in the sample

$\hat{p}_1 = \frac{x_1}{n_1}$ (Sample proportion)

$\hat{q}_1 = 1 - \hat{p}_1$

For Population 2, the corresponding notations are: $p_2, n_2, x_2, \hat{p}_2$ and $\hat{q}_2$.

Formula for Pooled Sample Proportion

The notation of pooled sample proportion is $\overline{p}$ and the formula is given by:

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$$
$$\overline{q} = 1 - \overline{p}$$

Z-Statistic for Differences between Two Proportions

The Z-Statistic for differences between two proportions is based on the *Sample proportion* and the *Pooled sample proportion*.

The Z-Statistic formula for differences between two proportions is given by:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}}$$

Where,

$p_1 - p_2$ is assumed to be 0

$\hat{p}_1 = \frac{x_1}{n_1}$ and $\hat{p}_2 = \frac{x_2}{n_2}$

$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$

$\overline{q} = 1 - \overline{p}$

$\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}$ is the standard error.

NOTES

P-Value

P-value is the probability of observing the sample statistic as extreme as the test statistic. Here the test statistic is a Z-statistic. The Standard Normal Distribution table is used to calculate the probability associated with the computed Z-statistic.

Basic Assumptions for Z-Statistic

1. The two samples must be independent, that is, the two samples must be drawn from two different populations, so that the samples have no effect on each other.
2. The samples must be large enough to use a normal sampling distribution and the difference of two population proportions should follow normal distribution approximately.
3. The samples must be randomly selected.
4. In both the samples, the number of successes as well as number of failures should be at least 5.

Computation of Z-Statistic Formula

Steps to calculate Z-Statistic formula are as follows:

Step 1: Calculate sample proportions: $\hat{p}_1$ and $\hat{p}_2$.

Where,

$$\hat{p}_1 = \frac{x_1}{n_1} \text{ and } \hat{p}_2 = \frac{x_2}{n_2}$$

Step 2: Calculate the difference between the two sample proportions: $\hat{p}_1 - \hat{p}_2$.

Step 3: Calculate the pooled sample proportion: $\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$.

Step 4: Calculate the standard error: $\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}$.

Step 5: Divide the result of Step 2 by the result from Step 4. Then, the obtained test statistic is as follows:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}}$$

Example 5: Consider the following data:

	Sample 1	Sample 2
Rate	26.7%	29.0%
Total number	13200	13433

Find x_1, x_2, $\overline{p}$, $\overline{q}$.

NOTES

Solution: Calculate x_1 in the following manner:

$$x_1 = \frac{26.7 \times 13200}{100}$$
$$= 3524 \text{ (Rounded Up)}$$

Similarly,

$$x_2 = \frac{29 \times 13433}{100}$$
$$= 3896 \text{ (Rounded Up)}$$

Now, calculate the pooled sample estimate $\overline{p}$ as shown below:

$$\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$$
$$= \frac{3524 + 3896}{13200 + 13433}$$
$$= 0.2786$$

And,

$$\overline{q} = 1 - 0.2786$$
$$= 0.7214$$

Example 6: Use the dataset of Example 1 to find whether there is any significant treatment effect. Use $\alpha = 0.01$ for one tail.

Solution: Now, the null hypothesis H_0 is,

$H_0: p_1 = p_2$ (Original claim of equality)

The alternative hypothesis H_1 is,

$H_1: p_1 > p_2$

The significance level is $\alpha = 0.01$.

Now, calculate the value of 'Test Statistic' as follows:

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}}$$
$$= \frac{\left(\frac{3524}{13200} - \frac{3896}{13433}\right) - 0}{\sqrt{\frac{(0.2786)(0.7214)}{13200} + \frac{(0.2786)(0.7214)}{13433}}}$$
$$= -4.19$$

The negative standard normal table gives *P*-value that is equal to 0.0001.

The *P* value is less than the level of significance 0.01, so we reject the null hypothesis of $p_1 = p_2$.

NOTES

Confidence Intervals (CI) for Z-Statistics

Confidence interval contains a range of values that are centered around the sample statistic. For this method, the confidence interval is computed to estimate the difference between two population proportions $p_1 - p_2$.

CI uses the standard deviation based on the estimated values of the population proportions, but the hypothesis testing method involves standard deviation based on the assumption that the two population proportions are equal.

Formula of CI for Z-Statistics

The margin of error is given by:

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Where, $Z_{\alpha/2}$ is the critical value directly obtained from the standard normal table corresponding to confidence level.

The CI estimate is given by:

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

Example 7: Consider the following data:

	Sample 1	Sample 2
Number of samples	126	205
Total number	331	331

Construct a 95% confidence level.

Solution: With 95% confidence level, $Z_{\alpha/2} = 1.96$ from the standard normal table.

Calculate the margin of error E as follows:

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$= 1.96\sqrt{\frac{\left(\frac{126}{331}\right)\left(\frac{205}{331}\right)}{331} + \frac{\left(\frac{205}{331}\right)\left(\frac{126}{331}\right)}{331}}$$

$$= 0.0739$$

Hence, $E = 0.0739$

Where,

$\hat{p}_1 = \frac{x_1}{n_1}$ and $\hat{p}_2 = \frac{x_2}{n_2}$ (Sample proportion)

$\overline{q} = 1 - \overline{p}$

p_1 = Population proportion

n_1 = Size of the sample

x_1 = Number of successes in the sample

Construct the 95% confidence Interval as follows:

$$(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$$

$$(0.3806 - 0.619) - 0.0739 < (p_1 - p_2) < (0.3806 - 0.619) + 0.0739$$

$$-0.3123 < (p_1 - p_2) < -0.1645$$

NOTES

2.7 HYPOTHESIS TESTING FOR COMPARING A VARIANCE

Sometimes we want to check if the variance of a population is statistically different from some hypothesized value that we have in mind. For example, we might want to test if the variance of the heights of men is significantly different from zero. An investor may want to test if the variance of a stock is statistically different and some cutoff value that may reflect his risk awareness. His decision to include such a stock in his portfolio is dependent on whether the variance of the stock calculated from available data is statistically different from his risk level. For an economist, this statistical concept holds importance because he may want to judge the variance of income for individuals in a region and thus formulate policy schemes to reduce income inequalities if the variance is statistically different from 'the acceptable' level of income inequality.

For testing the assumption about population variance, we use the χ^2 test that is based on the χ^2-distribution, which is defined as follows:

$$\chi^2 = (n-1)\frac{s^2}{\sigma^2}$$

Where $n - 1$ is the sample variance, n is the sample size and σ^2 is the population variance that we are hypothesizing. Note that $n - 1$ is the degrees of freedom of this distribution. It is this test statistic that we shall be computing in this case.

The general procedure for testing hypothesis about population variances can be thus outlined as follows:

- Set up the null hypothesis, that may be $\sigma^2 = c$ where c is the variance of the population. This is what we want to test.
- Set up the alternative hypothesis which is complimentary to the null hypothesis. So, if the null is $\sigma^2 = c$, then the alternative can be $\sigma^2 \neq c$. It could also be $\sigma^2 > c$ or $\sigma^2 > c$.
- Choose the appropriate level of significance α.
- Compute the relevant test statistic, χ^2 in this case.
- Find out the critical value; say $\chi^2_{\alpha/2, n-1}$ where $n - 1$ is the degrees of freedom.
- Use the decision rule to accept or reject the null. If we are doing a two tailed test, the decision rule is, if $\chi^2 > \chi^2_{\alpha/2, n-1}$, then reject the null hypothesis, otherwise do not reject it. If we are doing a one tailed test, such that the alternative hypothesis is $\sigma^2 > c$, the two conditions arise:
  - If $\chi^2 > \chi^2_{\alpha/2, n-1}$, then reject the null hypothesis otherwise do not reject it.
  - If $\chi^2 > \chi^2_{\alpha/2, n-1}$, then reject the null hypothesis otherwise do not reject it.

NOTES

F statistic can be written as:

$$F = \frac{S_1^2}{S_2^2}$$

Where,

$$S_1^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n_1 - 1} = \frac{\sum_{i=1}^{n} X_i^2 - n_1\bar{X}}{n_1 - 1}$$

$$S_2^2 = \frac{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n_2 - 1} = \frac{\sum_{i=1}^{n} Y_i^2 - n_2\bar{Y}}{n_2 - 1}$$

F statistic follows F distribution with $(n_1 - 1)$ and $(n_2 - 2)$ degrees of freedom under H_0.

If $S_1^2 > S_2^2$, then F statistic is calculated as follows:

$$F = \frac{S_1^2}{S_2^2}$$

We test $S_1^2 > S_2^2$ with $(n_1 - 1),(n_2 - 2)$.

If $S_1^2 < S_2^2$, then F statistic is calculated as follows:

$$F = \frac{S_2^2}{S_1^2}$$

We test $S_1^2 < S_2^2$ with $(n_2 - 1),(n_1 - 2)$.

Basic Assumptions for *F*-Test

Few assumptions for F-Test are as follows:

1. Samples drawn from normal population.
2. Samples are randomly drawn.
3. The observations of samples are independent.
4. Measurement error is absent.

Critical Region

For F-test, alternative hypothesis H_1 is two tailed, hence; the critical region lies in both the sides of the distribution as shown in the Figure 2.1.

Hence,

If $F \geq F_{\alpha/2}$ or $F < F_{1-\alpha/2}$, we reject H_0 at level of significance $\alpha\%$, otherwise we accept H_0.

NOTES

X	X^2	Y	Y^2
4.8	23.04	4.5	20.25
5	25	4.2	17.64
5.1	26.01	4.3	18.49
4.7	22.09	4.8	23.04
4.2	17.64	4.1	16.81
4.6	21.16	-	-
5	25	-	-
$n_1 = 7$ $\bar{X} = 4.7714$ $\sum X^2 = 159.94$		$n_2 = 5$ $\bar{Y} = 4.38$ $\sum Y^2 = 96.23$	

Hence,

$$S_1^2 = \frac{\sum_{i=1}^{n} X_i^2 - n_1 \bar{X}}{n_1 - 1}$$

$$S_1^2 = \frac{159.94 - (7 \times 4.7714)}{7 - 1}$$

$$S_1^2 = 21.09$$

And, $$S_2^2 = \frac{\sum_{i=1}^{n} Y_i^2 - n_2 \bar{Y}}{n_2 - 1}$$

$$S_2^2 = \frac{96.23 - (5 \times 4.38)}{5 - 1}$$

$$S_2^2 = 18.5825$$

We calculate F-test as follows:

$$F = \frac{S_1^2}{S_2^2}$$

$$F = \frac{21.09}{18.5825}$$

$$F = 1.1349$$

Hence, test statistic is $F = 1.1349$.

Critical Value

Degrees of freedom = $(n_1 - 1), (n_2 - 1) = (6, 4)$ and level of significance $\alpha = 5\%$, hence, critical value is 6.1631 (from F-table).

NOTES

Conclusion

Test statistic $F = 1.1349$ is less than critical value 6.1631, hence, we fail to reject null hypothesis H_0. The two variances for the normal populations are equal.

Hypothesis Testing of Correlation Coefficients

To test the hypothesis of correlation coefficient, we use formulae as follows:

1. Simple Correlation Coefficient

In simple correlation coefficient, we test the relation between variable in the population, i.e., variables are independent or unrelated. Simple correlation coefficient is calculated as follows:

$$t = r_{yx}\sqrt{\frac{n-2}{1-r_{yx}^2}}$$

Where,

r_{yx} = The correlation coefficient of random samples.

n = Pairs of observation (X_i, Y_i) drawn from Bivariate normal population.

If $|t| \geq t_{\alpha/2}$, we reject null hypothesis H_0 at level of significance $\alpha\%$, otherwise we accept H_0.

2. Partial Correlation Coefficient

Suppose we have three variables X_1, X_2 and X_3 and n observations. The correlation between X_1 and X_2 with respect to X_3 on each X_1 and X_2 is called partial correlation. Formula of partial correlation coefficient is as follows:

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}}$$

Where,

$$r_{12} = \frac{\sum X_1X_2/n - \bar{X}_1\bar{X}_2}{\sqrt{\left(\frac{\sum X_1^{\,2}}{n} - \bar{X}_1^{\,2}\right)\left(\frac{\sum X_2^{\,2}}{n} - \bar{X}_2^{\,2}\right)}}$$

We can compute r_{13} and r_{23} same as r_{12}.

3. Multiple Correlation Coefficient

Suppose we have three variables X_1, X_2 and X_3 has n observations. Multiple correlation coefficient is a simple correlation coefficient between X_1 and joint effect of X_2 and X_3 on X_1. It is represented as follows:

NOTES

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{\left(1-r_{23}^2\right)}}$$

Where,

$$r_{12} = \frac{\sum X_1X_2/n - \bar{X}_1\bar{X}_2}{\sqrt{\left(\frac{\sum X_1^{\,2}}{n} - \bar{X}_1^{\,2}\right)\left(\frac{\sum X_2^{\,2}}{n} - \bar{X}_2^{\,2}\right)}}$$

We can compute r_{13} and r_{23} same as r_{12}.

Example 11: Given that $r_{12} = 0.6, r_{13} = 0.7$ and $r_{23} = 0.65$, determine $R_{1.23}$ and $R_{12.3}$.

Solution:

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{0.} \times r_{13} \times r_{23}}{1-r_{23}^2}}$$

$$= \sqrt{\frac{(0.6)^2 + (0.7)^2 - 2(0.6)(0.7)(0.65)}{1-(0.65)^2}}$$

$$= 0.73$$

And,

$$R_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1-r_{13}^2}\sqrt{1-r_{23}^2}}$$

$$= \frac{0.8-(0.65)(0.7)}{\sqrt{1-(0.65)^2}\sqrt{1-(0.7)^2}}$$

$$= 0.635$$

2.9 STATISTICAL TECHNIQUES OF HYPOTHESIS TESTING

Sometimes, we want to test the claims related to population using samples. For example, we have to test if the proportion of literacy is same or different allover India. It means we are interested in testing such claims and for that, we have already learnt the technique called hypothesis testing. Hypothesis testing involved two terms; a hypothesis of no difference is called null hypothesis denoted by H_0 and hypothesis complimentary to null hypothesis denoted by H_1.

Let us learn hypothesis testing of means and hypothesis testing of difference of means.

NOTES

2.9.1 Hypothesis Testing of Means

Here, we define different situations for hypothesis testing of means.

Testing Population Mean (μ) Equal to Specified Value (μ_0)

For Normal, Infinite Population

Let us assume that we have a random sample $X_1, X_2, \ldots X_n$ drawn from normal population with mean μ and known variance σ^2. Here we test if the populations mean μ is equal to specified mean μ_0.

Case 1: Two Tailed Test

In this case, hypothesis is as follows:

$H_0 : \mu = \mu_0$ against
$H_1 : \mu \neq \mu_0$

Test Statistic

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Under H_0 test statistic follow normal distribution.

Critical Region

For level of significance $\alpha\%$, we reject null hypothesis H_0 if $z > z_\alpha$ and $z < -z_\alpha$, otherwise we accept H_0.

Case 2: One Tailed Test

In this case hypothesis is as follows:

$H_0 : \mu = \mu_0$ against
$H_1 : \mu < \mu_0$ or $\mu > \mu_0$

Test Statistic

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

Under H_0 test statistic follow normal distribution.

Critical Region

If $H_1 : \mu > \mu_0$ then for level of significance $\alpha\%$, we reject H_0 if $z > z_\alpha$.

If $H_1 : \mu < \mu_0$ then for level of significance $\alpha\%$, we reject H_0 if $z < -z_\alpha$.

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For Normal, Finite Population

For normal finite population test statistic is as follows:

$$Z = \frac{\bar{X} - \mu}{\left(\sigma/\sqrt{n}\right) \times \left[\sqrt{(N-n)/(N-1)}\right]}$$

When population is non normal and sample size is sufficiently large then by using Central Limit Theorem (CLT), the test statistic follow normal distribution.

Example 12: For industrial process 150 random sample cans of edible oil are drawn from a population. Machine is set filling with 5 kg of edible oil with standard deviation of 0.5 kg and average weight 4.9 kg. Test hypothesis using level of significance 5%.

Solution: Let us state the null hypothesis H_0

$H_0 : \mu = 5$ against
$H_1 : \mu \neq 5$

Here,

n = Sample size = 150
$\bar{X}$ = Sample mean = 4.9
σ = Standard deviation=0.5

Test Statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$$

$$Z = \frac{4.9 - 5}{0.5/\sqrt{150}}$$

$$Z = -2.4494$$

Critical Value

At 5% level of significance, $Z_\alpha = 1.96$.

Conclusion

Test statistic $Z = -2.4494$ is more than 1.96 and less than -1.96, hence, we reject null hypothesis H_0.

Example 13: Given $N = 1000, n = 100, \bar{X} = 47, \mu = 50, \sigma = 12$ test hypothesis of means with 5% level of significance.

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Solution: Test Statistic

$$Z = \frac{\bar{X} - \mu}{\left(\sigma/\sqrt{n}\right) \times \left[\sqrt{(N-n)/(N-1)}\right]}$$

$$Z = \frac{47 - 50}{\left(12/\sqrt{100}\right) \times \left[\sqrt{(1000-100)/(1000-1)}\right]}$$

$$Z = -0.00264$$

With level of significance 5%, critical value is 1.96.

Conclusion

Test statistic is -0.00264 is less than 1.96 and more than -1.96, hence, we fail to reject null hypothesis H_0.

Testing Population Mean (μ) when Variance σ^2 is Unknown

For Normal, Infinite Population

Let us assume that we have a random sample $X_1, X_2, \ldots X_n$ drawn from normal population with mean μ and unknown variance σ^2. Here, we test population mean, μ, which is equal to specified mean μ_0 (Sample size may be small or large).

Case 1: Two Tailed Test

In this case hypothesis is as follows:

$H_0 : \mu = \mu_0$ against
$H_1 : \mu \neq \mu_0$

Test Statistic

Here, we replace σ^2 by S^2 in the formula of Z for mean equal to specified value to get

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Where,

$$S = \sqrt{\frac{\sum\left(X_i - \bar{X}\right)^2}{n-1}}$$

Under H_0, test statistic follows normal distribution.

Critical Region

For level of significance $\alpha\%$, we reject null hypothesis H_0 if $Z > Z_\alpha$ and $Z < -Z_\alpha$ otherwise we accept

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Case 2: One Tailed Test

In this case hypothesis is as follows:

$H_0 : \mu = \mu_0$ against
$H_1 : \mu > \mu_0$ or $\mu < \mu_0$

Test Statistic

$$Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

Where,

$$S = \sqrt{\frac{\sum\left(X_i - \bar{X}\right)^2}{n-1}}$$

Under H_0 test statistic follow normal distribution.

Critical Region

If $H_1 : \mu > \mu_0$, then for level of significance $\alpha\%$, we reject H_0 if $Z > Z_\alpha$.

If $H_1 : \mu < \mu_0$, then for level of significance $\alpha\%$, we reject H_0 if $Z < -Z_\alpha$.

For Normal, Finite Population

For normal finite population, test statistic is as follows:

$$Z = \frac{\bar{X} - \mu}{\left(S/\sqrt{n}\right) \times \left[\sqrt{(N-n)/(N-1)}\right]}$$

2.9.2 Hypothesis Testing for Difference between Means

We draw two independent samples from two populations. From the first population, we select a sample of size n_1 with mean μ_1 and variance σ_1. Let $\bar{X}_1$ be the mean of first sample. Similarly, from the second population, we select a sample of size n_2 with mean μ_2 and variance σ_2. Let $\bar{X}_2$ be the mean of the second sample.

Two Tailed Test

Let us state the null hypothesis H_0 for differences between means.

$H_0 : \mu_1 = \mu_2$ against
$H_1 : \mu_1 \neq \mu_2$

Test Statistic

Let $T = \bar{X}_1 - \bar{X}_2$

Hence, the expectation of T is,

NOTES

$$E(T) = E(\bar{X}_1 - \bar{X}_2)$$
$$= \mu_1 - \mu_2$$

And variance is,

$$Var(T) = V(\bar{X}_1 - \bar{X}_2)$$
$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Hence,

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Critical Region

For a level of significance to the tune of $\alpha\%$, we can reject the null hypothesis H_0, if $Z > Z_\alpha$ and $Z < -Z_\alpha$, otherwise we can accept H_0.

One Tailed Test

Let us state the null hypothesis H_0 for differences between means.

$H_0 : \mu_1 = \mu_2$ against
$H_1 : \mu_1 > \mu_2, \mu_1 < \mu_2$

Test Statistic

Let $T = \bar{X}_1 - \bar{X}_2$

Hence, the expectation of T is,

$$E(T) = E(\bar{X}_1 - \bar{X}_2)$$
$$= \mu_1 - \mu_2$$

And variance is,

$$Var(T) = V(\bar{X}_1 - \bar{X}_2)$$
$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

Hence,

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Critical Region

NOTES

If $H_1 : \mu_1 > \mu_2$, then for a level of significance $\alpha\%$, we reject H_0 if $Z > Z_\alpha$.

If $H_1 : \mu_1 < \mu_2$ then for a level of significance $\alpha\%$, we reject H_0 if $Z < -Z_\alpha$.

If σ_1^2 and σ_2^2 are unknown, replace them with S_1^2 and S_2^2.

Where,

$$S_1^2 = \frac{\sum(X_i - \bar{X}_1)}{n_1 - 1}$$

$$S_2^2 = \frac{\sum(Y_i - \bar{X}_1)}{n_2 - 1}$$

Example 14: For the first population sample of 100 screws that are drawn for this sample the mean is 1.9 cm and the variance is 0.2 cm. Samples of 100 screws are drawn from the second population with mean 1.95 cm and variance is 0.3 cm. Using 5% level of significance, test whether the mean for the two populations are same or different.

Solution: Here, the hypothesis is,

$H_0 : \mu_1 = \mu_2$ against
$H_1 : \mu_1 \neq \mu_2$

It is given that:

$\bar{X}_1 = 1.9, \bar{X}_2 = 1.95, n_1 = 100, n_2 = 100$
$\sigma_1^2 = 0.2$ and $\sigma_2^2 = 0.3$

The test statistic is,

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$Z = \frac{1.9 - 1.95}{\sqrt{\frac{0.2}{100} + \frac{0.3}{100}}}$$

$Z = -0.7071$

Critical Value

For 5% level of significance, the critical value is 1.96.

Conclusion

Test statistic $Z = -0.7071$ is less than 1.96 and more than -1.96. Hence we fail to reject the null hypothesis H_0. Therefore, the mean for different populations are same.

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Large Sample Drawn from the Same Population

In this situation, the test statistic changed as follows.

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

And when σ^2 is unknown, we use pooled variance.

$$\sigma^2 = \sqrt{\frac{n_1\left(\sigma_1^2 + D_1^2\right) + n_2\left(\sigma_2^2 + D_2^2\right)}{n_1 + n_2}}$$

Where

$$D_1 = \bar{X}_1 - \bar{X}_{1.2}$$

$$D_2 = \bar{X}_2 - \bar{X}_{1.2}$$

$$\bar{X}_{1.2} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

$\bar{X}_{1.2}$ = Combined mean of both samples.

Check Your Progress

7. What is the significance of hypothesis in research?
8. What does hypothesis testing mean?
9. Why is hypothesis tested?
10. What is Z-test?
11. Why researchers often use hypothesis testing for comparing two population parameters?
12. What does null hypothesis state?
13. What does confidence interval estimate?
14. What is P-value?
15. Which test is used to test the assumption about population variance?
16. What are the types of hypothesis testing?

ACTIVITY

1. Give an example of every type of hypothesis mentioned in the unit.
2. Prepare a hypothesis keeping in mind all the characteristics of a good hypothesis.

Did You Know

Etymologically, hypothesis is made up of two words *'hypo'* (less than) and *'thesis'*, which means less than a thesis. It is the presumptive statement of a proposition or a reasonable guess, based upon the available evidence, which the researcher seeks to prove through his study.

2.10 SUMMARY

- Educational research or research in education refers to a range of methods which helps individuals to evaluate different aspects of education. Educational researchers have accredited that the educational research must be performed in a rigorous and systematic way.
- Accomplishing or conducting any educational research is based on exploration, description, explanation or prediction of educational phenomenon using systematic data collection and analysis procedures.
- Research objectives can be defined as the systematic method of collecting data from selected sample or on the phenomenon under study and analysing the information in order to achieve the end result and to check the validity of the hypothesis that was formulated before the research was started.

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- A 'Research Problem' is considered as the initial step and the most significant prerequisite in the research process. It provides the groundwork or foundation of a research study because if the research problem is well prepared then the anticipated fine research study will pursue.
- Review of the related literature; besides, allowing the researcher to acquaint himself/herself with current knowledge in the field or area in which he/she is going to conduct his/her research also serves the specific purposes. The review of related literature enables the researcher to define the limits of his/her field. It also helps the researcher to delimit and define his/her problem.
- The first step in reviewing the related literature is identifying the material that is to be read and evaluated. The identification can be made through the use of primary and secondary sources available in the library. In the primary sources of information, the author reports his/her own work directly in the form of research articles, books, monographs, dissertations or theses. In secondary sources of information, the author compiles and summarizes the findings of the work done by others and gives interpretation of these findings.
- A microfiche is a sheet of film that contains microimages of a printed manuscript or book. Its development has been one of the most significant contributions to library and information services by providing economy and convenience of storing and distribution of long runs of scholarly materials.
- The card catalog is the index to the entire library collection. It lists the details of publications found in the library, with the exception of serially published periodicals. Generally, the card catalog contains author, title and subject cards arranged alphabetically.
- Library classification systems provide ingenious ways of systematizing the placement and location of books. Every system is based upon a methodology that is logical and orderly to the smallest detail. The two principal systems of library classification in the US are the 'Dewey Decimal' system and the 'Library of Congress' system.
- Encyclopedias serve as a store house of information, and usually contain well-rounded discussion and selected bibliographies that are prepared by specialists. Encyclopedias are arranged alphabetically by subject, and for each field of research, they present a critical evaluation and summary of the work that has been done.
- Abstracts include brief summaries of the contents of the research study or article. They serve as one of the most useful reference guides to the researcher and keep him/her abreast of the work being done in his own field and also in the related fields.
- The hypotheses provide a framework for organizing the related literature. If the study has several aspects or is investigating more than a single hypothesis, this is done separately for each facet of the study.
- The organization of the related literature involves recording the essential reference material and arranging it according to the proposed outline of the study.
- The basic information in the bibliography card should include name of the author with last name first; title of the book or article; name of the publication (for articles); name of the publisher; date of publication; volume number, page numbers; and library call number (for books). If some of this information is not available,

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the specified space should be left blank so that the missing information can be included immediately upon locating the references.

- A hypothesis is an approximate assumption that a researcher wants to test for its logical or empirical consequences. It can contain either a suggested explanation for a phenomenon or a proposal having deductive reasoning to suggest a possible interrelation between multiple phenomena. A deductive reasoning can be defined as a type of reasoning that can be derived from previously known facts.
- A hypothesis should be reliable and consistent with established and known facts. A hypothesis should be stated in the most simple and clear terms to make it understandable.
- The hypothesis provides the researcher with rational statements, consisting of elements expressed in a logical order of relationships, which seek to describe or to explain conditions or events that have not yet been confirmed by facts.
- Hypothesis testing means to determine whether or not the hypothesis is appropriate. This involves either accepting or rejecting a null hypothesis. The researcher has to pursue certain activities contained in the procedure of hypothesis.
- A hypothesis relates theory to observation and vice-versa. Hypotheses when tested are either rejected or accepted and help to infer the conclusion, which helps in theory building. Being a specific statement of prediction, a hypothesis describes in concrete (rather than theoretical) terms what you expect will happen in your study. Not all studies have hypotheses.
- Z-test is based on normal probability distribution. It is mostly used to judge the significance of mean as a statistical measure. This is the most frequently used test in research studies. It is generally used to compare the mean of a sample with the hypothesized mean of the population.
- T-test is based on *t*-distribution and is aptly considered to judge the significance of a sample mean or the difference between the means of two small samples when population variance is not known.
- χ^2 test is based on a chi-square distribution and is used for comparing a sample variance to a theoretical population variance.
- F-test is based on F-distribution and is also used to compare the variance of two independent samples. It is also used to compare the significance of multiple correlation coefficients.
- Researchers often use hypothesis testing for comparing two population parameters based on the corresponding statistics from each population. In this method two separate scores are to be obtained for each individual sample where the data in each sample set is related in some special way.
- The null hypothesis states that the population parameter is equal to the claimed value and is denoted by H_0. It is used for comparing statistics with the help of mean, μ. For example, if the average time taken by the student to complete his homework is 5 hours, then, $H_0: \mu = 5$.
- Confidence interval gives an estimate about a range of values that are centered around the sample statistic. It is calculated by using sample mean so that it can be confidently estimated that the value of the parameter lies in the interval of the known population.

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- The two estimated proportions may be different due to a difference in the populations. A hypothesis test helps in determining if there is a difference in the estimated proportions: $p_1 - p_2$ which reflects a difference in the population proportions.
- *P*-value is the probability of observing the sample statistic as extreme as the test statistic. Here the test statistic is a Z-statistic. The Standard Normal Distribution table is used to calculate the probability associated with the computed Z-statistic.
- For testing the assumption about population variance, we use the χ^2 test that is based on the χ^2-distribution.
- In simple correlation coefficient, we test the relation between variable in the population, i.e., variables are independent or unrelated.
- Hypothesis testing involved two terms; a hypothesis of no difference is called null hypothesis denoted by H_0, and hypothesis complimentary to null hypothesis denoted by H_1.
- Typically, the chi-square test is any statistical hypothesis test, in which the test statistics has a chi-square distribution when the null hypothesis is true. It is performed on different samples (of people) who are different enough in some characteristic or aspect of their behaviour that we can generalize from the samples selected.
- The population proportion is given by: $\hat{p}_1 = \frac{x_1}{n_1}$
- The pooled sample proportion $\overline{p}$ is given by: $\overline{p} = \frac{x_1 + x_2}{n_1 + n_2}$
- The formula for *Z*-statistic is given by: $Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}}$
- The estimated standard error is computed by: $\sqrt{\frac{\overline{pq}}{n_1} + \frac{\overline{pq}}{n_2}}$
- The null hypothesis specifies: $p_1 - p_2 = 0$
- The margin of error is given by $E = Z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$
- The confidence interval is computed by: $(\hat{p}_1 - \hat{p}_2) - E < (p_1 - p_2) < (\hat{p}_1 - \hat{p}_2) + E$

2.11 KEY TERMS

- **Review of related literature:** It enables the researcher to define the limits of his/her field and also helps the researcher to delimit and define his/her problem

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- **Card catalog:** It is the index to the entire library collection and lists the details of publications found in the library, with the exception of serially published periodicals; generally it contains author, title and subject cards arranged alphabetically
- **Encyclopedias:** These serve as a store house of information and usually contain well-rounded discussion and selected bibliographies that are prepared by specialists; the Encyclopedias are arranged alphabetically by subject and for each field of research, they present a critical evaluation and summary of the work that has been done
- **Hypothesis:** It is an approximate assumption that a researcher wants to test for its logical or empirical consequences and the deductive reasoning can be defined as a type of reasoning that can be derived from previously known facts
- **Z-test:** This kind of test is based on normal probability distribution and is mostly used to judge the significance of mean as a statistical measure and to compare the mean of a sample with the hypothesized mean of the population
- ***t*-test:** This test is based on *t*-distribution and is aptly considered to judge the significance of a sample mean or the difference between the means of two small samples when population variance is not known
- **χ^2:** This test is based on a chi-square distribution and is used for comparing a sample variance to a theoretical population variance
- ***F*-test:** This test is based on *F*-distribution and is also used to compare the variance of two independent samples. It is also used to compare the significance of multiple correlation coefficients
- **Null hypothesis:** It states that the population parameter is equal to the claimed value and is denoted by H_0 and is used for comparing statistics with the help of mean, μ
- ***P*-value:** It is the probability of observing the sample statistic as extreme as the test statistic where the test statistic is a Z-statistic

2.12 ANSWERS TO 'CHECK YOUR PROGRESS'

1. The first step in reviewing the related literature is identifying the material that is to be read and evaluated. The identification can be made through the use of primary and secondary sources available in the library. In the primary sources of information, the author reports his/her own work directly in the form of research articles, books, monographs, dissertations or theses. In secondary sources of information, the author compiles and summarizes the findings of the work done by others and gives interpretation of these findings.
2. A microfiche is a sheet of film that contains microimages of a printed manuscript or book. Its development has been one of the most significant contributions to library and information services by providing economy and convenience of storing and distribution of long runs of scholarly materials.
3. The card catalog is the index to the entire library collection. It lists the details of publications found in the library, with the exception of serially published periodicals. Generally, the card catalog contains author, title and subject cards arranged alphabetically.
4. Encyclopedias serve as a store house of information, and usually contain well-rounded discussion and selected bibliographies that are prepared by specialists. Encyclopedias are arranged alphabetically by subject and for each field of research, they present a critical evaluation and summary of the work that has been done.
5. Abstracts include brief summaries of the contents of the research study or article. They serve as one of the most useful reference guides to the researcher and keep him/her abreast of the work being done in his own field and also in the related fields.
6. The organization of the related literature involves recording the essential reference material and arranging it according to the proposed outline of the study.
7. In research, a hypothesis is an approximate assumption that a researcher wants to test for its logical or empirical consequences. It can contain either a suggested explanation for a phenomenon or a proposal having deductive reasoning to suggest a possible interrelation between multiple phenomena. A deductive reasoning can be defined as a type of reasoning that can be derived from previously known facts.
8. Hypothesis testing means to determine whether or not the hypothesis is appropriate. This involves either accepting or rejecting a null hypothesis. The researcher has to pursue certain activities contained in the procedure of hypothesis.
9. Hypothesis is tested to identify the errors occurred in the statements and concepts used in hypothesis. Hypothesis testing can be broadly divided into two types, which are parametric tests or standard tests of hypothesis and non-parametric tests or distribution-free tests of hypothesis.
10. Z-test is a kind of test that is based on normal probability distribution. It is mostly used to judge the significance of mean as a statistical measure. This is the most frequently used test in research studies. It is generally used to compare the mean of a sample with the hypothesized mean of the population.
11. Researchers often use hypothesis testing for comparing two population parameters based on the corresponding statistics from each population. For instance, researchers might want to check if the two populations have the same mean, which they can test with the help of hypothesis testing. In this method two separate scores are to be obtained for each individual sample where the data in each sample set is related in some special way.
12. The null hypothesis states that the population parameter is equal to the claimed value and is denoted by H_0. It is used for comparing statistics with the help of mean, μ. For example, if the average time taken by the student to complete his homework is 5 hours, then, H_0: $\mu = 5$.
13. Confidence interval gives an estimate about a range of values that are centered around the sample statistic. It is calculated by using sample mean so that it can be confidently estimated that the value of the parameter lies in the interval of the known population.
14. *P*-value is the probability of observing the sample statistic as extreme as the test statistic. Here the test statistic is a Z-Statistic. The Standard Normal Distribution table is used to calculate the probability associated with the computed Z-statistic.
15. For testing the assumption about population variance, we use the χ^2 test that is based on the χ^2-distribution.

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16. Hypothesis testing involved two terms; a hypothesis of no difference is called null hypothesis denoted by H_0 and hypothesis complimentary to null hypothesis denoted by H_1.

2.13 QUESTIONS AND EXERCISES

Short-Answer Questions

1. What is the importance of reviewing the related literature?
2. What are the six types of 'Readers' that are used for reading microfilms or microfiche?
3. What does library classification system provide?
4. Which periodicals a researcher must consult?
5. What basic information is included in the bibliography card?
6. List any three characteristics of a valid hypothesis.
7. When is hypothesis tested?
8. When is *F*-test used?
9. What are basic assumptions for Z-statistic?
10. Write down the definition of related sample *t*-test. Give two or three real life examples of this kind of study.
11. Mention two advantages as well as two disadvantages of comparing two related samples using *t*-test.
12. What are the basic assumptions underlying a hypothesis test with the related measures?
13. A related samples test lowers sample variability and increases the chances of obtaining significant results. Is this statement TRUE or FALSE?
15. Find out the statistic of *F*-test if (i) Sample 1: $n_1 = 10, S_1^2 = 3.5$ and (ii) Sample 2: $n_2 = 10, S_2^2 = 3.2$
16. Find out the statistic of *F*-test for $S_1^2 < S_2^2$

    Sample 1: $n_1 = 20, S_1^2 = 50$

    Sample 2: $n_2 = 20, S_2^2 = 100$
17. Find out the critical value for *F*-test with 5% level of significance.

    $n_1 = 25, n_2 = 20$

Long-Answer Questions

1. Discuss the significance of review of related literature in research specifying the purposes it serves.
2. Explain the Van Dalen's valuable guidelines for a researcher.
3. Explain the various references that are useful to a researcher in the field of education research.
4. Discuss the important directories of US and UK that are used by researchers.
5. Explain the ways in which the related literatures are organized.
6. Discuss the significance of hypothesis in research. Also discuss the characteristics of valid hypothesis and need for hypothesis formulation.
7. Discuss the significance of hypothesis formulation and its testing in a research process. Also explain the various sources of hypothesis and procedures of hypothesis testing.
8. Explain the various parametric and non-parametric tests of hypothesis giving suitable examples.
9. A sample of 36 packets is drawn from a population having average 811 gm. The population mean is 800 gm and the standard deviation is 16 gm. Test the hypothesis of mean at 1% level of significance.
10. A sample of 100 tins was examined and the average volume was found to be 1.94 liter.
11. A machine is designed for filling 2 liter oil, with a standard deviation of 0.1 liter. Test the hypothesis of mean using a 5% level of significance.
12. A random sample of 400 packet of rice is drawn from the first population having mean 1225 kg with standard deviation 42 kg. The second sample of 200 packet of rice is drawn from a second population having mean 1265 kg with standard deviation 60 kg. Test whether the two population have the same mean.
13. First sample has $n_1 = 400, \bar{X} = 170$ and $\sigma_1 = 6$. The second sample has $n_1 = 800, \bar{X} = 178$ and $\sigma_1 = 8$. Use hypothesis testing of difference between means.
14. Use one sample test concerning variance to find the Chi-square value.

    72 74 68 70 61 63 69 73 71

    Given $H_0 : \sigma^2 = 36$ against $H_1 : \sigma^2 \neq 36$
15. Use $\alpha = 0.05$, two-tailed to test whether there is a significant population proportion difference for the given data:

	Sample 1	Sample 2
Number of samples	136	215
Total number	500	555

16. Construct a 95% CI for the following data:

	Sample 1	Sample 2
Number of samples	228	229
Total number	456	500

2.14 FURTHER READING

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UNIT 3 METHODS OF EDUCATIONAL RESEARCH

Structure

3.0 INTRODUCTION

Educational research has changed dramatically since its introduction in the late 19th century. The earliest researcher on this subject was John Dewey, who worked to combine philosophy, psychology and education. Next came Judd whose preference for quantitative data collection and analysis and his emphasis on the scientific method with a particular focus on psychology, had an influential impact on educational research during the early 20th century. Another researcher was Thorndike, who did not encourage the collection

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of data for census purposes but rather for the production of statistics and precise measurements that could be analyzed. Thorndike became a very influential educational scholar and his approach to educational research became popular both in the United States and abroad.

In this unit, various techniques of educational research are discussed in detail. Each technique has its advantages and limitations, and it is up to the researcher to ascertain which technique would be best suited for a specific situation.

3.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Define qualitative and quantitative research
- Elaborate on the methods of educational research
- Identify the steps of research
- Discuss the design of experimental methods
- Explain the ethnographic method and the case study method

3.2 QUALITATIVE TECHNIQUE

Qualitative technique seeks to describe or explain psycho-social events from the point of view of people involved. It is not easy to provide explanations if there are no defined thoughts or ideas to start with. The researcher takes this into consideration and has an open mind while undertaking collection and analysis of data. In the qualitative technique, the data that is collected is usually derived from interviews that are conducted individually or in groups, participant or non-participant observations, notes in diaries and other documented studies or analysis.

The qualitative technique provides depth and detailed information for a research. Depth and detail emerge through direct questioning and careful descriptions. The extent of depth and detail will vary depending upon the nature and purpose of a particular study. The responses to open-ended questions in a questionnaire are detailed and comprehensive. These responses are neither systematic nor standardized. However, they permit the researcher to understand situations as seen and felt by him. Since the responses to open-ended questions are longer and detailed, they help the researcher to understand in depth the points of view of other people, their level of emotions, their characteristics, their attitudes and values, and their experiences.

The data gathered through participant observation or interviews are also descriptive in nature. These strategies are most comprehensive for understanding fully the complexities of a particular situation. Participant observation provides detailed first hand information to the researcher about a social event. Data gathered through participant observation generally includes: (i) description of the setting of the social situation; (ii) activities that take place in the setting; and (iii) description about people who participated in the activities and their extrinsic behaviour during the activities. The descriptions may be in the form of field notes, specifying some basic information pertaining to the place where the observation takes place, the persons present during the observation, nature of the settings, type and nature of various types of interactions and activities during the observation. The field notes taken during observation contain direct quotations from the people who participated in the observation as well as the observer's own feelings and reactions.

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It is not possible to find out what is in other individuals' minds while observing their extrinsic behaviour. Through participant observation, it is difficult for an observer to know the feelings, thoughts and intentions of others and also about the behaviours that took place in earlier situations. However, through open-ended/unstructured interviews, it is possible to find out what had happened earlier or what could not be observed during the participant observation. It provides a framework within which the researcher should be able to gather information from people conveniently and accurately. The information mostly pertains to a programme, the reaction of participants about the programme and the type of change the participants perceive in them after their involvement in the programme. The data are mostly in the form of responses to structured and unstructured questions put to the respondents by the researcher during an informal conversation. The responses are generally direct quotations from respondents in their own words and provide details about situations, events, people, experience, behaviours, values, customs, etc. The information gathered during or after an interview includes notes taken by an interviewer along with his detailed comments about what people say about their experiences, what they think and feel about the phenomena under study, and what they know about the phenomena.

Social sciences researchers use several qualitative methods by which they explore diverse issues. These are:

- ***Phenomenology*:** This is a philosophy or a method of inquiry that is used in education. Phenomenology entails the researcher trying to access individuals' 'life worlds'—their world of experiences. It is where consciousness exists.
- ***Ethnography*:** This is derived from anthropology and usually involves observation of participants and obtaining information through natural inquiry. It reveals a very comprehensive understanding of behaviours and interactions, which are set within specific social and cultural contexts.
- ***Narrative Analysis*:** This is a method that is deployed to study the structure and the content of the stories that people narrate about the important events in their lives. It helps us to understand the ways people arrive at meaning in their lives.
- ***Grounded Theory*:** This method was created to be used in sociology. It is based largely on interviews but may also rely on observation and documentary sources in order to develop new theoretical accounts of social situations and interactions. It provides a well defined approach for data analysis.

Organization of Qualitative Data

The amount of qualitative data gathered by using an open-ended questionnaire, participant observation and in-depth interviews is huge. It needs to be structured and categorized into specific patterns, types, and descriptive units to avoid any confusion. However, before any such classification, it is advisable to make some copies of all the data. Patton (1982) suggests that one should make four copies and store one complete copy in a safety deposit box as this data is priceless and unique. In case of loss of any data, it is impossible to recapture what was actually observed during participant observation or what exact conversation took place during an open-ended in-depth interview. The second copy should be used for further treatment of the data throughout. It is essential to ensure that the field notes taken during participant observation or open-ended interview are complete. The

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third copy may be used to fill the missing gaps, if any, identified during their scrutiny by the researcher. Additional notes can also be recorded in this copy. The organization of qualitative data involves a lot of cutting and pasting for which the fourth copy may be used.

Actual classification or organization can begin only after the copies are made. There are no formal or universal rules which a researcher may follow in organizing the data in various units, patterns, or categories. It requires a creative approach and a lot of perseverance to give a meaningful look to the data. The contents of field notes about the interview or observation may be read carefully by the researcher and he may note down his comments in the margins or attach small pieces of paper with his written notes using staples or tags. The next step is to arrange the data in topics for which the researcher may use abbreviations. The abbreviated topics are written either in the margins of the relevant data or on a slip of paper which may be attached with the relevant page. However, it may be noted that the researcher should read the information or data several times before it is indexed. The process of classifying or labelling various kinds of data and preparing a data index is the essential step in the organization of qualitative data. Sometimes, there are large amounts of data and it is not possible to develop a simple classification system. In such situations, the data is organized with the help of computers. Computers help in developing systematic and comprehensive classification schemes using code numbers for different categories and sub-categories. The computerized classification system permits the use of organized data by several groups of people over a long period of time. It permits easy cross-classification and cross comparison of descriptive narrations for complex analysis.

Analysis and Interpretation

Analysis of qualitative data means studying the organized material in order to discover inherent facts. These data are studied from as many angles as possible either to explore the new facts or to reinterpret already known existing facts. Content analysis, inductive analysis and logical analysis are mostly used in analysis of qualitative material.

Content Analysis

Content analysis is concerned with the classification, organization and comparison of the content of a document or communication. In the context of communication research, Berelson (1952) remarked that **content analysis** is a research technique for the objective, systematic, and quantitative description of the manifest content of communication.

Cartwright (1970) used the terms 'content analysis' and 'coding' interchangeably as both the processes involve objective, systematic, and quantitative description of any symbolic behaviour. Since content analysis is concerned with the classification, evaluation and comparison of the content of communication or document, it is sometimes referred to as 'documentary activity' or 'information analysis'. The communication may be in the form of responses to open-ended questionnaire, conversation as a result of interview, or description of an observed activity. It may also be in the form of official records (*census*, birth, accident, crime, school, institutional and personal records), judicial decisions, *laws*, budget and financial records, cumulative records, courses of study, content of *textbooks*, reference words, newspapers periodicals or journals, prospectus of various educational institutions or universities, etc., direct quotations, and notes from an interview.

Steps in Content Analysis

The steps involved in the process along with some issues related to this operation are:

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(i) Defining the Unit of Analysis: The material may be confined to single words, phrases, complete sentences, paragraphs, or to even larger amounts of materials such as articles or complete books. Either of these can be considered an entity whose specified characteristics can be determined and analysed. Hayman (1968) suggested that the unit should be comprehensive enough to provide meaning through some content at least, but small enough not to allow subjectivity in its use.

(ii) Specifying Variables and Categories: After the unit of analysis is defined, the researcher then analyses it with the purpose of creating data which is objective and can be reproduced. This can be deployed for scientific treatment as well as generalization beyond merely the actual set of symbolic material analysed. For converting this symbolic material into objective data, the 'variables' need to be explicit in terms of which descriptions are to be made. The variables are sometimes referred to as 'dimensions' or 'types of attributes'. A few examples of such variables are: number of words, percentages of personal pronouns, attitude towards privatization, attractive traits of teachers, degree of confidence in a friend, etc.

After the selection of a variable, viz., degree of confidence in a friend, there are many ways in which this variable may be broken down into categories as: (a) Unqualified confidence, (b) Qualified confidence, (c) Confidence and mistrust equally balanced, (d) Qualified mistrust, (e) Unqualified mistrust, (f) Question not asked by interviewer, (g) Question asked, but answer not classifiable in above categories. A second classification of categories of the same variable may be: (a) High, (b) Low, (c) Not classifiable in either. It may be pointed out that if two independent persons were to code the same material, one using the first set of categories and the other using the second, they would come out with different descriptions of the same material. Hence, explicit specification of the system of categories to be used with each variable is necessary for reproducible analysis.

(iii) Frequency, Direction and Intensity: Once the unit of analysis is defined, and the variables and their categories specified, the analysts will then classify units as well as well as the material to be analysed, according to their frequency, direction and intensity.

For frequency, the analyst merely counts the number of units which fall into each of his categories. Cartwright (1970) refers to it as an 'unit of enumeration'. The 'unit of enumeration' and 'recording unit' are not necessarily the same. But when the analyst merely counts the number of recording units which get a certain categorization, the recording unit is exactly the same as the enumeration unit. For example, in the analysis of public speech by an economist, it can be the number of times 'privatizations of higher education' may be employed as an 'argument' for a certain policy of government. In this case, an 'argument' is taken both as the recording unit and the enumeration unit. Another example would be of an analyst analysing an editorial on 'privatization of higher education' for its favourableness or unfavourableness. For purposes of quantification, he counts the number of column inches of the whole editorial. In this case, a column inch would be the unit of enumeration, whereas the editorial as a whole would be the recording unit, and hence the two units are not identical.

In certain situations, it is useful to further classify the units according to direction and intensity. Direction refers to whether the reference was favourable, unfavourable, or neutral. It might be pleasant/unpleasant, interesting/uninteresting, and threatening/non-threatening. Intensity indicates the emotional impact of the unit analysed. Is it large or small, and in what direction? Judging direction and intensity is more subjective than merely counting for frequency.

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(iv) Contingency Analysis: The contingency analysis takes into account the content within which the unit exists. A researcher should assess the unit in light of the entire communication so that its actual meaning is not lost.

(v) Sampling: One of the major and practical problems in content analysis is sampling. For the results to be generalized, the analysed unit must be representative of the total material with which the researcher is concerned. Invariably, a researcher undertakes the analysis of a specific content in order to reveal something about the universe of data than just those symbolic materials with which he deals.

(vi) Preparation of Content Analysis Outline: The following are six steps for arriving at a satisfactory content analysis outline:

Step 1: Specify Needed Data

In laying out a satisfactory analysis outline, a researcher should clearly specify the data that are required by him in the total research design so that he may face fewer difficulties in the long run. The specification of needed data is helpful in planning the final tables which the researcher may use at later stages of content analysis.

Step 2: Map out Plans for Tabulation

A researcher can avoid a number of problems if he makes clear plans for the tabulation of coded data. He should decide well in advance whether the coded data are to be tabulated by hand or computers.

Step 3: Lay out the Outline

The researcher should list the variables in terms of which the content is to be coded. For example, if the study pertains to analysing interviews, these variables will be used to classify not only various features of the answers to questions about the psychological make-up of the respondent but also such matters as his age, qualification, income, marital status, and other demographic and behavioural characteristics. In listing the variables to be included in the outline, the researcher should ensure that all the information needed has been entered in the computer via a Microsoft Excel spreadsheet. The outline should contain provision for coding the name of the study, the number of each enumeration unit (interview, issue of journal or newspaper, etc.), the name of coder, and any other relevant information.

Step 4: Fill in Categories for Each Variable

A researcher should use a classification which is thorough with mutually exclusive categories. Its categories are mutually exclusive if there is one and only one place to put an item within that system of categories. After defining all the categories in a structure, a manual of instructions should be prepared with operational definitions of the categories.

Step 5: Establish Procedure for Unitizing Material

It is essential for a researcher to establish a procedure for unitizing material. Specific working definitions to be used in the content analysis should be formulated in such a way that various coders can all use the same material in the same way. These definitions should be written down as part of the coding instructions.

Step 6: Try out the Analysis Outline and Unitizing Procedure

The analysis outline and unitizing procedure on a sample of material should be given a tryout in order to discover what modifications are needed. This trying out of the coding procedures is also used as training for those who are to be involved in the final coding. Coding procedures may include:

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- The coder must be a sensitive person, well differentiated with respect to symbolic materials. He must be able to detect slight differences of meaning but also to neglect differences that do not have an effect on the specific purpose. In other words, he must be able to make use of the genotypic categories required by the analysis outline. In most psycho-social research, this means that the coder must be acquainted with the concepts of social psychology. If the analysis outline requires only phenotypic categories or categories defined in terms of everyday usage, the coder may well be an intelligent layman. A reasonably good level of intelligence is the minimal requirement of any content analysis.
- The successful and meaningful use of a well-developed outline depends upon the selection of efficient coders and their effective training in the outline being used under good supervision so that the proper procedures of coding are followed. If the content is large, the process of coding involves the repetitive application of the analysis outline to the material. It demands the same operational definition of categories, the same frame of reference, the same degrees of differentiation, etc. throughout the entire coding operation. In such cases, a person who is easily satisfied with repetitive work should not be selected as a coder.
- After the selection of efficient coders, it is necessary to train them in the use of analysis outline so that they have a full understanding of the objectives of the project.
- At the stage of final coding, the coder can add new categories to some of the variables of the outline if he comes across a recording unit for which there is no category. However, the merit of adding a new category should be assessed by determining whether a new category would be meaningful within the rationale of the system of categories. It is also desirable to hold periodic discussions among the coders to ensure that the same frame of reference and operational definitions of categories are maintained throughout the coding period.

Validation of Qualitative Analysis

This section is concerned with the major strategies that are helpful for validating and verifying the result of qualitative analysis.

(i) Rival Explanations

Once the researcher has described the patterns and their explanations, it is important to look for rival or competing themes and explanations both inductively and logically. Inductively means looking for other ways of organizing the data that might lead to a different result. Logically, it involves searching for other logical possibilities and then finding if those possibilities can be supported by the data. However, it may be noted that when considering rival hypotheses and competing explanations, the strategy to be employed by the researcher is not one of attempting to disprove the alternatives, but to look for data that supports alternative explanations. In this strategy, the researcher should give due weightage to supporting evidence and look for the best 'fit' between data and analysis.

(ii) Negative Cases

The search for negative cases and instances that do not fit within the identified pattern and their understanding is also important in the verification and validation of the result.

(iii) Triangulation: Comparing Multiple Qualitative Data Sources

This type of triangulation involves comparing and cross-checking consistency of data derived by different means at different times using qualitative methods. It means:

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(a) Comparing observational data with interview data (b) Comparing observational data with questionnaire data (c) Comparing what participants of a programme say in public with what they say in private (d) Checking for the consistency the opinion of the participants abut a programme over a period of time, and (e) Comparing the opinions of the participants of a programme with others who were associated with the programme in one capacity or the other. The triangulation of data sources within qualitative methods will seldom lead to a single totally consistent picture. But such triangulations are helpful to study and understand when and why there are differences.

(a) Reconciling Qualitative and Quantitative data

This type of triangulation aims at comparing data collected through some kind of quantitative methods with data collected through same kind of qualitative methods. It is highly likely that qualitative methods and quantitative methods will eventually lead to different findings and not to a single and well integrated picture of the situation. It is because qualitative data are commonly used for 'generating hypotheses' or 'describing hypotheses' and quantitative data are used to 'analyse outcomes', or 'verify hypotheses'. However, in endorsing the notion of triangulation, Trend (1978), quoted by Patton (1980), maintains that it is useful to bring a variety of data and methods to bear on the same problem in order to reduce system basis by supplementing a qualitative approach with quantitative analysis.

(b) Multiple Perspectives from Multiple Observers

The aim of this kind of triangulation is to involve triangulating observers using several interviewers so as to reduce the potential bias or subjectivity as a result of observation by a single observer.

(iv) Design

The nature of research design and methodology also contribute to distortion in results. Sampling gives rise to three types of errors. These are: (a) distortion in the situations that were sampled for observation, (b) distortion introduced by the time periods during which observations took place, and (c) distortion because of selectivity in the people who were sampled either for observation or interviews. Thus, the researcher must be careful to limit results of his study to those situations, time periods, people, and contents for which the data are applicable.

(v) Evaluator Effects

The presence of the researcher during the observation or interview can distort the results of the study. The distortion may be due to: (a) reactions of programme participants and others associated with it to the presence of researcher, (b) changes in the researcher during the process of observation or interview, (c) biases of researcher, and (d) incompetence of the researcher. The presence of a researcher during an observation or interview may create a halo effect, and consequently the participants of the programme are motivated to 'show off'. Their deviation from normal behaviour will lead to distorted findings. It is desirable to undertake long-term observations for minimizing the halo effect. Researchers sometimes become personally involved with programme participants and therefore lose their sensitivity to the full range of events occurring during the process of observation of interview. A record of the changes in the researcher, field notes and conversation with the people associated with the programme are helpful to overcome evaluator effects.

The basic objective of qualitative analysis is to provide useful, meaningful and objective answers to the research questions of researchers, decision-makers and information users. To be useful, answers provided must relate directly to the questions that have been asked; to be meaningful the answers must be understandable and clearly presented; to be objective the researcher must demonstrate that answers (findings) will stand the test of careful scrutiny.

Check Your Progress

1. What is the function of the qualitative technique?
2. Name one way in which computers help in organizing data.

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3.3 QUANTITATIVE TECHNIQUE

In this technique, the data are studied from a variety of angles to explore the new facts. Analysis requires an observant, flexible and open-mind. It is worthwhile to prepare a plan of analysis before the actual collection of data. Good, Barr and Scates (1941) suggest four helpful modes to get started on analysing the gathered data:

(i) To think in terms of significant tables that the data permits.

(ii) To examine carefully the statement of the problem and earlier analysis and to study the original records of the data.

(iii) To get away from the data and to think about the problem in layman's terms, or to actually discuss that problem with others.

(iv) To attack the data by making various statistical calculations.

The exploratory modes may prove very helpful in the analysis of data of any research study and no seminaries, differences, trends and significant factors would go unnoticed by the researcher.

Statistical techniques have contributed greatly in gathering, organizing, analysing and interpreting numerical data. The processing of numerical data through statistics calls for competence in the use of statistical methods and for understanding of concepts that underline their development and their application. The researcher must know the strengths and the weaknesses of the statistical methods which he uses so that he may not mislead or be misled by such methods.

A discussion of two major areas of statistics, descriptive statistics and inferential statistics, is presented in some detail. The main purpose of such discussion is to help the researcher develop an understanding of statistical terminology, and the concepts necessary to study with understanding the literature dealing with educational research. It also serves to help the student develop competence and know-how to conduct investigations using simple types of statistical analysis.

Organization of Quantitative Data

Organization of data includes editing, classifying and tabulating quantitative information. Editing implies checking of the gathered raw data for accuracy, usefulness and completeness. Classification refers to dividing of the data into different categories, classes, groups or heads. For this, the researcher is guided by the nature of the problem, the hypotheses to be verified, or by the responses or characteristics of the samples he has selected. If the problem or hypotheses, for example, involved the difference between attitudes of men and women teachers towards co-education at the secondary school stage, the categories male and female serving in government and private aided schools would be clearly indicated. In some situations when the group is sufficiently homogeneous, no breakdown into categories or subgroups is necessary and it is desirable to describe the group as a whole. However, in the situations where the group is sufficiently heterogeneous, it is desirable to divide the group into homogeneous sub-groups or

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categories that have in common some distinctive attributes significant for the purpose of analysis.

(i) **Tabulation** is the process of transferring classified data from data-gathering tools to the tabular form in which they may be systematically examined. This process may be performed in a number of ways. In simple and less sophisticated types of research, hand-sorting and tabulating procedures are usually employed. More extensive and sophisticated investigations make use of the card-tabulating process.

- ***Hand-Sorting and Hand-Tabulation:*** Hand-sorting and hand-tabulation require careful planning. It includes the method of hand-sorting and recording on tabulation sheets in accurate mathematical terms by marking and counting frequency tallies for different items on which information is sought. The sorting of response sheets in case of psychological tests or scales in various categories must be done before the tabulation of responses. At times without proper planning, a researcher may waste his time and energy by tabulating the responses first before it occurs to him that it would be interesting to compare the responses of the various sub-groups comprising the sample under investigation. This process would require another handling of the response sheets, scales or opinionnaires and would involve reticulating the responses.
- ***Modern Computational Mechanical Aids:*** Modern computational mechanical aids are a boon to the modern researcher. They are used to save time and effort, and to minimize error during the organizing and analysis of research data. The increasing and popular use of these computational devices has advanced educational research in terms of both quality and quantity. The computational mechanical aids commonly used are 'calculators' and 'computers'.

(ii) **Calculators**

The most common computational mechanical device available to the researcher is the calculator. Its principal advantages are speed and accuracy in performing addition, subtraction, multiplication and division tasks. These operations are performed easily, merely by the pressing of the necessary keys to enter the data and another key to begin the desired operation. The calculations involving combinations of the fundamental operations can also be performed by setting their order as required in computational problem. The desk calculator provides reliable results. At times, improper input of the data or incorrect operations of the machine, or both, furnish erroneous result.

The electromechanical calculators perform the calculations by electrically operated mechanical devices. On the other hand, electronic calculators developed recently operate electronically and perform calculations without the use of mechanical counters and with greater speed. Some of these electronic calculators are capable of performing operations beyond the four basic operations of addition, subtraction, multiplication and division. These additional operations include interpolation, extraction of square roots and reciprocals.

The manufacturers of calculators usually provide instruction manuals with them for the use of their operators. These manuals provide directions even for simple operations. If they are studied carefully, the user may not face any difficulty in performing any operation.

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(iii) **Computers**

A computer system operates in accordance with specific instructions. Each instruction defines an operation to be performed. It also specifies the data, device, or mechanism needed to carry out the operation. These instructions are referred to as a program. A computer is useless until a programmer writes a detailed set of instructions to be loaded into its internal storage (memory) unit. There has been a revolution in the field of information technology in recent times. Simultaneously, programming of computers has made it easy to analyse data. Statistical Programming in Social Sciences (SPSS) is used by researchers to analyse and interpreting the results. Another program is EXCEL which can analyse large volumes of data.

The researcher should keep the following factors in view while interpreting the results:

- **Influence of Unstudied Factors:** In any type of educational research, the researcher is generally guided by the factors or variables which he has studied during the research process. He totally ignores the influence or effect of unstudied factors while interpreting the results of his study. To totally ignore the unstudied factors and ascribe the findings of the research to the occurrence of studied factors alone may be misinterpreting the actual truth, for the findings in any research are conditioned not by one or two but innumerable variables. It is truer in the case of experimental or causal-comparative type of research in which the researcher studied a very limited number of variables. For example, a researcher, finds that a group of eighth class students following programmed instruction material in social studies has performed better compared with another group of students of the same class taught through lecture method. If he were to ascribe the better achievement of the first group to the method alone and ignore the other possible determining factors like high general mental ability, high achievement motivation, better study habits, interest in the subject and better socio-economic conditions found among the higher achieving group, he will be misinterpreting the truth.
- **Selective Factors:** A researcher may hideously misrepresent the truth if he ignores the selective factors. This is more evident in the studies where a selective group is made the subject of investigation or where a particular factor is operating in the situation studied. For example, if a researcher finds that the boys of a particular tribe are mostly low in intelligence and then concludes, therefore, that the boys of all tribes have a low intelligence, is ignoring the fact that there exist outside the particular tribe, many tribal boys with average or high intelligence. Similarly, to find that in a particular secondary school, the number of the tenth class students failing in mathematics is greater than the number of students failing in other subjects and to conclude from this that mathematics is comparatively more difficult than other subjects of study is ignoring the fact that the students of mathematics did not receive good instruction in the subject.
- **Expected Results:** While interpreting the expected results, the researcher has to keep in mind that he does not go beyond his data support and that he does not forget the limitations of the study. The researcher has to be cautious in reporting all such factors which could account for the results.

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- **Negative Results:** Researchers, often, on arriving at results contrary to what they had hypothesized, jump to develop a sort of defiance mechanism by exaggerating all the factors that could have possibly vitiated the results. They often list shortcomings in terms of the use of inadequate tools or sample fluctuations. These things may be true and there is no harm in reporting all such factors which come in the way of making the study precise. Nevertheless, it is not always correct to get results that confirm hypotheses. Hypotheses arise from guesswork and cannot be accepted as correct without being tested for confirmation. Only after the research is completed is the researcher in a position to declare his results with certainty. When the results contradict the original hypothesis of the study, the interpretation and discussion of results should include the researcher's reconsideration of the original hypothesis in the light of his findings. At times, researchers are reluctant to discuss results that contradict the existing known facts. This attitude is not fair and is likely to impede the progress of research. It must be noted that hypotheses are tentative and results can differ from them.
- **Results when the Null Hypothesis is Retained:** A retained null hypothesis may occur when:
  (a) There is no relationship between the variables; or the experimental variable is not more effective than the control variable.
  (b) The null-hypothesis is false, but the internal validity problems of the data contaminated the investigation so badly that the actual relationship between variables could not be established.

Table 3.1 provides a comparison between the qualitative and quantitative techniques.

Table 3.1 Comparison of the Quantitative and Qualitative Techniques of Educational Research

Qualitative Technique	Quantitative Technique
'All research ultimately has a qualitative grounding' - Donald Campbell	'There's no such thing as qualitative data. Everything is either 1 or 0' - Fred Kerlinger
The aim is to compile a comprehensive and detailed description.	The aim is to classify and count features, and then construct statistical models in order to explain the observations.
The researcher will only have a rough idea in advance of what he is looking for.	The researcher knows exactly what he is looking for in advance.
This is usually recommended for use during the early phases of research projects.	Usually recommended during later phases of research projects.
The design of the analysis gets formulated progressively as the study unfolds.	All the aspects of the study are carefully highlighted up front, before data is collected.
The researcher himself is the data gathering instrument.	The researcher uses tools like questionnaires and equipment in order to collect numerical data.
Data takes the form of words, pictures or objects.	Data is in the form of numbers and statistics
Subjective — individuals. interpretation of events is important ,e.g., uses participant observation, in-depth interviews etc.	Objective — more scientific, seeks precise measurement and analysis of target concepts, e.g., uses surveys, questionnaires etc.
Qualitative data is more 'rich', time consuming, and less able to be generalized.	Quantitative data is more efficient, able to test hypotheses, but may miss contextual detail.
Researcher tends to become subjectively immersed in the subject matte..	Researcher tends to remain objectively separated from the subject matter.

Check Your Progress

3. Name two methods of tabulation.
4. Name two facts the researcher should keep in mind while interpreting results.

3.4 HISTORICAL RESEARCH

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History is a meaningful record of past events. It is a valid integrated account of social, cultural, economic and political forces that had operated simultaneously to produce historical events. It is not simply a chronological listing of events but an integrated assessment of the relationship between people, events, times and places. It is used to understand the present on the basis of what we know about past events and developments.

Historical research attempts to establish facts so as to arrive at a conclusion concerning past events. It is a process by which a researcher is able to come to a conclusion as to the likely truth of an event in the past by studying objects available for observation in the present. Historical research is a dynamic account of the past, which seeks to interpret past events in order to identify the nuances, personalities and ideas that have had an influence on these events.

According to Kerlinger: 'Historical research is the critical investigation of events, developments, and experience of the past, the careful weighing of the evidence of the validity of sources of information of the past, and the interpretation of the weighed evidence.'

According to Gay (1981): 'Historical research is the systematic collection and objective evaluation of data related to past occurrences in order to test hypotheses concerning causes, effects, or trends of those events which may help to explain present events and anticipate future events.'

Therefore, it can be concluded that true historical research is a process of reconstructing the past through systematically and objectively collecting, evaluating, verifying and synthesizing evidence relating to the past events to establish facts and defensible conclusions, often in relation to particular hypotheses (if appropriate), to arrive at a scholarly account of what happened in the past.

3.4.1 Nature and Value of Historical Research

The main aim of historical research is to obtain an exact account of the past to gain a clearer view of the present. Historical research tries to create facts to arrive at conclusions concerning past events. It is usually accompanied by an interpretation of these events at the end of their relevance to present circumstances and what might happen in the future. This knowledge enables us, at least partially, to predict and control our future existence.

- Historical research as many other types of research, includes the delimitation of a problem, formulating hypothesis or tentative generalization, gathering and analysing data, and arriving at conclusions or generalizations, based upon deductive-inductive reasoning. However, the historian faces greater difficulties than researchers in any field.
- The job of the historian becomes more complicated when he derives truth from historical evidence. The major difficulty lies in the fact that the data on which historical facts are based cannot be substantiated and is relatively inadequate.
- It may be difficult to determine the date of occurrence of a certain historical event partly because of changes brought in the system of calendar and partly due to incomplete information. The historian lacks control over both treatment and measurement of data.

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Historical research has great value in the field of educational research because it is necessary to know and understand educational achievements and trends of the past in order to gain perspective on present and future direction. Knight (1943), Good, Barr and Scates (1941) have given the following analysis of the value of historical research:

Knowledge of the history of schools and other education agencies is an important part of the professional training of the teacher or school administrator.

(i) Much of the school work is traditional. The nature of work is restrictive and tends to foster prejudices in favour of familiar methods. The history of education is the 'sovereign solvent' of educational prejudices.

(ii) The history of education enables the educational worker to delete facts and drills in whatever form they appear, and it serves as a necessary preliminary to educational reforms.

(iii) Only in light of their origin and growth can the numerous educational problems of the present be viewed sympathetically and without bias by the teacher, administrator or public.

(iv) The history of education shows how the functions of social institutions shift and how the support and control of education have changed.

(v) It inspires respect for and reverence for great teachers.

The history of education serves to present the educational ideas and standards of other times, and it enables social worker to avoid mistakes of the past.

3.4.2 Types of Historical Research

- **Legal Research:** It is of immense value and interest to educational administrators. It seeks to study the legal basis of educational institutions run by different religions and castes, central and state schools, school finance, etc. But this type of researches need special training in the field of law. Anybody without this training is not competent to do this type of research.
- **Biographic Research:** It aims at determining and presenting truthfully the important facts about the life, character and achievements of famous and important educators, e.g., contributions of Dr. Radha Krishnan, Prof. B.K. Passi, Prof. L.C. Singh, etc.
- **Studying the History of Ideas:** This involves the tracing of major philosophical or scientific thoughts from their origins through their different stages of development. It aims at tracing changes in popular thought and attitudes over a given period of time.
- **Studying the History of Institutions and Organizations:** While studying such history, the same general method applies as for the study of a University. For example, one may study the history of the growth and development of National Law Universities, IIMs, etc.

3.4.3 Advantages and Disadvantages of Historical Research

The advantages of historical research are:

- The researcher is not physically involved in the situation under study.
- No danger of experimenter-subject interaction.
- Documents are located by the researcher, data is gathered, and conclusions are drawn out of sight.
- Historical method is much more synthetic and eclectic in its approach than other research methods, using concepts and conclusions from many other disciplines to explore the historical record and to test the conclusions arrived at by other methodologies.
- Perhaps more than any other research method, historical research provides librarians with a context. It helps to establish the context in which librarians carry out their work. Understanding the context can enable them to fulfil their functions in society.
- It provides evidence of ongoing trends and problems.
- It provides a comprehensive picture of historical trends.
- It uses existing information.

Historical research suffers from several limitations, some are natural due to the very nature of the subject and others extraneous to it and concerning the capabilities of the researcher.

- Good historical research is not lazy. It is slow, painstaking and exacting. An average researcher finds it difficult to cope with these requirements.
- Historical research requires a high level of knowledge, language skills and art of writing on the part of the researcher.
- Historical research requires a great commitment to methodological scholarly activity.
- Sources of data in historical researches are not available for the direct use of the researcher and historical evidence is, by and large, incomplete.
- Interpretation of data is very complex.
- Through historical research, it is difficult to predict the future.
- Scientific method cannot be applied to historical evidence.
- Modern electronic aids (like computers) have not contributed much towards historical research.
- It is not possible to construct 'historical laws' and 'historical theories'.
- Man is more concerned with the present and future and has a tendency to ignore the past.
- Time-consuming.
- Resources are scarce.
- Data can be contradictory.
- The research may not be conclusive.
- Gaps in data cannot be filled as there are no additional sources of information.
- Gaps in data cannot be filled as there are no additional sources of information.

A historian can generalize but not predict or anticipate, can take precautions but not control; can talk of possibilities but not probabilities.

3.4.4 Process of Historical Research

Historical research includes the delimitation of a problem, formulating hypothesis or tentative generalizations, gathering and analysing data, and arriving at conclusions or generalizations based upon deductive-inductive reasoning. However, according to Ary, *et al.*, (1972) the historian lacks control over both treatment and measurement of data. He has relatively little control over sampling and he has no opportunity for replication. As historical data is the closed class of data located along a fixed temporal locus, the

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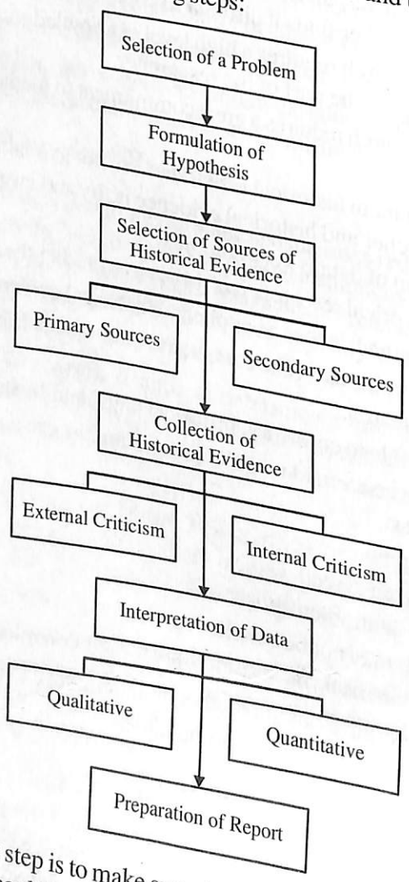
historian has no choice of sampling his data. He is supposed to include every type of data that comes his way. Historical research is not based upon experimentation, but upon reports of observation, which cannot be authenticated. The historian handles data which are mainly traces of past events in the form of various types of documents, relics, records and artefacts, which have a direct or indirect impact on the event under study.

In deriving the truth from historical evidence, the major difficulty lies in the fact that the data on which historical research is based are relatively inadequate.

It may be difficult to determine the data of occurrence of a certain historical event partly because of changes brought out in the system of calendar and partly due to incomplete information.

Historical research attempts to establish facts to arrive at a conclusion concerning the past events.

***Steps in Historical Research*:** The steps involved in undertaking a historical research are not different from other forms of research. But the nature of the subject matter presents a researcher with some peculiar standards and techniques. In general, historical research involves the following steps:



Step1: The first step is to make sure the subject falls in the area of the history of education. One topic could be the study of the various educational systems and how they have changed with the passing of time. On the other hand, studying 'contributions of education' as a component of national history can be of interest to a researcher. The researcher may be interested in a historical investigation of those aspects of education

that have not been touched upon by any studies yet. Moreover, the researcher may be interested in re-examining the validity of current interpretations of certain historical problems which have already been studied.

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Step 2: This necessitates that a thought is given to the various aspects of the problem and various dimensions of the problem are identified. Hypothesis also needs to be formulated. The hypothesis in historical research may not be able to be tested, they are written as explicit statements that tentatively explain the occurrence of events and conditions. While formatting a hypothesis, a researcher may formulate questions that are most appropriate for the past events he is investigating. Research is then directed towards seeking answers to these questions with the help of the evidence.

Step 3: Collection of historical evidence involves two sub-steps.

(i) Selection of sources of historical evidence

(ii) Cutting out the historical evidence from them

Historical evidence is hidden broadly in two types of historical sources and is useful to the researcher in many respects. The primary sources, however, are closest to the researcher's heart and kept at the highest pedestal.

Step 4: Historical evidence collected must be truthful; hence for establishing the validity of these sources, the dual processes of external and internal criticism are used. External criticism is undertaken to establish the authenticity of the documents of source, correctness of author or builder, data or period to which it belongs, etc. Internal criticism is done to judge the correctness of the contents of sources.

Step 5: Though statistical testing of hypothesis is not possible, the relationship among various facts still needs to be established, and synthesis and integration of the facts in terms of generalization needs to be done.

Three strategies are used to analyse educational concepts. These are:

(i) **Generic Analysis:** Identifies the essential meanings of a concept and isolates those elements that distinguished the concept from other words.

(ii) **Differential Analysis:** Is used when a concept means to have more than one standard meaning and the basis for differentiating between meanings is unclear.

(iii) **Conditions Analysis:** Involves identification of the context condition in which it can be safely said that the concept was present. Such conditions are rejected, revised and new conditions added.

In this type of investigation, the researcher must be very cautious while dealing with the 'cause and effect' relationship.

Step 6: The final stage of the study is the preparation of a systematic and comprehensive report. It is not just the data which is of significance in such a study. Of prime relevance are the ideas and insights of the researcher, particularly his assessment of the interaction between the data and the ideas that are used to explain the data.

3.4.5 Sources of Data in Historical Research

In this section, we discuss the three sources of data in historical research: (i) primary sources, (ii) secondary sources, and (iii) tertiary sources.

(i) Primary Sources: Primary sources are eye witness accounts and are the only firm basis of historical enquiry. Good, Barr and Scates (1941) have called them the 'first witness to a fact'.

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Direct observation, and reporting or recording of the same, comprise primary sources of data. These provide first-hand information about events that have occurred in the past. Some of the main types of primary sources are:

- Verbal narratives written by the participants or observers. These may take various forms such as official minutes or records, biographies, letters, contracts, deeds, wills, certificates, magazines or newspaper accounts, maps, pictures, books, etc.
- Personal primary sources which are typically a person's observation of events in which he has participated.
- Physical artefacts like museum collections, artefacts in historical spots such as remains or relics, as well as various other types of institutions.
- Mechanical artefacts represent information that is observed through the medium of non-natural items like photographs, films, and audio cassettes.

(ii) Secondary Sources: Secondary sources of data basically refer to information that is obtained second-hand. For instance, the person from whom information is obtained neither participated nor witnessed the events. Some types of secondary sources are magazine and newspaper articles, interviews referred to in the articles, research papers, research reports, documentaries, etc.

While carrying out historical studies, primary sources of data have highest credibility when they are used to authenticate presented facts. However, second-hand information that is available, should also be considered in order to develop a more holistic view.

Advantages of Secondary Sources

(a) They may acquaint a researcher with major theoretical issues in his field and to the work that has been done in the area of study.

(b) They may suggest possible solutions of the problem and working hypotheses and may introduce the researcher to important primary sources.

Some type of data may be primary sources for some purposes and secondary sources for another. For example, a high school textbook in Indian history will be ordinarily classified as secondary source, but the book would be a primary source of data if one were making a study of the changing emphasis on national integration in high school history textbooks.

(iii) Tertiary Sources: These sources include bibliographies, catalogues and indexes that guide a researcher to primary and secondary sources.

3.4.6 Evaluation of Data

The main feature of historical research is the evaluation of historical data. The backbone of historiography is the authenticity of data collected through different sources. Even when the data are collected through different sources, doubts can be raised about their validity, reliability and relevance. The process of judging validity, reliability and relevance of data is carried out through two devices viz., (a) external criticism and (b) internal criticism.

(a) External Criticism

External criticism is also known as lower criticism. It involves testing the sources of data for integrity, i.e., every researcher must test the information received to ensure that any

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source of data is in fact what it seems to be. External criticism helps to determine whether it is what appears or claims to be and whether it reads true to the original so as to save the researcher from being the victim of fraud. On the whole, the general criteria followed for such criticism depends on:

- A good chronological sense, a versatile intellect, common sense, an intelligent understanding of human behaviour, and plenty of patience and persistence on the part of the researcher.
- Recent validation of the quality of the source.
- A good track record of the source.

This information may be found in relevant literature. Thereafter, these literary sources can be verified for genuineness of content by verifying signatures, handwriting, writing styles, language, etc. Further, material sources of information can be verified through physical and chemical tests on the ink, paint, paper, cloth, metal, wood, etc.

(b) Internal Criticism

After the integrity of the data sources are established, the actual data content is subject to verification—this process is known as the internal criticism of the data. It is also called higher criticism which is concerned with the validity, truthfulness, or worth of the content of document.

At the outset, the information obtained through a particular source is examined for internal consistency. The higher the internal consistency, the greater the accuracy. The researcher should establish the literal as well as the real meaning of the content within its historical context.

This is followed by an evaluation of the external consistency of the data. This is important because, although the authorship of a report is established, the report may comprise distorted pictures of the past. For verifying that the content is accurate, the researcher should. firstly compare the information received through two independent sources, and secondly match new information obtained with the information already on hand which has been tested for reliability. Fox (1969) suggested three major principles that need to be followed in order to establish external consistency of the data: (i) data from two independent sources to be matched for consistency, (ii) data must have been obtained from at least one independent primary source, and (iii) data should not be gathered from a source that has a track record of providing contradictory information. It is recommended that the researcher apply his professional knowledge and judgment to make a final evaluation in case it is not possible to find matching information from two comparable sources.

The following series of questions have been listed by Good, Barr and Scates (1941) to guide a researcher in the process of external and internal criticism of historical data:

- Who was the author, not merely what his name was but what his personality, character and position were like, etc.?
- What were his general qualifications as a reporter — alertness, character and bias?
- What were his special qualifications as a reporter of the matters here treated?
- How was he interested in the events related?
- Under what circumstances was he observing the events?

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- Had he the necessary general and technical knowledge for learning and reporting the events?
- How soon after the events was the document written?
- How was the document written, from memory, after consultation with others after checking the facts, or by combining earlier trial drafts?
- How is the document related to other documents?
- Is the document an original source—wholly or in part? If the latter, what parts are original, what borrowed? How credible are the borrowed materials? How accurately is the borrowing done? How is the borrowed material changed and used?

Perpetually, the researcher needs answers for all these questions and, therefore, he has to depend, somewhat, upon evidence he can no longer verify. At times, he will have to rely on the inferences based upon logical deductions in order to bridge the gaps in the information.

3.4.7 Purpose of Historical Research

Historical research is carried out to serve the following purposes:

- **To discover the context of an organizational situation:** In order to explore and explain the past, a historian aims to seek the context of an organization/a movement/ the situation being studied.
- **To answer questions about the past:** There are many questions about the past to which we would like to find answers. Knowing the answers can enable us to develop an understanding of past events.
- **To study the relationship of cause and effect:** There is a cause and effect relationship between two events. A historian would like to determine such a relationship.
- **To study the relationship between the past and the present:** The past can often help us get a better perspective about current events. Thus, a researcher aims to identify the relationship between the past and the present, whereby we can get a clear perspective of the present.
- **To reorganize the past:** A historian reconstructs the past systematically and objectively, reaching conclusions that can be defended.
- **To discover unknown events:** There are some historical events that could have occurred in the past that are not known. A historian seeks to discover these unknown events.
- **To understand significance of events:** There may be significant events that could have been responsible for shaping the organization/movement/situation/ individual being studied by a historian.
- **To record and evaluate the accomplishments of individuals, institutions and other kinds of organizations:** Historians are greatly interested in recording and evaluating the accomplishments of leading individuals and different kinds of organizations including institutions and agencies as these influence historical events.
- **To provide understanding of the immediate phenomenon of concern:** A researcher may be investigating a phenomenon. Historical perspective can enable him to get a good understanding of the immediate phenomenon of concern.

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The students and teachers in the discipline of education can develop the following competencies through a study of history and conducting historical research:

(i) Undertaking of dynamics of educational change
(ii) Increased undertaking of the relationship between education and the culture in which it operates
(iii) Increased understanding of contemporary educational problem
(iv) Understanding the functions and limitations of historical evidence in analysing educational problems
(v) Development of elementary ability in locating, analysing and appraising historical evidence
(vi) Development of a sense of dignity and responsibility of the teaching profession

3.4.8 Problems in Historical Research

- **Amount of data:** Often, it is difficult to decide as to how much data is sufficient to reach meaningful conclusions.
- **Selection of data:** A historian must avoid improper or faulty selection of data which may be the result of relying too heavily on some data, ignoring other data, etc. This can result in a bias in the study.
- **Evaluation of historical data and their sources:** Inadequate evaluation of data and their sources can lead to misleading results.
- **Synthesis of data into a narrative account:** Due to the very nature of historical research, it becomes most fruitful, if a researcher is able to successfully synthesize or integrate the facts into meaningful generalizations. Thus, a failure on the part of a researcher to interpret data adequately is considered a serious setback.

There are four problems at the stage of synthesis and in report preparation as given below:

(i) The ability to establish causation from interrelated events is the first problem. It is incorrect to infer that one event caused the other just because they occurred simultaneously.
(ii) The second problem is to accurately define the keywords and terms such that ambiguity is avoided and the correct connotation is established.
(iii) Distinguishing between evidence indicating how people should behave *vs* how they did behave is the third problem.
(iv) The fourth problem involves distinguishing between the intent and the outcome. This means that educational historians ensure that the consequences of some activity or policy were actually the intended consequences.

Historical synthesis and interpretation are considered an art, which is subjective in nature. This raises a serious problem of subjectivity. 'Historical synthesis is necessarily a highly subjective art. It involves the intuitive perception of patterns and relationships in the complex web of events, as well as the art of narrative writing. Explanations and judgments may be called for, that will involve the historian's own personality, experience, assumptions, and moral values. Inevitably there are personal differences among historians in this respect, and prolonged academic disputes among historians of different schools or nationalities have arisen over practically every event. The initial reduction of complex events of the recent past to comprehensible pattern is particularly difficult and

subjective…'. Since the very process of writing a narrative is a human one, therefore, total objectivity is almost impossible. As a consequence, bias and distorting of facts to fit preconceived notions or ideas are not unusual. It may also be kept in mind that historical conclusions are conditioned by place, time and the author. In order to overcome some of these inherent weaknesses, the writer must clearly indicate the underlying assumptions in his approach. In case he belongs to a particular school of thought, the same must be stated clearly.

3.5 DESCRIPTIVE SURVEY RESEARCH

One of the basic functions of research is to understand the nature of the problem and to identify various factors that contribute towards the occurrence of events. Descriptive research surveys are prominent methods of conducting exploratory research in the area of social sciences.

The terminology of descriptive survey research is designed such that it compels the researcher to obtain relevant and accurate information about the current status of the phenomena and to draw out valid generalizations from the established facts without any interference or control over the situation. Such type of studies are not limited to fact finding and may often lead to the development of key principles of knowledge, as well as solutions for major problems that concern local, state, national and international issues.

According to Best, 'descriptive survey describes and interprets what is concerned with conditions and relationships that exist; practices that prevail; beliefs, points of view or attitudes that are held; processes that are going on, effects that are being felt, or trends that are developing'.

Descriptive surveys can tell us about what exists at present by determining the nature and degree of existing conditions. This is the most popular and most widely used research method in education.

Descriptive research, thus, is concerned with the present and attempts to determine the status of the phenomenon under investigation.

Descriptive surveys collect and provide three types of information:

- What exists, with respect to variables or conditions in a situation.
- What we want by identifying standards or norms with which to compare the present conditions or what experts consider to be desirable.
- How to achieve a goal, by exploring possible ways and means on the basis of the experience of others or opinions of experts.

In simple words, the purpose of descriptive research is to:

- identify present conditions and point to present needs
- study the immediate status of a phenomenon
- find facts
- examine the relationship of traits and characteristics (trends and patterns)

3.5.1 Characteristics of Descriptive Survey

The characteristics of descriptive survey are as follows:

- They are non-experimental, for they deal with relationships between non-manipulated variables in a natural, rather than an artificial setting.
- They may involve hypothesis formulation and testing.
- They use the logical method of indicative-deductive reasoning to arrive at generalizations.
- They are characterized by disciplined inquiry, requiring expertise, objectivity and careful execution.
- They often employ a method of randomization so that error may be estimated when inferring population characteristics from observation of samples.
- The variables and procedures are described as accurately and thoroughly as possible so that the study can be replicated by other researches.
- They use techniques of observation, description and analysis.
- Descriptive surveys ask questions about the nature, incident or distribution of educational variables.
- Descriptive surveys are a primitive type of research and do not aspire to develop an organized body of scientific laws.
- Descriptive surveys investigate phenomena in their natural setting. Their purpose is both immediate and long range.

3.5.2 Steps of Descriptive Survey

The steps of descriptive survey are as follows:

(i) **Statement of the Problem:** This statement identifies the variable to be involved in the study and specifies whether the study is merely seeking to determine the status of these variables or whether it will also investigate relationships between the variables.

(ii) **Identification of Information Needed to Solve the Problem:** The research lists the information to be collected, states whether this information is of a qualitative or a quantitative nature and identifies the form the information will take.

(iii) **Selection or Development of Instrument for Gathering Data:** Questionnaires, interviews, tests and scales of various types are the most frequently used instruments for descriptive research. Already existing tools can be used or new tools can be devised.

(iv) **Identification of Target Population and Determination of Any Necessary Sampling Procedure:** The researches determine the group about which information is being sought—an adequate sample is selected that will adequately represent the population.

(v) **Design of the Procedure for Data Collection:** The researcher lays out the practical schedule obtaining the sample and using the instrument.

(vi) **Collection of Data:** This involves detailed planning, including getting permission from the school or persons concerned, time tabling, etc. This time table helps to collect data in an organized manner.

(vii) **Analysis of Data:** Statistical techniques to be used will have to be planned. The researcher should be familiar with the different categories of measurement scales and use the most suitable so the appropriate tests can be carried out.

(viii) **Preparation of the Report:** The researcher decides the final format of his descriptive study.

Check Your Progress

5. Define 'historical research'.
6. What are the various sources of historical data?
7. What are two types of historical research?
8. Give two advantages of historical research.

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Data in Descriptive Research

Descriptive research is designed to obtain pertinent and precise information concerning the current status of phenomena, and wherever possible to draw valid general conclusions from the facts discovered. They may often result in the formulation of important principles of knowledge and solution of significant problems concerning local, state, national, and international issues. They involve measurement, classification, analysis, comparison and interpretation. They collect and provide three types of data:

- What exists with respect to variables or conditions in a situation.
- What we want by identifying standards or norms with which to compare the present conditions or what experts consider desirable.
- How to achieve gaps by exploring possible ways and means on the bases of the experience of others or the opinion of experts.

3.5.3 Values of Descriptive Research in Education

Due to the following reasons, the descriptive method has been the most popular and widely used research method in education.

- **Description in Present:** It helps to explain educational phenomena in terms of the conditions or relationships that exists, opinions that are held by the students, teachers, parents and experts processors that are going on , effects that are evident or trends that are developing.
- **Easy and Direct:** Due to the apparent simplicity and directness of the method, a researcher can gather information in terms of individual's opinion about some issue, by a simple questionnaire.
- **Only Means:** At times, descriptive surveys are the only means through which opinions, attitudes, suggestions for important educational practices and instruction, and other data can be obtained.
- **Problem Solving:** They are of immense value in solving problems about children, school organizations, supervision and administration, curriculum, teaching methods and evaluation.
- **Keeping Abreast with Changes:** The problems in education directly involve people and the situations precipitating these problems are constantly in a state of change. To keep abreast of changes, descriptive surveys conducted at different intervals with representative groups of people will be immensely helpful.
- **Development of Data Gathering Tools:** Descriptive research is useful in development of data gathering instruments and tools like checklists, schedules, score questionnaires and rating scales.
- **Provision of Ideas and Data:** It provides the background ideas and data from which many more refined or controlled studies of causal relations are made.

3.5.4 Types of Surveys

These include census surveys, sample surveys, longitudinal surveys, cross sectional surveys, comparative surveys, evaluation surveys and documentary surveys.

- **Census Survey:** Census survey means gathering pertinent information about all the units of population viz., people, institutions, householders, etc. As you know, population may consist of persons, institutions, objects, attributes, qualities, families, etc. A population is a well defined group of many of these. For instance, the Census Survey of India, which takes place once in ten years, gathers benchmark data about each and every household of India. Since it concentrates on each and every household, it restricts its scope to certain surface level demographic data like age, sex, income, education, lands possessed, cattle, nature of house, domestic facilities available, etc. The studies are conducted through a quick survey in a stipulated period. However, coverage of units is very exhaustive. The census survey as a method of research in education can be employed to understand educational problems and make policy decisions.

***Strength of Census Survey*:** The strength of the census survey is associated with generalized characteristics of data. Description of population data acts as a major source of identifying several pertinent issues and questions for research. It is very useful in making a trend analysis of different events. Moreover, hard database system of the entire population is very useful in development of strategic planning and policy-making of education at the micro level as well as at the macro level.

***Limitations of Census Survey*:** As discussed, each and every unit of population is covered under census survey. However, data is gathered only under limited headings. Also, this data is only surface level information. Through a census survey one can gather nominal data. Thus the researcher cannot ask questions in depth.

Many times such data is gathered mechanically where the investigators are not well trained about cross examining the evidence at the field level. In such cases, the probability of getting valid data is also minimized. Census surveys involve employment of huge manpower and monetary resources. This method is also time consuming. Getting each respondent to cooperate for data collection is very difficult. Hence, the feasibility of conducting census studies is very limited. Moreover, because of sample surveys many questions can be well answered by saving time, money and human resources hence, one may look for census studies with limited focus of research.

- **Sample Survey:** Sample survey means gathering relevant information about a smaller representation of the population under study. The data gathered through sample surveys are generalized to the population of the study. For example, the opinion of a sample of distance learners drawn from a particular district towards usefulness of media in open and distance education can be generalized as the opinion of all distance learners of that district. Educational research invariably makes use of sample surveys.

***Reasons for Conducting Sample Surveys*:** Sample surveys are preferred over census surveys on the following grounds:

(a) ***Greater Range*:** Unlike census studies where limited information is gathered from the whole population, sample surveys cover a wide range of data on different dimensions of the study.

(b) ***Greater Correctness*:** With reduced volume of work using expert and trained personnel, and application of appropriate monitoring mechanism of data collection and analysis, there is a greater chance of gathering valid data and its appropriate processing. Moreover, sampling is particularly more important in obtaining accurate results about phenomena which are undergoing rapid changes such as opinions about political and social issues and their impact on education.

(c) ***Reduced Expenditure*:** Data collected from a small fraction of population involves lesser expense than that of a census survey.

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(d) ***Greater Pace*:** Since the size of respondents is smaller than the whole population, the volume of data is smaller. Hence, it is economical in terms of gathering evidence, tabulating them and processing them quickly.

Characteristics of Sample Surveys: Sample surveys have specific characteristics, viz., application of appropriate sampling techniques with a view to ascertaining representativeness of units under investigation; use of appropriate data gathering tools, mainly, questionnaires, interview schedules, observation schedules and checklists for record surveys; and use of appropriate techniques of data analysis for drawing inferences about population.

Examples

(i) Attitude of parents towards *Ladli* scheme in Delhi schools.

(ii) Causes of dropouts among distance learners as perceived by teachers of IGNOU.

(iii) Attitude of university students towards Information and Communication Technology based education.

(iv) Teaching competencies of primary school teachers.

(v) Physical facilities of primary schools in backward districts of a state.

- **Cross-Sectional Survey:** Cross-sectional survey can be understood as a kind of sample survey where standardized information is gathered from a sample drawn from a cross section of pre-determined population at one point of time. Hence, the sample respondents must represent different distinct segments of population or stages of development of events. The segments may indicate different stages of school education like primary, secondary and higher or different modes of education like face-to-face mode and distance mode. It may include the schools covered under a specific scheme like mid-day meals and the schools yet to be covered under the scheme. It may cover a sample of trained teachers and in-service teachers yet to be trained. The sampling techniques like random, stratified and cluster sampling are used in identifying cross-sectional representation of population of the study. Hence the basic feature of cross sectional survey is associated with varieties of respondents covering different stages/status in the study. Another main feature of cross-sectional study is that the information is collected at one point of time. Here, one point of time refers to single slot of data collection stretching over a few days or months or more.

Types of Cross-Sectional Surveys: Cross-sectional surveys are classified into two types. These are:

(i) Description of status of single variables.

(ii) Exploring relationships between two or more variables.

(i) ***Single Variable Study:*** In such kind of studies, the researcher is interested in describing the status of any one variable as explored through investigation of sample respondents. For example, in an academic assessment study, the researcher explores needs of secondary and higher secondary students towards pursuing various kinds of academic programmes. Data may be gathered through questionnaires or interview techniques, and analysed descriptively highlighting the degree of different academic needs of students' background-wise at the secondary and higher levels. Such kinds of studies are conducted to describe the characteristics of a cross section of a population. It may include the opinion of people, attitude of students, teacher competencies, etc. The analysis of data in such a kind of study will indicate how characteristics of one sample group are different from that of another sample group.

(ii) ***Many Variables Study:*** In such studies, at least two variables are picked up for investigation. It may also be more than two variables. For example, you may be interested in exploring the relationship between achievement and academic interest of students at different stages of education. Hence, you go one step ahead by describing the achievement level of a group of students and academic interest of the same group of students respectively. In exploring relationships of two variables, you are to plot two sets of data, i.e., achievement scores and academic interest scores of the same group of sample respondents. Then you will have to use appropriate statistical techniques for measuring the level of relationships or co-efficient of correlation, multiple correlation, etc.

- **Longitudinal Survey:** Through longitudinal surveys, one explores the status of variables as investigated at different points in time. Through such studies, the changes in the status of the variables over a period of time are explored. Time-ordered associations of one variable's status at different periods of time are also studied. Unlike a cross-sectional study where past data is recollected by the respondents at one point in time, the longitudinal study deals with gathering actual evidences at different points in time. Hence, it is assumed that the accuracy of data is a higher in longitudinal studies than in that of cross sectional time-ordered studies. There are three kinds of longitudinal surveys:

(i) ***Trend Studies:*** In such kinds of studies, data collection takes place at different phases. Different groups of sample respondents drawn from the general population are contacted for different phases of data collection. For instance, in the case of the impact study of the mid-day meal scheme at the primary stage, the researcher may collect evidence from a fresh batch of primary school students on an annual basis for a few years. The researcher identifies the trend of response patterns of each batch of fresh students over a period of time.

(ii) ***Panel Studies:*** In such studies, the same group of sample respondents is studied over a period of time. For example, the attitude of a particular group of students towards school education is studied over a period of time. In another case, the attendance rate and the achievement level of students enrolled in class I can be studied longitudinally till the same group of students clear the class V examination in a school. The problem of retention of sample respondents is associated with absenteeism or non-availability/non-cooperation of the same group of sample respondents in different phases of data collection.

(iii) ***Cohort Studies*:** In the cohort study method, a specific population is taken for study over a period of time. Different samples are drawn from a specific population over different phases of data collection corresponding to different phases/stages of development of events. For example, in a district, prior to the introduction of decentralized management system of primary education, the teachers' attitude towards management of the school system was studied. In the second phase, immediately after introduction of the new management system, the attitude of teachers was studied. In the third phase, the attitude of the teachers was studied after completion of five years of introduction of the new management system. In this case, the teacher population was

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restricted to all the primary schools of a district covered under District Primary Education Project (DPEP). Even though the population of the study was specific, the sample respondents representing the population varied from one phase of study to another.

- **Comparative Survey:** In comparative surveys, the purpose is to compare the status of two or more number of variables, institutions, strategies adopted or groups of respondents, etc. In other words, one may be interested in comparing the achievement level of students enrolled in two different institutions assuming that the students of the two different institutions belong to the same population. In another study, you may like to compare the reaction of teachers serving in government and private management schools towards the leadership behaviour of their principals. Moreover, you may be interested to determine the superior nature of an innovative teaching-learning strategy adopted in certain schools with a traditional approach of teaching adopted in the same institutions in terms of achievement of learners. Though comparative survey studies have many limitations, and they often do not produce the precise and reliable knowledge that can be gained through rigorous experimental studies, they provide means of tackling problems that cannot be probed in laboratory situations. Furthermore, such studies yield valuable information and clues concerning the nature of the phenomena and are admirably suited to many types of field studies seeking to establish causal relationships.
- **Evaluative Survey:** Evaluative survey is conducted with the purpose of evaluating a programme, a curriculum, a policy, etc. When one intends to conduct empirical studies on identifying the effectiveness of any programme functioning or programme output, one may undertake an evaluative survey. Evaluation studies lead towards arriving at a value judgment about a programme or policy or institution being worthwhile. Two purposes can be served in the evaluation of programmes through surveys: (a) judging the effectiveness of the programme and (b) taking a decision for the future course of action. On the basis of such a study, policymakers or decision-making bodies may identify the strength and loopholes of a programme and take an appropriate decision to improve the situation.
- **Documentary Survey:** Various research questions need to be answered through analysis of data already available in the form of printed text viz., books, official records, research reports, review articles, research papers, information bulletins, handouts, prospectus, annual reports, periodicals, progress reports, experience of individuals, news items, etc. For instance, the researcher is interested in comparing the existing status of teacher education programmes in Asian countries. The researcher makes an analysis of available literature on teacher education programmes of countries under investigation like policy documents on education in general, and teacher education in specific curriculum frameworks of teacher education, role and functions of quality control bodies on education, planning documents on teacher education, researches conducted on existing practices of teacher education programmes, committee reports. minutes of meetings, recommendations of education commissions, guidelines and norms prescribed by statutory bodies in education, recommendations of professional organizations and forums, seminar reports and proceedings on teacher education, etc. There can be innumerable sources of obtaining documentary evidences and conducting relevant studies.

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Difference between Documentary Survey and Historical Survey: The difference is simple and is seen in the context of time of occurrence of events under investigation. As a documentary surveyor one would be interested in analysing present events from available records, whereas as a historical researcher you may analyse past records to explore the incidences which occurred in the distant past. The similarity of both the approaches exists in the context of analysis procedure adopted in historical documents and present documents. For instance, the surveyor as well as the historian follows similar approaches of identifying relevant records for investigation, judging authenticity of documents, doing content analysis of evidence, codifying and organizing contents/information, reviewing the content, making interpretations, etc.

Purpose of Documentary Studies: The documentary surveys serve different purposes which have a lot of significance for educational research.

(i) ***To describe the existing structure and functions of educational system or conditions that exists in the educational field:*** For example, the existing practices of primary education in view of achieving the goals of universalization of elementary education or the status of distance education programme in teacher preparation can be included in such a category of documentary surveys.

(ii) ***To discover the relative importance of certain problems and identifying future trends of different developments in the field of education*:** The trend analysis of growing demand for certain areas of education, and analysing the corresponding need for expansion of education can best fit in with such a kind of documentary study.

Another example is Survey of Research in Education. We will come across different Surveys of Research in Education conducted chronologically by the late Professor M.B. Buch and NCERT. In such surveys, researches conducted in different areas of education have been presented in the form of abstracts. Such studies have been analysed by experts. Projections have been made about the future. Developments and priorities of research have also been analysed area-wise. In a closer sense, review of research studies conducted by a research student in identifying his problem of investigation also comes under the category of documentary survey study.

(iii) ***To analyse curriculum of different stages of education or to compare curriculum of different states or countries*:** For example, we may be interested in analysing the curriculum of B.Ed education adopted B.Ed colleges and universities vis-à-vis NCTE. Similarly, we may be interested in pointing out the logical flow of curriculum of DIET, SCERT, and higher education of a particular state or country.

(iv) ***To analyse and review study materials/evaluation items*:** Analysis, review and evaluation of textbooks, study materials, reference books, examination question papers, assessment of assignments, internal assessment of students performance, evaluation of answer books, etc., fall under this category of research. We will come across a number of studies where the researcher may be interested in content analysis and evaluation of self study materials of an open university, examination papers of a board of secondary education, nature of assignments of distance education programme, nature of feedback given by tutors, marking procedure of assignments, etc. Such kinds of analysis

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depend on the availability of original documents, adoption of standard parameter of evaluation and the researcher's judgment capacity. On many occasions such kinds of evaluation studies are treated as part of formative research which have major potentials for programme development.

(v) ***To analyse the literacy style, concepts, beliefs and ideology of a writer:*** In the case of research in the field of literature and social science, such analysis is given importance. Moreover, in the area of philosophy of education, analysis of original text prepared by an author and ideas/comments of others about the author are analysed for drawing inferences which have significance for education.

Limitations of documentary survey studies: While conducting documentary analysis, you may find certain limitations inherent in the method itself.

(a) First, our analysis exclusively depends on documentary evidence. Conclusions drawn on the basis of documentary data may not give a complete picture of the phenomenon under investigation. For example, while analysing curriculum we may depend on text materials. However, the curricular practices as presented in textual form may not reveal the complete picture of the process dimensions. The investigation remains incomplete without incorporating observation based evidence.

(b) Second, data presented in the form of records or publications may not be available in a particular order. It may be available in an incomplete form. Moreover, evidence gathered through available documents may not represent the population of the study. The views, opinions or reactions of people already available in published documents may not be representative in nature. A particular segment of the population who may be expressing their views on certain incidents may not be a true sample of the population under study. Hence, generalizations of documentary evidences have major limitations.

(c) Third, we may doubt the authenticity of data available in printed text. We may cross-examine the information available through one source with that of the other. Moreover, we will have to be careful about the trustworthiness of sources of data. Unless we become careful about the objectives of our study and try to trace necessary documentary evidence, many a time documentary evidence creates confusion and leads to complicating the process of investigation. Different records may use different parameters of present data. Unless we trace these parameters of classification of such data, we will find it difficult to find a meaningful base of data analysis. For example, the boundary of some units of analysis, e.g., school districts, age associates, dropout ratio, etc., can differ from document to document. Different records pertaining to these data may not have used a common parameter of classifications or definition of terms. Hence, it is always advisable to be cautious while adopting documentary analysis with caution of internal and external criticism of data, meaningfulness of information and correlating documentary data with other methods of data collection procedures with a view to getting the total picture of the reality.

Check Your Progress

9. Define 'survey method of research'.
10. What is 'longitudinal survey'?
11. Name one type of information collected by descriptive surveys.
12. Give one reason why the descriptive method is the most widely used research method in education.

3.6 EXPERIMENT

and structured basis for answering questions. The experimental researchers manipulate the environment, stimuli or applications and observe the impact of this manipulation on the condition or behaviour of the subject. The manipulation that they undertake is deliberate and systematic.

Experimentation is the testing of hypotheses. Once the experimenters have defined a situation or issue, they formulate a preliminary solution or hypothesis. They then apply their observations of the controlled variable relationships in order to test, and then confirm or reject the hypothesis.

Experimentation is the classic method of experimenting in a science laboratory where elements are manipulated and effects observed can be controlled. It is the most sophisticated, exacting and powerful method for discovering and developing an organized body of knowledge.

According to J.W. Best, 'Experimental research is the description and analysis of what will be or what will occur under carefully controlled condition'.

3.6.1 Characteristics of Experimental Research

Experimental research is based on highly rigorous procedures and aims at producing reliable and valid conclusions. By looking at the various designs and procedures used, one can formulate some essential characteristics of experimental research which distinguish it from other types of research methods like survey and historical.

- **Pre-Experimental Statistical Equivalence of Subjects in Different Groups:** This pre-condition is achieved by random selection and assignment of subjects to different groups. This procedure is essential to meet the threat of selection differences to the internal validity of the results.
- **Use of At Least Two Groups or Conditions that can be Compared:** An experiment cannot be conducted with one group of subjects or one condition at a time. The intent of the experimenter is to compare the effect of one condition on one group with the effect of a different condition on another equivalent group. An experiment may take the shape of a comparison of the effect of one condition on a group of subjects and the effect of another condition on the same group.
- **Manipulation of the Independent Variable:** It is perhaps the most distinct feature of experimental research. Manipulation stands for the process of assignment of different values or magnitudes or conditions or levels of the independent variable to different groups.
- **Measurement of Dependent Variable in Quantifiable Form:** This distinguishes experimental research from descriptive, qualitative or analytical research.
- **Use of Inferential Statistics:** This is done to make probability statements about the results, and thus meet the requirements of imperfect measurements on which the behavioural sciences base their generalization.
- **Control of Extraneous Variables:** Though applicable to any other type of research, control of extraneous variables is the [illegible] designs and the [illegible]

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3.6.2 Steps in Experimental Research

The steps in experimental research are as follows:

(i) **Survey of the Literature Relating to the Problem:** In experimentation, the researcher needs to acquire up-to-date information relating to the problem.

(ii) **Selection and Definition of the Problem:** It needs a rigorous logical analysis and definition of the problem in precise terms. The variables to be studied are defined in operational terms clearly and unambiguously. It helps the researcher to convert the problem into a hypothesis that can be verified or refuted by the experimental data.

(iii) **Statement of Hypotheses:** Hypotheses are the heart of experimental research. They suggest that an antecedent condition or phenomenon is related to the occurrence of another condition, phenomenon, event or effect. To test a hypothesis, the researcher attempts to control all the conditions except the independent variable. Therefore, he should give sufficient attention to the formulation of hypotheses. The experimental plant and statistical procedures help him in the testing of hypotheses and contribute little in the development of theories or advancement of knowledge. However, the hypotheses developed or derived from existing theories contribute to the development of new theories and knowledge.

(iv) **Construction of Experimental Plan:** Experimental plan refers to the conceptual framework within which the experiment is to be conducted. According to Van Dalen, an experimental plan represents all elements, conditions, phenomena, and relations of consequences so as to:

- Identify the non-experimental variables.
- Identify the most appropriate research design.
- Identify a sample of subjects that will suitably represent the target population, form groups of these subjects and decide on the experiments which will be conducted on each group.
- Choose or develop an instrument that can be deployed to measure the results of the experiment.
- Lay out the data collection process and conduct a pilot study to test the instrument and the research design and state the hypotheses.

3.6.3 Variables

A **variable** is any feature or aspect of an event, function or process that, with its presence and nature, affects some other event or process which is being studied. According to Kerlinger, 'variable is a property that takes on different value'.

Types of Variables

- ***Independent Variables*:** These are conditions or characteristics that are manipulated by the researcher in order to identify their relationship to observed phenomena. In the field of educational research, for instance, a specific teaching method or a variety, of teaching material are types of independent variables.

The two kinds of independent variables are:

(i) ***Treatment Variables*:** These are variables which can be manipulated by the researcher and to which he assigns subjects.

(ii) ***Organism or Attribute Variables*:** These are factors such as age, sex, race, religion etc., which cannot be manipulated.

- ***Dependent Variables*:** Dependent variables represent characteristics that alter, appear or vanish as a consequence of introduction, change or removal of independent variables. The dependent variable may be a test score or achievement of a student in a test, the number of errors or measured speed in performing a task.

- ***Confounding Variables*:** A confounding variable is one which is not the subject of the study but is statistically related with the independent variable. Hence, changes in the confounding variable track the changes in the independent variable. This creates a situation wherein subjects in a particular condition differ unintentionally from subjects in another condition. This is not a good result for the experiment which is attempting to create a situation wherein there is no difference between conditions other than the difference in the independent variable. This phenomenon enables us to conclude that the manipulation undertaken directly causes differences in the dependent variable. However, if there is another variable besides the independent variable that is also changing, then the confounding variable is the likely cause of the difference. An example of a common confounding variable is that when the researcher has not randomly assigned participants to groups, and some individual difference such as ability, confidence, shyness, height, looks, etc., acts as a confounding variable. For instance, any experiment that involves both men and women is naturally afflicted with confounding variables, one of the most apparent being that males and females operate under diverse social environments. This should not be confused to mean that gender comparison studies have no value, or that other studies in which random assignment is not employed have no value; it only means that the researcher must apply more caution in interpreting the results and drawing conclusions.

Let us consider an instance wherein an educational psychologist is keen to measure how effective is a new learning strategy that he has developed. He assigns students randomly to two groups and each of the students study materials on a specific topic for a defined time period. One group deploys the new strategy that the psychologist has developed, while the other uses any strategy that they prefer. Subsequently, each participant takes a test on the materials. One of the obvious confounding variables in this study would be advance knowledge of the topic of the study. This variable will affect the test results, no matter which strategy is used. Because of an extraneous variable of this nature, there will be a level of inconsistency within and between the groups. It would obviously be the preferred situation if all students had the exact same level of pre-knowledge. In any event, the experimenter, by randomly assigning the groups, has already taken an important step to ensure the likelihood that the extraneous variable will equivalently affect the two groups.

Let us imagine an experiment being undertaken to measure the effect that noise has on concentration. Assume that there are 50 subjects each in quiet and noisy environments. Table 3.2 below illustrates the ideal or perfect version of this experiment. 'IV' and 'EV' represent the independent variable and external variables respectively. Note that (as shown in the table), the only difference between the two conditions is the IV, which indicates that the noise level varies from low to high in the two conditions. All the other variables are controlled and are exactly the same for the two conditions. Therefore, any difference in the concentration levels of subjects between the two conditions must have been caused by the independent variable.

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Table 3.2 Determining the Impact of Internal and External Variables

Variables	Quiet Condition N = 50	Noisy Condition N = 50
Noise Level (IV)	Low	High
IQ (EV)	Average	Average
Room temperature (EV)	68 degrees	68 degrees
Sex of subjects (EV)	60 per cent F	60 per cent F
Task difficulty (EV)	Moderate	Moderate
Time of day (EV)	All different times between 9–5	All different times between 9–5
Etc. (EV)	Same as noisy environ.	Same as quiet environ.
Etc. (EV)	Same as noisy environ.	Same as quiet environ.

An Ideal Experiment

Now consider another version of this experiment wherein some of the other variables differ across conditions. These are confounding variables (highlighted below) and the experiment being conducted is not ideal. In this experiment, if the concentration levels of subjects vary between the two conditions this may have been caused by the independent variable, *but it could also have been caused by one or more of the confounding variables*. For instance, if the subjects in the noisy environment have lower concentration levels, is it because it was louder, too hot or because they were tested in the afternoon? It is not possible to tell and therefore, this is less than ideal.

Variables	Quiet Condition	Noisy Condition
Noise Level (IV)	Low	High
IQ (EV)	Average	Average
Room temperature (EV)	68 degrees	82 degrees
Sex of subjects (EV)	60 per cent F	60 per cent F
Task difficulty (EV)	Moderate	Moderate
Time of day (EV)	Morning	Afternoon
Etc. (EV)	Same as noisy environment	Same as quiet environ.
Etc. (EV)	Same as noisy environment	Same as quiet environ.

A Non-Ideal Experiment

Controlling the Confounding Variables

There are ways by which the extraneous variables may be controlled to ensure that they do not become confounding variables. All people-related variables can be controlled through the process of random assignment which will most likely ensure that the subjects will be equally intelligent, outgoing, committed, etc. Random assignment *does not* necessarily ensure that this is the case for every extraneous variable in every experiment. However, when a sample is large, it works very well and the researcher's motives for using this method will never be questioned.

One of the way in which situation variables or task variables can be controlled is basically by keeping them constant. For instance, in the noise-concentration experiment above, we could adjust the thermostat and thereby keep the room temperature constant and test all the subjects in the same room. We would, of course, hold the difficulty of the tasks constant by giving all subjects in both environments the same task. It is common practice for instructions to be written or recorded and presented to each subject in exactly the same way.

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At time, the researcher cannot hold a situation or task variable constant. In these situations too, random assignment can be of great help. Consider a situation where the same room is not available for testing the two groups and, in fact, one group is tested on a Monday in Room 1 and the other group on a Tuesday in Room 2. In this situation, we can use random assignment which can result in half the Monday subjects in Condition A and the rest in Condition B, and the same for the Tuesday subjects. Hence both conditions will have roughly the same percentage of subjects tested in Room 1 and 2. On the other hand, consider what would happen if we did not use random assignment and instead tested the Monday subjects in Condition A and the Tuesday subjects in Condition B. In this situation, we have two confounding variables. Subjects in Condition A were tested on different days of the week and in different rooms from those in Condition B. Any difference in the results could have been caused by one or more of the independent variable, the day of the week, or the room.

In other words, confounding variables are those aspects of a study or sample that might influence the dependent variable and whose effect may be confused with the effects of the independent variable. Confounding variables are of two types:

(a) ***Intervening Variables*:** In many types of behavioural research, the relationship between independent and dependent variables is not a simple one of stimulus to response. Certain variables that cannot be controlled or measured directly may have an important effect on the outcome. These modifying variables intervene between the cause and the effect. For example, in a classroom language experiment, a researcher is interested in determining the effect of immediate reinforcement on learning the parts of speech. He suspects that certain factors or variables other than the one being studied may be influencing the result, even though they cannot be observed directly. These factors may be anxiety, fatigue or motivation. These factors cannot be ignored. Rather they must be controlled as much as possible through the use of appropriate design. For example, a variable (as memory) whose effect occurs between the treatment in a psychological experiment (as the presentation of a stimulus) and the outcome (as a response) is difficult to anticipate or is unanticipated, and may confuse the results.

(b) ***Extraneous Variables*:** These are variables that are not the subject of an experiment but may have an impact on the results. Hence, extraneous variables are uncontrolled and could significantly influence the results of a study. Often we find that research conclusions need to be questioned further because of the influence of extraneous variables. For instance, a popular study was conducted to compare, the effectiveness of three methods of social science teaching. Ongoing, regular classes were used, and the researchers were not able to randomize or control the key variables as teacher quality, enthusiasm or experience. Hence, the influence of these variables could be mistaken for that of an independent variable.

For instance, in a study which attempts to measure the effect of temperature in a classroom on students' concentration levels, noise coming into the class through doors or windows can influence the results and is therefore an extraneous variable. This may be controlled by soundproofing the room, which illustrates how the extraneous variable may be controlled in order to eliminate its influence on the results of the test.

The following are the types of extraneous variables:

- Subject variables pertain specifically to the people being studied. These people's characteristics such as age, gender, health status, mood, background, etc., are likely to affect their actions.

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- Experimental variables pertain to the persons conducting the experiment. Factors such as gender, racial bias, or language influence how a person behaves.
- Situational variables represent the environment factors which were prevalent at the time when the study or research was conducted. These include the temperature, humidity, lighting, and the time of day, and could have a bearing on the outcome of the experiment.
- Continuous variable is one wherein, any value is possible within the range of the limits of the variable. For instance, the variable 'time taken to run the marathon' is continuous since it could take 2 hours 30 minutes or3 hours 15 minutes to run the marathon. On the other hand, the variable 'number of days in a month that a worker came to office' is not a continuous variable since it is not possible to come to office on 14.32 days.
- Discrete variable is one that does not take on all values within the limits of the variable. For instance, the response to a five-point rating scale must only have the specific values of 1, 2, 3, 4, or 5. It cannot have a decimal value such as 3.6. Similarly this variable cannot be in the form of 1.3 persons.
- Quantitative variable is any variable that can be measured numerically or on a quantitative scale, at an ordinal, interval or ratio scale. For example, a person's wages, the speed of a car, or the person's waist size are all quantitative variables.
- Qualitative variables are also known as categorical variables. These variables vary with no natural sense of ordering. They are therefore measured on the quality or characteristic. For example, eye colour (black, brown, or blue) is a qualitative variable, as are a person's looks (pretty, handsome, ugly, etc.). Qualitative variables may be converted to appear numeric, but this conversion is meaningless and of no real value (as in male = 1, female = 2).

3.6.4 Experimental Designs

The various experimental designs have been discussed in this section.

(a) Single Group Design: In this design study is carried out on a single group. Experiments can be conducted in the following ways:

(i) ***The One-Shot Case Study*:** This is a single group studied only once. A group is introduced to a treatment or condition and then observed for changes which are attributed to the treatment. This is like an ex-post facto method in which on the basis of a dependent variable, an independent variable is looked for.

(ii) ***One Group Before after Design*:** This design entails the inclusion of a pre-test in order to establish base level scores. For instance, to use this design in a study of college performance, we could compare college grades prior to gaining the experience to the grades after completing a semester of work experience. In this design, we subtract the score of pre-test from post-test and see the differences. This difference is seen using a 't' test.

(iii) ***Time Series Designs*:** Time series designs refer to the pre-testing and post-testing of one group of subjects at different intervals. In this design, continuous observation is carried out till a clear result is not seen. The purpose is to establish the long-term effects of treatment and can often lead to the number of pre- and post-tests varying from just one each, to many. At times, there is a period of interruption between tests so as to assess the strength of the treatment over a long time frame.

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(iv) ***Counterbalanced Design*:** Experiments that use counterbalanced design are effective ways to avoid the pitfalls of repeated measures, where the subjects are exposed to treatments one after the other.

Typically in an experiment, the order in which the treatments are administered can affect the behaviour of the subjects. It may also elicit a false response due to fatigue or any other external factors which may have a bearing on the behaviour of the subjects. To control or neutralize this, researchers use a counterbalanced design, which helps to reduce the adverse effects of the order of treatment or other factors on the results.

Counterbalancing helps to avoid confounding among variables. Take for example an experiment in which subjects are tested on both, auditory reaction time task and visual reaction time task. If each and every subject were first tested on the auditory reaction time task and then on the visual reaction time task, the type of task and the order of presentation would be confounded. If the visual reaction time was lower, we would not be sure whether reaction time to a visual stimulus is 'really' faster to an auditory stimulus, as it is quite likely that the subjects would have learned something while performing the auditory task which led to an improvement of their performance on the visual task.

(b) Two Equivalent Group Design

(i) ***Static Group Comparison Study:*** This design attempts to make up for the lack of a control group but falls short in relation to showing if a change has occurred. In this group, no treatment is given but only observation is carried out in a natural way of two groups, e.g., observation of the monkeys living in a city and observation of other monkeys living in the jungle. It is fair to mention here that in these groups nothing is manipulated as this design does not include any pre-testing and therefore any difference between the two groups prior to the study is unknown.

(ii) ***Post-Test Equivalent Groups Design*:** Randomization as well as the comparison of both the control and experimental group, are used in studies of this nature. Each group is chosen and assigned randomly and presented with either the treatment, or a type of control. Post-tests are subsequently administered to each subject to establish whether or not a difference exists between the two groups. While this is close to being the best possible method, it falls short on account of its lack of a pre-test measure. It is not possible to establish if the difference that seems to exist at the end of the study actually represents a change from the difference at the beginning of the study. Hence, while randomization mixes the subjects well, it does not necessarily create an equivalency between the two groups.

(iii) ***Pre-Test Post-Test Equivalent Groups Design*:** This is the most effective as well as the most difficult method in terms of demonstrating cause and effect. The pre-test post-test equivalent groups design ensures the presence of a control group as well as a measure of change. Importantly, it also adds a pre-test thereby assessing any differences that existed between the groups prior to the study taking place. In order to apply this method, we select students at random and then segregate them into one of two groups. We would subsequently evaluate the previous semester's grades for each group in order to arrive at a mean grade point average. The treatment (work experience) would be applied to one group, whereas a control would be applied to the other.

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It is critical that the two groups be treated similarly in order to control for variables such as socialization, so the control group may participate in an activity such as a softball league while the other group participates in the work experience programme. The experiment ends at the end of the semester, and the semester's grades are compared. If it is found that the grade change for the experimental group was significantly different from the grade change of the control group, one could conclude that a semester of work experience results in a significant difference in grades when compared to a semester of non-work related activity. programme.

(iv) ***Counterbalanced Randomized Two Groups Design:*** In this design, the group is divided in two parts on a random basis. This design is also called 'rotation design'.

The simplest type of counterbalanced measure design is used when there are two possible conditions, A and B. As with the standard repeated measures design, the researchers want to test every subject for both conditions. They divide the subjects into two groups — one group is treated with condition A, followed by condition B, and the other is tested with condition B followed by condition A as shown in Figure 3.1.

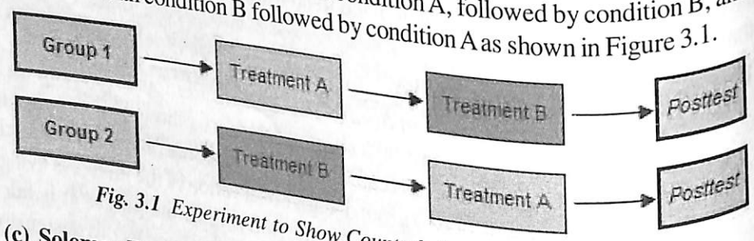

Fig. 3.1 Experiment to Show Counterbalanced Measure Design

(c) Solomon Four Group Design: The sample is randomly divided into four groups. Two of the groups are experimental samples, whereas the other two groups experience no experimental manipulation of variables. Two groups receive a pre-test and a post-test. Two groups receive only a post-test. Table 3.3 shows the effect of a particular teaching method on the following groups.

Table 3.3 Solomon Four Group Design

Group		Pre-test	Treatment	Post-test
a)	R	No	No	No
b)	R	No	Yes	No
c)	R	Yes	No	No
d)	R	Yes	Yes	No

Table 3.4 shows a teaching experiment using the Solomon design where testing before and without treatment have similar results, whilst results after teaching are significantly improved. This indicates that the treatment is effective and not subject to priming or learning effects.

Table 3.4 Pre-and Post-Testing

Group		Pre-test	Treatment	Post-test	Pre-result	Post-result
a)	R	No	No	No	3	10
b)	R	No	Yes	No	4	5
c)	R	Yes	No	No		9
d)	R	Yes	Yes	No		3

Check Your Progress

13. Define 'experimental research'.
14. What is a pre-test and a post-test?

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3.6.5 Internal and External Validity in Experimental Research

Internal Validity

Internal validity is considered as a property of scientific studies which indicates the extent to which an underlying conclusion based on a study is warranted. This type of warrant is constituted by the extent to which a study minimizes systematic error or 'bias'. If a causal relation between two variables is properly demonstrated then the inferences are said to possess internal validity. A fundamental inference may be based on a relation when the following three criteria are satisfied:

1. The 'cause' precedes the 'effect' in time (temporal precedence).
2. The 'cause' and the 'effect' are related (covariation).
3. There are no plausible alternative explanations for the observed covariation (non-spuriousness).

Internal validity refers to the ability of a research design for providing an adequate test of an hypothesis and the ability to rule out all plausible explanations for the results but the explanation being tested. For example, let us consider that a researcher decides that a particular medication prevents the development of heart disease because he found that research participants who took the medication developed lower rates of heart disease than those who never took the medication. This interpretation of the study's results is likely to be correct, however, only if the study has high internal validity. In order to have high internal validity, the research design must have controlled the directionality and third-variable problems, as well as for the effects of other extraneous variables. In short, the researcher would have needed to perform an experimental study in which:

- Participants were randomly assigned to the experimental and control groups.
- Participants did not know whether they were taking the medication.

The most internally valid studies are experimental studies because they are better than correlational and case studies at controlling for the directionality and third-variable problems, as well as for the effects of other extraneous variables.

Threats to Internal Validity

The following are the various threats to internal validity:

Ambiguous Temporal Precedence: Lack of precision about the occurrence of variable, i.e., which variable occurred first, may yield confusion that which variable is the cause and which is the effect.

Confounding: Confounding is a major threat to the validity of fundamental inferences. Changes in the dependent variable may rather be attributed to the existence or variations in the degree of a third variable which is related to the manipulated variable. Rival hypotheses to the original fundamental inference hypothesis of the researcher may be developed where spurious relationships cannot be ruled out.

Selection Bias: It refers to the problem that, at pre-test, differences between the existing groups that may interact with the independent variable and thus be 'responsible' for the observed outcome. Researchers and participants bring to the experiment a myriad of characteristics, some learned and others inherent. For example, sex, weight, hair, eye, and skin color, personality, mental capabilities and physical abilities, etc. Attitudes like motivation or willingness to participate can also be involved. If an unequal number of test subjects have similar subject-related variables during the selection step of the research study, then there is a threat to the internal validity.

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Repeated Testing: It is also referred to as testing effects. Repeatedly measuring or testing the participants may lead to bias. Participants of the testing may remember the correct answers or may be conditioned to know that they are being tested. Repeatedly performing the same or similar intelligence tests usually leads to score gains instead of concluding that the underlying skills have changed for good. This type of threat to internal validity provides good rival hypotheses.

Regression toward the Mean: When subjects are selected on the basis of extreme scores (one far away from the mean) during a test then this type of threat occurs. For example, in a testing when children with the bad reading scores are selected for participating in a reading course, improvements in the reading at the end of the course might be due to regression toward the mean and not the course's effectiveness actually. If the children had been tested again before the course started, they would likely have obtained better scores anyway.

External Validity

External Validity is considered as the validity of generalized (causal or fundamental) inferences in scientific studies. It is typically based on experiments as experimental validity. In other words, it is the degree to which the outcomes of a study can be generalized to other situations and people.

If inferences about cause and effect relationships which are based on a particular scientific study may be generalized from the unique and characteristics settings, procedures and participants to other populations and conditions then they are said to possess external validity. Causal inferences possessing high degrees of external validity can reasonably be expected to apply:

- To the target population of the study, i.e., from which the sample was drawn. It is also referred to as population validity.
- To the universe of other populations, i.e., across time and space.

An experiment using human participants often employ small samples which are obtained from a single geographic location or with characteristics features is considered as the most common threat to external validity. Due to this reason, one cannot be certain that the conclusions drawn about cause and effect relationships do actually apply to people in other geographic locations or without these particular features.

External validity refers to the ability of a research design for providing outcomes that can be generalized to other situations, especially to real-life situations. For instance, if the researcher in the hypothetical heart disease medication study found that the medication, under controlled conditions, prevented the development of heart disease in research participants, he would want to generalize these findings to state that the medication will prevent heart disease in the general population. However, let us consider that the research design required the elimination of many potential participants, such as people who abuse alcohol or other drugs, suffer from diabetes, weigh more than average for their height, and have never suffered from a mood or anxiety disorder. These are common risk factors for heart disease and, by eliminating these factors; the outcomes of the study would provide little evidence that the medication will be effective for people with these risk factors. In other words, the study would have low external validity and, hence, its outcomes to the general population could not be generalized.

This commonly happens in tests of antidepressant medications. Because researchers want to make sure that the antidepressant effects of the medications being tested are not hidden by the effects of extraneous variables, they often have excluded potential participants with one or more of the following characteristics:

- People who are addicted to alcohol or illicit drugs.
- People who take various medications.
- People who have anxiety disorders (such as, phobic disorders).
- People who suffer from depression with psychosis.
- People with mild depression (because they would show only a small response to the medication).

If a study excluded people with these characteristic features, then most of the participants suffering from depression would be excluded from the final pool of participants. The outcomes of the study, therefore, would provide little information about how most depressed people will respond to the medication.

Threats to External Validity

A threat to external validity is an explanation of how you might be wrong in making a generalization. Usually, generalization is limited when the cause, i.e., independent variable depends on other factors; therefore, all threats to external validity interact with the independent variable.

- **Aptitude-Treatment Interaction:** The sample may have specific characteristic features that may interact with the independent variable, limiting generalization. For example, inferences based on comparative psychotherapy studies often employ specific samples (e.g. volunteers, highly depressed, no comorbidity). If psychotherapy is found effective for these sample patients, will it also be effective for non-volunteers or the mildly depressed or patients with concurrent other disorders?
- **Situation:** All situational features, such as treatment conditions, time, location, lighting, noise, treatment administration, investigator, timing, scope and extent of measurement, etc. of a study potentially limit generalization.
- **Pre-Test Effects:** If cause and effect relationships can only be found when pre-tests are carried out, then this also limits the generality of the findings.
- **Post-Test Effects:** If cause and effect relationships can only be found when post-tests are carried out, then this also limits the generality of the findings.
- **Reactivity (Placebo, Novelty and Hawthorne Effects):** If cause and effect relationships are found they might not be generalized to other situations if the effects found only occurred as an effect of studying the situation.
- **Rosenthal Effects:** Inferences about cause-consequence relationships may not be able to generalize to other investigators or researchers.

3.7 QUALITATIVE RESEARCH

Qualitative research is a specific method of inquiry used in various different educational disciplines. Qualitative researchers aim to collect an in-depth perceptive of human activities and the reasons that govern such activities. Typically, the qualitative method exceptionally investigates the 'why' and 'how' of decision-making along with 'what', 'where' and 'when'. Consequently, the researchers use the smaller but focused samples rather than large samples.

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Qualitative researchers have several alternatives related to data collection, such as qualitative research interview, grounded theory practice, narratology, storytelling, classical ethnography or shadowing. Besides, the qualitative researchers categorize the data into specific modules for organizing and analysing results. Typically, the qualitative researchers rely on the following methods for collecting required information:

- Participant Observation
- Non-Participant Observation
- Field Notes
- Reflexive Journals
- Structured Interview
- Semi-Structured Interview
- Unstructured Interview
- Analysis of Documents and Materials

The techniques of participating and observing may differ from situation to situation. Some of the distinctive qualitative methods include the use of focus groups and key informant interviews. One of the traditional and specialized forms of qualitative research is termed as cognitive testing or pilot testing which is specifically used to develop quantitative survey objects. Survey objects are piloted on study participants to test the reliability and validity of the objects.

The qualitative researchers use various different research approaches or research designs in the educational researches. Some of the most commonly used qualitative research approaches are as follows:

- Basic/Generic/Pragmatic Qualitative Research
- Ethnographic Research
- Grounded Theory
- Phenomenological Research
- Philosophical Research
- Critical Social Research
- Ethical Inquiry
- Foundational Research
- Historical Research

Qualitative research is thus a generic term specifically used for investigative methodologies. It emphasizes the use of the significant variables in their natural surroundings.

3.7.1 Phenomenological Research

Applied to research, phenomenology is the study of phenomena: their nature and meanings, i.e., the descriptive study of how individuals experience a phenomenon. The phenomenological research describes the 'subjective reality' of an event as perceived by the population under study; thus it is the study of a phenomenon. The underlying principle of the phenomenological approach is to elucidate and identify the specific phenomena through how they are perceived by the actors in the case of human samples, this usually involves

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through inductive, qualitative methods, such as interviews, discussions and participant observation, and then representing it from the perspective of the research participant(s).

Phenomenology is typically concerned with the study of experience from the perspective of the individual, 'bracketing' taken-for-granted assumptions and standard techniques of perceiving. Epistemologically, phenomenological approaches are exclusively based in a paradigm of individual knowledge and subjectivity, and emphasize the significance of personal perspective and interpretation. As such these are considered as powerful tools that help to understand subjective experience, gain insights into people's motivations and actions, and evaluate taken-for-granted assumptions and conventional wisdom.

Phenomenological approaches can be characteristically applied to single cases or to serendipitous or intentionally selected samples. Several methods can be used in phenomenological based research, including interviews, conversations, participant observation, action research, focus meetings and analysis of personal details.

The objective of qualitative phenomenological research is to describe a 'lived experience' of a phenomenon. Since this is a qualitative analysis of narrative data, hence methods used to analyse the data must also be quite different as compared to traditional or quantitative methods of research.

Data Collection

To collect data for processing meaningful information in a phenomenological study, the participant can depict their own live phenomenal experience. The research tools used can be an interview to collect the participants' explanations of their own experience or the participants' written or oral self-report, or even their aesthetic expressions, such as, art, poetry, narratives, etc. The researcher should attempt to be non-directive as much as possible in giving the instructions. Encourage the participant to provide a full description of their own experience which may include their thoughts, feelings, images, sensations, memories along with a description of the circumstances in which the experience actually happened.

Data Analysis

The data analysis is also very significant aspect of any research because if the data is not analysed logically and accurately then the purpose of the research conducted will be unsuccessful. The most important principle of analysis of phenomenological data is to utilize an emergent policy that will support the technique of analysis to pursue the data analysis process in such a way that the original nature of the data remains intact. For example, the approach used for analysing artistic depictions of experience must be different from narratives or interview data. Though in all types of cases under analysis, the focus should be on to evidently understand the significance and meaning of the description.

Foundational Concepts for Research

The phenomenological researcher aims to provide affluent textured description of live experiences. The term live experience typically refers to a key perception and the focus of investigation for phenomenological study.

The live

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and the world of experienced horizons within which we meaningfully dwell together." It can be described as the world that is lived and experienced, i.e., a world that emerges significantly to consciousness in its qualitative approach. This lived world is considered pre-reflective as it automatically occurs before a person think about it or put it into narrative language. Typically, the lived world includes the day-to-day world that is completely filled with multifaceted meanings which structure the background of everyday actions and interactions.

Phenomenology inquires, "What is this kind of experience like?", "What does the experience mean", "How does the lived world present itself to anyone general or to any participant under study?" The phenomenological researchers face the following two challenges:

- How to help participants express their world as directly as possible?
- How to explicate these dimensions such that the lived world is revealed?

Implications revealed by the researcher essentially depend on the researcher's attitude and the approach the researcher poses questions. Especially, the researcher intends to '**bracket**' or suspend earlier assumptions or realizations to understand and analyse the phenomenon as it appears.

3.7.2 Ethnographic Research

Ethnography is a qualitative research method that is used by anthropologists to describe a culture of a group, e.g., what are the characteristics of a particular group.

Culture is defined in many ways but usually comprises origins, values, roles, as well as material items linked to a particular group of people. Ethnography research, therefore, seeks to comprehensively describe a large number of aspects of a cultural group in order to enhance the understanding of the subjects of the study.

Ethnographic research focuses on local as well as foreign cultures and seeks to understand native people — those who are isolated from modern civilization. One of the famous anthropologists, who undertook research of this nature, was Margaret Mead. Her renowned study of three New Guinea cultures explored the gender characteristics and roles of these cultures. By examining a large number of cultural norms, gender characteristics and roles, this type of research enables scientists to categorize key characteristics of each gender. Several ethnographic studies have provided significant detail of cultural roles that challenge the Western perspectives of gender characteristics.

The orientation or mindset of the researcher undertaking ethnographic studies is termed 'ethic' or 'emic'. The ethic orientation refers to the view from the perspective of an outsider.

Assumptions

Research that follows the *critical approach* differs from research that follows the *descriptive* or *interpretive* approaches. The latter have historically adopted a more detached, objective and value-free assessment of knowledge, although there is some degree of convergence between the critical and descriptive approaches in contemporary ethnography. Critical approaches are aligned with the post-enlightenment philosophical tradition which believes in situating research within its social context. This enables the researcher to consider how knowledge is influenced by the values of human beings and communities, implicated where there are power struggles, and critical in the process of democratizing relationships as well as institutions. The critical approach questions

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dichotomies such as the separations of theory and method, interpretation and data, subjective and objective, and ethics and science. The method also specifically questions the treatment of the second term in each pair as constituting valid research. Critical ethnography views these binary constructs as being interconnected and making mutual contributions to the body of knowledge.

Ethnography accepts a complicated theoretical orientation toward culture. Culture, expressed by collections of humans of varying characteristics and magnitude such as educational institutions, student bodies or classes, or activity groups, is treated as heterogeneous, conflicted, negotiated, and evolving, as opposed to unified, cohesive, fixed and static. It should also be noted that while cultures carry the 'different-but-equal' view, critical ethnography openly assumes that cultures are not positioned equally in power relations. Further, critical ethnography assumes that the descriptions of culture are shaped by the biases of the researcher, the project sponsors, the audience, or the dominant communities. Hence, cultural representations are deemed to be partial and partisan. Studies that adopt the ethnographic approach should be conducted against the backdrop of the theoretical assumptions behind this research initiative.

Data

- Provides evidence of cohabitating or spending considerable time with people who are in the study setting, by observing and recording their activities as they unfolded through notes or journals, (Emerson, Fretz and Shaw, 1995), audio and video recordings, or both. One of the trademarks of ethnography is the extended and first-hand participant observations of their interactions with participants in the study setting.
- Records participants' beliefs as well as their attitudes through typical means such as notes or transcriptions of informal conversation and interviews, as well as participant journals (Salzman, 2001).
- Includes multiple sources of data. Besides observation and interactions with participants, these sources can include life histories (Darnell, 2001) or narrations (Cortazzi, 2001), photography, audio or video recordings (Nastasi, 1999), written documents (Brewer, 2000), data that describes historical trends, as well as questionnaires and surveys (Salzman, 2001).
- Often called for in critical ethnography (and also in several cases of descriptive or interpretive ethnography), to use additional sources of data and reflection including:
  (i) Evidence to show how the differences in power between you and the informants or subjects were addressed. It is idealistic to assume that differences in power may be totally eliminated, and hence what must be addressed is how these differences were managed, amended, or moved and also the influence that they had on the data gathered.
  (ii) The attitudes as well as biases towards the community and its culture. There needs to be a record of how perspectives got modified as the research progressed and how these modifications impacted the data that was collected.
  (iii) The impact that your behaviour and activities have had on the community. One must state if one was personally involved in the ethical, social, or political challenges faced by the community. The data should also contain the manner in which this involvement could have provided deeper insights or impacted the research (and also the manner in which the tensions were addressed).

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to adopt this case study method in his work with delinquents. Among the historians who used this method, mention may be made of Arther S. Urik, George E. Mowry and John Gunther. Some of the anthropologists and ethnologists have utilized this method for their detailed description of primitive and modern cultures.

According to P.V. Young, 'case study is a method of exploring and analysing the life of a social unit'. Charles Horton Cool says that, 'case study depends on our perception and gives us clearer insight into life'.

According to H. Odum, 'the case study method is technique by which an individual factor whether it be an institution or just an episode in the life of an individual or a group is analysed in its relationship of any other in the group'.

3.8.1 Characteristics of Case Study

Like every other method in social research, the case study method has some important characteristics which are briefly discussed below:

(i) First important characteristic of this method is that the social unit which is picked up for study is comprehensively studied in its entirety. All aspects are deeply and thoroughly studied. Various factors which act or interact on that unit are carefully studied. In this study, both qualitative and quantitative aspects are given full weight and consideration is given to family, group and community life of the individual, group or even of the community as a whole.

(ii) In this method, the approach of study is direct and neither abstract nor indirect.

(iii) Another characteristic of this method is that it studies both 'what and why'. In the first instance, the researcher investigator tries to describe complex behaviour patterns of a unit. After that has been done, his next attempt is to discover such factors which will rationally account for them. In other words, it can be said that he aims to describe as well as explain the units which are studied. Not only this, he also tries to explain the behaviour of the unit in social surroundings and set-up.

(iv) In this method, the whole approach is not quantitative but qualitative. The idea is not merely to collect information but to go in depth into the life of the person concerned.

(v) In this method the researcher can pick up for his study only one social unit. While studying, effort is made to know the mutual inter-relationship of several factors.

This method tries to integrate the individual units in a manner that its integrity and wholesomeness can be preserved. In this, all unifying bonds are created so that diverse elements are brought together in a well knit unit system. In this system, the individual, family, institution or group is considered as a unified whole.

The six types of case studies are:

(i) A group or a community case study.
(ii) Casual comparative studies.
(iii) Activity analysis.
(iv) Content or document analysis.
(v) A follow-up study.
(vi) Trend studies.

Criteria for a Good Case Study

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The essential characteristics of a satisfactory case study are as follows:

- **Continuity:** There should be desirable continuity of information provided by two successive psychological examinations and by an elementary school record in relation to performance in high school.
- **Completeness of data:** In so far as possible, the potential range of data or information includes symptoms, examination results and history.
- **Validity of data:** A doubtful birth date may be verified through the bureau of vital statistics and employment record by referring to employers.
- **Confidential recording:** Educational workers have something to learn from medicine with respect to the confidential nature of professional records, the difficulties of individual teachers or pupil in relation to discipline, failure, achievement should be recorded as professional problems to be treated in a confidential manner.
- **Scientific synthesis:** This is an interpretation of the evidence that is more than a mere enumeration of data secured, it embraces diagnosis in identifying casual factor, and prognosis in looking toward treatment or developmental procedure.

3.8.2 Stages of a Case Study

In the case study method, there are normally three stages, which need to be completed before the whole study is complete. These are briefly discussed as under:

- If the unit is to be studied as a whole, it is essential that there should be a broad array of data, no matter whether such a unit is a social relationship, a group or a person. In the words of Goode and Hatt: 'Although mere quantity of data is not sufficient since the collection must be guided by research problem, there is greater opportunity to grasp the pattern of an individual's life if a substantial body of data concerning many facts of that is available.'
- The case study method is further distinguished by the use of data from other abstract levels than the purely sociological. Recourse to other levels gives added dimensions to the individual being studied. It is rightly pointed out that when an individual is seen in his total network of relationship, it is more difficult to lose sight of him as unit.
- Goode and Hatt are of the view that in the case study method, indices and types are most important and as such proper care should be taken in their formation. According to them: 'However, the most important technique in preserving the wholeness of social unit is the development of typologies and indexes; so that the various traits are actually used in characterizing the units. It is equally essential that the data collected should be timely processed.'

The steps to be followed for a case study are:

(i) Statement of the Problem: In this respect, it is essential that the problem should be clearly stated and specified. It should be clearly stated as to what types of cases have been selected for the study and how many of these cases have actually been picked up for the study. The statement should also show as to what are the types of units which have been picked up, and what is the scope of analysis once the data has been collected.

(ii) **Description of Events:** The next step is description of events. It is a very essential step and implies that every unit must be carefully and clearly studied. As already mentioned, carrying out a case study means in-depth extensive and quantitative study of a subject. Accordingly, in this method, no event can either be partially or completely missed or its study can be slightly overlooked, neglected or underestimated in any manner.

(iii) **Factors of Influence:** There are different factors which influence a social unit. Nobody can underestimate the importance of these factors, which can be both general and particular. Such factors can be those which may have influenced a social unit directly or indirectly. If a case study is to be considered complete, it is essential that in the first instance these factors should be identified and then each such factor should be very carefully and thoroughly studied. Usually, indirect factors are given less importance as compared with direct factors. It is essential that this tendency should be avoided in case it is desired that the study should be completed and perfect.

(iv) **Analysis of Data:** In this method of study, a social unit can be an individual/ group or a particular community. The investigator collects data from his social unit and makes records taking into consideration various influences and factors. Once the data has been collected, it should be carefully and properly recorded and analysed. As in the case of every other method of social research, in this case also, both at the data collection and analysis stage no biases or prejudices should be introduced.

3.8.3 Sources of Information for a Case Study

Case studies can be prepared with the help of personal documents, life histories and other similar sources. Personal documents, as the name itself indicates, relate to the personal lines of people. These give a lot of information about episodes connected with the lives of the individuals, the place(s) at which the episodes occurred also help in knowing concepts, attitudes and experiences of the persons concerned. The documents also help in knowing the circumstance of those happenings and stages through which a person had to pass. With the help of personal documents, ample light is thrown on the personal life of the individual, which otherwise would have remained obscure. Information about the mental make-up and psychological thinking of the person with whom the documents are connected are also provided as is information about social relationships and whether the person concerned was socially placed in an advantageous and disadvantageous position. If the person concerned has been frank in his personal documents, then many secrets, which can be very [illegible] purposes, can come to light. In short, in the case stud[illegible] important role.

In addition to these two direct and important sources, there are certain indirect sources of information which include books and magazines, literature and previous studies, government documents and communications which are issued from time to time and made public.

3.8.4 Comparing the Statistical and Case Study Techniques

In some respects statistical techniques and case study method differ from each other, whereas in many other respects these frequently supplement each other. While discussing the relationship between the two, P.V. Young says: 'A case study seeks to determine social processes, it reveals the complexity of factors and indicates their sequences and their interrelationship. Statistical studies on the other hand, deal with relatively few factors but are able to provide scope by indicating extent, frequency, trends and degree of association.'

Also, statistics confirm or disprove hypothesis in studies of social changes, social adjustments and maladjustments. These also help in determining existing correlations and aid in avoiding conclusions based on exceptional cases. Statistical studies of human behaviour can be enriched if supplemented by individual case studies.

But there are certain basic differences between the case study and statistical techniques. One difference between the two is that statistical techniques are quantitative, whereas case study techniques are qualitative and do not much care about quantity. Another difference is that in the statistical method there is no in-depth study. Particular aspects of numerous methods are only studied. On the other hand, in case studies there is a deep and thorough study of each aspect of the case and thus there is all-round probing.

In so far as a case study is concerned, there is no question of sample size because a sample is picked up only when it is not possible to study the whole universe of study thoroughly. Since in a case study, the whole case is completely and thoroughly studied from all angles and proper care is taken of the past, present and future, the question of samples does not arise. On the other hand, in the statistical method samples are studied and thus there is a difference between the two in this regard as well.

In the statistical method there is no stress on emotional aspects of the subject matter because emotions cannot be quantified. Thus, in the statistical method there is only a formal and generalized approach to the whole problem. On the other hand, in the case study method, emotions play a very important role. In fact, it is difficult to study a case without taking into account the emotions which influenced a person when he was performing a particular action. Without properly accounting for emotions, research cannot properly stud[illegible]

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a final questionnaire and finally, both analysis and processing of material has to be treated statistically to confirm or reject the hypothesis and to determine a more precise correlation. In the word of M.H. Gopal, 'the case study has to turn to statistics, if its data are frequencies, types, trends, uniformities, patterns and so on'.

3.8.5 Advantages and Disadvantages of the Case Study Method

Case study method has its own value and difficulties. Some advantages are as follows:

- **Helps in the Formulation of Hypothesis:** One important advantage of this method is that case studies help in the formulation of hypotheses. Every researcher, before finalizing his hypothesis and questionnaire tries to get a proper analysis of many cases which centre around his subject problem. Preliminary study of the process is greatly facilitated by case documents, incorporating the pertinent data which will guide the later stages of research.
- **Intensive Study Possible:** Another advantage is that with the help of this method, it becomes possible to intensively study a unit from all aspects and this leads to deep probing, which is very essential for research work.
- **Helps in Understanding Behaviour Patterns:** One more advantage of this method is that being an exhaustive study, this method enables the researcher to understand behaviour patterns of the concerned unit directly and he can obtain information about the inner workings of the person concerned.
- **No Need of Sampling:** Sampling, as we know, involves many serious problems. Usually, there is a complaint that some biases and personal prejudices have found entry in the sample. It is a method in which there is no need to pick up any sample and as such all the defects of the sampling method do not get introduced in this method of study.
- **Study of Subjective Aspects:** It is with the help of this method that subjective aspects can be studied. This method lays stress on the psychology of the person concerned, behaviour of the individual, circumstances in which he placed himself and his reaction to certain situations of life and so on. It is with the help of the study that subjective aspects can be studied as thoroughly, as possible.
- **Helps Studying Social Changes:** The case study method also helps in studying social changes by thoroughly studying different facets of social units and changes which came with the passing of time by making certain inferences. It is an important method for understanding the past of a social unit and suggests measures for improvement in the present context.
- **Helps in Removing Defects of Research:** This is a method in which the case studied is the same throughout. It then becomes possible to find out, locate and identify possible defects which may creep in a full-scale study. In this way, this method helps in removing many defects of a full-fledged study.
- **Helps in Comparative Studies:** With the help of this method, two different sets of cases can be picked up and compared. Though in actual practice it is not possible to pick up two types of cases, but once that has been done, it becomes easy to compare the two and as such comparative study becomes both easy and possible.
- **Increases Knowledge:** When a researcher or an investigator studies all aspects of a social unit, both past, present and likely future and no aspect of the problem is left out, then his knowledge helps both in that particular study and in all his subsequent research work. In this way this method provides a very good training ground for social science researches.

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The case study method has its advantages but along with it there are certain disadvantages as well. The difficulties in the operation of this method need careful study. They are as follows:

- **False Sense of Prestige:** A difficulty is that the researcher develops a false sense of prestige. In the words of Goode and Hatt: 'The researcher comes to feel a false sense of certainty about his own conclusions. The danger then does not lie in any technical weakness of this approach but to social processes or individual as a whole.' The result of this certainty is that there is a temptation to ignore basic principles of research designs. As the researcher feels so very certain about the area of experience he is investigating, he feels no need to check the overall design of study.
- **No Method of Checking:** In this method, an investigator is assigned the responsibility of preparing a case history and study a particular case. He collects some information with some labour and provides that to society. But there is no method of checking the information, because no other investigator is working on the same or similar case. Thus, whatever information is supplied by the researcher that must be accepted. This is not healthy for an objective research.
- **Danger of Subjectivity:** In this method of study, there is every danger of subjectivity because in it the researcher is too closely associated with the social unit under study. The greater the rapport, the more subjective is the whole study. The subject then is more likely to be self-justificatory than factual.
- **Comparison Not Possible:** In social research, the investigator is to deal with human beings. They are quite different from each other. There values, attitudes, behaviours, reactions, circumstance, etc., very widely differ from each other. Accordingly, it is very difficult to find out two identical cases. As such it is not possible to compare two case studies, because no two case studies can be similar.
- **It is Time Consuming:** A case history can be prepared after a lot of labour. In fact, the whole process is very time consuming. The investigator has to study the case from all aspects and also its past, present and likely future. Not only this, his behaviour, approaches and attitudes on the one hand and the circumstances on the other are also required to be studied.
- **Possibility of Wrong Conclusions:** In this method, the investigator tries to arrive at certain conclusions on the basis of a case study. But it is always possible that the conclusions drawn on the basis of one case study may be wrong. In this way, the researcher may draw wrong conclusions to the disadvantage of society as a whole.
- **Reliability of Source Material:** A case study is prepared with the help of diaries, memories and personal papers and records of the persons concerned. Usually when a person records his life events, he does not give the true picture and considerable colouring is added. Usually more stress is laid on the strong and brighter side, whereas the weaker side is either concealed or underestimated and in many cases even sidelined and ignored. It is more or less impossible to remove colours from the events with the result that the conclusions drawn are coloured ones and thus undependable.

3.9 SUMMARY

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- The qualitative technique provides depth and detailed information for research. Depth and detail emerge through direct questioning and careful descriptions and will vary depending upon the nature and purpose of a particular study.
- In the quantitative technique, the data are studied from a variety of angles to explore the new facts. Analysis requires an observant, flexible and open-mind. It is worthwhile to prepare a plan of analysis before the actual collection of data.
- Historical research attempts to establish facts so as to arrive to conclusions concerning past events. It is a process by which a researcher is able to reach a conclusion as to the likely truth of an event in the past, studying objects available for observation in the present.
- The terminology of descriptive survey research is designed to obtain pertinent and precise information concerning the existing status of phenomena and, whenever possible, to draw valid generalizations for the facts discovered without making any interference or control over the situation. Such type of studies are restricted not only to fact finding but may often result in the formulation of important principles of knowledge and solution of significant problems concerning local, state, national and international issues.
- In experimental research, variables are manipulated and their effect upon other variables is studied. Experimental research provides a systematic and logical method for answering the question. Experimenters manipulate certain stimuli, treatment or environmental conditions and observe how the condition or behaviour of the subject is affected or changed. Their manipulation is deliberate and systematic.
- Qualitative research is a specific method of inquiry used in various different educational disciplines. Qualitative researchers aim to collect an in-depth perceptive of human activities and the reasons that govern such activities.
- The qualitative method exceptionally investigates the 'why' and 'how' of decision-making along with 'what', 'where' and 'when'.
- The techniques of participating and observing may differ from situation to situation. Some of the distinctive qualitative methods include the use of focus groups and key informant interviews.
- Qualitative research is a generic term specifically used for investigative methodologies. It emphasizes the use of the significant variables in their natural surroundings.
- The phenomenological research describes the 'subjective reality' of an event as perceived by the population under study; thus it is the study of a phenomenon. The underlying principle of the phenomenological approach is to elucidate and identify the specific phenomena through how they are perceived by the actors/ samples in a situation.
- Phenomenology is typically concerned with the study of experience from the perspective of the individual, 'bracketing' taken-for-granted assumptions and standard techniques of perceiving.

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- Epistemologically, phenomenological approaches are exclusively based in a paradigm of individual knowledge and subjectivity, and emphasize the significance of personal perspective and interpretation.
- The objective of qualitative phenomenological research is to describe a 'lived experience' of a phenomenon. Since this is a qualitative analysis of narrative data, hence methods used to analyse the data must also be quite different as compared to traditional or quantitative methods of research.
- The phenomenological researcher aims to provide affluent textured description of live experiences. The term live experience typically refers to a key perception and the focus of investigation for phenomenological study.
- Ethnographic research studies various national and foreign cultures to gain an understanding about native people who are isolated from civilization.
- The case study method is a technique by which an individual factor whether it be an institution or just an episode in the life of an individual or a group is analysed in its relationship with any other in the group.

3.10 KEY TERMS

- **Content analysis:** A research technique for the objective, systematic, and quantitative description of the manifest content of communication
- **Tabulation:** The process of transferring classified data from data-gathering tools to the tabular form in which they may be systematically examined
- **Sample survey:** Gathering relevant information about a smaller representation of the population under study
- **Variable:** Any feature or aspect of an event, function or process that, with its presence and nature, affects some other event or process which is being studied
- **Ethnography:** A qualitative research method that is used by anthropologists to describe a culture of a group

3.11 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Qualitative technique seeks to describe or explain psycho-social events from the point of view of people involved.
2. Computers help in organizing data by developing systematic and comprehensive classification schemes using code numbers for different categories and sub-categories.
3. Two methods of tabulation are: (i) Hand tabulation and (ii) Modern mechanical aids like the computer.
4. While interpreting results, researchers should keep in mind the influence of unstudied factors and also not ignore selective factors.
5. Historical research has some unique characteristics. It is non-experimental research, so no variables are directly manipulated by the researcher. It relies heavily on source materials from the past.
6. The two sources of historical research are: (i) Primary source and (ii) Secondary source.

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7. Two types of historical research are: (i) Legal research and (ii) Biographical research.
8. Two advantages of historical research are that firstly, the researcher is not physically involved in the situation under study and secondly, there is no danger of experimenter-subject interaction.
9. Survey research can include a status quo study or a study in which the interrelationships of sociological or psychological variables are determined and summarized.
10. In longitudinal surveys, one explores the status of variables as investigated on different points in time in order. Through such studies the changes in the status of the variables over a period of time are explored.
11. Descriptive surveys provide information on how to achieve a goal, by exploring possible ways and means on the basis of the experience of others or opinions of experts.
12. Descriptive research is the most widely used research method in education because at times, descriptive surveys are the only means through which opinions, attitudes, suggestions for important educational practices and instruction, and other data can be obtained.
13. In experimental research, variables are manipulated and their effect upon other variables is studied. Experimental research provides a systematic and logical method for answering the question. Experimenters manipulate certain stimuli, treatment or environmental conditions and observe how the condition or behaviour of the subject is affected or changed. Their manipulation is deliberate and systematic.
14. A pre-test is the test that is administered to the subjects before the independent variable, and a post-test is the test that is administered to the subjects after the independent variable is applied.
15. The qualitative researchers rely on the following methods for collecting required information:
    - Participant Observation
    - Non-Participant Observation
    - Field Notes
    - Reflexive Journals
    - Structured Interview
    - Semi-Structured Interview
    - Unstructured Interview
    - Analysis of Documents and Materials
16. The live experiences include the world of objects that are around us as we perceive them and our own live experience about our self, body and relationships. Characteristically, it is the "Locus of interaction between ourselves and our perceptual environments and the world of experienced horizons within which we meaningfully dwell together." It can be described as the world that is lived and experienced, i.e., a world that emerges significantly to consciousness in its qualitative approach. This lived world is considered pre-reflective as it automatically occurs before a person think about it or put it into narrative language. Typically, the lived world includes the day-to-day world that is completely filled with

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multifaceted meanings which structure the background of everyday actions and interactions.

17. Ethnography is a qualitative research method that is used by anthropologists to describe a culture of a group.
18. The objective of ethnography is to develop a comprehensive understanding of how people embedded in specific contexts experience and react to their social and cultural worlds.
19. A 'case study' is an intensive investigation of a social unit that may be an individual, a student or any organization.
20. An advantage of the case study method is that it can remove the defects of research by in-depth study of the subject.

3.12 QUESTIONS AND EXERCISES

Short-Answer Questions

1. Write a short note on the qualitative methods used by researchers to explore diverse issues.
2. List the limitations of historical research.
3. Which are the three strategies used to analyse educational concepts?
4. What is the purpose of descriptive surveys?
5. Name some most commonly used qualitative research methods.
6. List the limitations of the case study method.

Long-Answer Questions

1. Differentiate between the quantitative and qualitative methods of research in detail.
2. What should the researcher keep in mind while interpreting results using the quantitative technique?
3. Elaborate on the steps involved in historical research.
4. Describe the different types of surveys in the descriptive method.
5. Explain the two most commonly used method for qualitative research.
6. What are the essential characteristics of a case study? Discuss with the help of examples.

3.13 FURTHER READING

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UNIT 4 TOOLS AND TECHNIQUES IN EDUCATIONAL RESEARCH

Structure

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4.0 INTRODUCTION

In the process of decision-making, data plays a vital role. A researcher requires various data gathering tools which facilitate original research investigations and observations, leading to useful and valuable results. Data collection is essentially an important part of the research process. Researchers generally collect evidence either for verifying new hypotheses or for checking current conclusions. To accomplish their objectives, researchers obtain data from documentaries or field sources.

In this unit, some of the most commonly used tools or techniques for data collection like observation, questionnaire, interviews and sociometry are discussed. Each of these tools differs in their nature and scope. The researcher has to bear in mind the suitability of these tools, i.e., relevancy and effectiveness depending upon the type of problem under consideration.

A research study is a tedious task and calls for an exhaustive investigation on the part of the researcher. This quite often leads to accumulation of bulk data obtained from the research study. Even if the concerned study results in brilliant hypotheses or a generalized theory, it is the responsibility of the researcher to format this bulk study into a pattern that is easy to understand. This is where report writing comes into play.

The main aim of research is to discover principles that have universal application. Generally, research in education includes all such assumptions that are based on a large number of samples/units/objects. It would be impractical if not impossible to test or observe each unit of population under controlled conditions in order to arrive at principles having universal validity. A 'population' is any group of individuals/units that have one or more characteristics in common which are of interest to the researcher, for a particular research. A 'sample' is a small percentage of the larger group who are selected for research. A sample can be statistically explained as being a subset of a population. The sample will be able to give an idea of the characteristics of the larger group from where it has been drawn. It is possible to make deductions about the larger population on the basis of the sample. This unit discusses the concept of population and sample, methods of sampling, sampling design, sampling distribution and sampling errors.

In this unit, you will learn the concept of a research proposal. You will also learn about written and oral reports. An effective written report is a creative activity that requires a lot of imagination. It requires a lot of effort and patience in order to write a report. It is impossible to think of the progress of an organization without effective written reports, since most of the business activities require sending letters, reports, etc. Effective oral report involves verbal communication of an idea to a listener. An oral report saves time and builds a healthy atmosphere in an organization by bringing the employees closer to each other.

4.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Discuss the importance of various tools of educational research like observation, questionnaires, interviews and sociometry
- Apply the tools of education research practically
- Describe various probability and non-probability sampling methods
- Explain the significance of sampling techniqles in education research
- Specify the steps involved in sampling
- Describe the types of sampling
- Explain the different methods of sampling distribution
- Describe the concept of sampling errors
- Explain the concept of a research proposal
- Discuss the essentials of a research process
- Learn the characteristics of a good report
- List the precautions for writing research reports
- Explain the significance of written and oral reports

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4.2 OBSERVATION

Observations have lead to some of the most important scientific discoveries in human history. Charles Darwin used his observations of animal and marine life at the Galapagos Islands to help him formulate his theory of evolution that he described in *On the Origin of Species*. Today, social scientists, natural scientists, engineers, computer scientists, educational researchers and many others use observations as a primary research method.

The kind of observations one makes depends on the subject being researched. Traffic or parking patterns on a campus can be observed to ascertain what improvements could be made. Clouds, plants, or other natural phenomena can be observed as can people, though in the case of the latter one may often have to ask for permission so as to not violate any privacy issue.

Observation may be defined as a process in which one or more persons monitor some real-life situation and record pertinent occurrences. It is used to evaluate the overt behaviour of the individual in controlled and uncontrolled situations.

According to Jahoda: 'Observation method is a scientific technique to the extent that it (a) serves a formulated research purpose, (b) is planned systematically rather than occurring haphazardly, (c) is systematically recorded and related to more general propositions than presented as a set of interesting curios, and (d) is subjected to checks and controls with respect to validity, reliability, and precision much as is all other scientific evidence.'

According to Good and Hatt: 'Observation may take many forms and is at once the most primitive and the most modern of research techniques. It includes the most casual, uncontrolled experiences as well as the most exact film records of laboratory experimentation.'

4.2.1 Types of Observation

Observations may be classified in two types:

- **Participant Observation:** In the process of 'participant observation' the observer becomes more or less one of the group members and may actually participate in some activity or the other of the group. The observer may play any one of the several roles in observation, with varying degrees of participation, as a visitor, an attentive listener, an eager learner, or as a participant observer.
- **Non-Participant Observation:** In the process of 'non-participant observation', the observer takes a position where his presence is not felt by the group. He may

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follow the behaviour of an individual or characteristics of one or more groups closely. In this type of observation, a one-way 'vision screen' permits the observer to see the subject but prevents the subject from seeing the observer.

Observations may also be classified into the following categories:

(i) ***Natural Observation:*** Natural observation involves observing the behaviour in a normal setting and in this type of observation; no efforts are made to bring any type of change in the behaviour of the observed. Improvement in the collection of information can be done with the help of natural observations.

(ii) ***Subjective and Objective Observation:*** All observations consist of two main components, the subject and the object. The subject refers to the observer whereas the object refers to the activity or any type of operation that is being observed. Subjective observation involves the observation of one's own immediate experience, whereas the observations involving an observer as an entity apart from the thing being observed are referred to as the 'objective observation'. Objective observation is also known as the 'retrospection'.

(iii) ***Direct and Indirect Observation:*** With the help of the direct method of observation, one comes to know how the observer is physically present, in which type of situation is he present and then this type of observation monitors what takes place. Indirect method of observation involves studies of mechanical recording or the recording by some of the other means like photographic or electronic. Direct observation is relatively straightforward as compared to indirect observation.

(iv) ***Structured and Unstructured Observation:*** Structured observation works according to a plan and involves specific information of the units that are to be observed and also about the information that is to be recorded. The operations that are to be observed and the various features that are to be noted or recorded are decided well in advance. Such observations involve the use of special instruments for the purpose of data collection that are also structured in nature. But in the case of unstructured observation, its basics are diametrically against the structured observation. In such observations, the observer has the freedom to note down what he feels is correct and relevant to the point of study. This approach of observation is very suitable for exploratory research.

(v) ***Controlled and Non-Controlled Observation*:** Controlled observations are the observations made under the influence of some external forces. Such observations rarely lead to improvement in the precision of the research results. However, these observations can be very effective if these are made to work in coordination with mechanical synchronizing devices, film recordings, etc. Non-controlled observations are made in the natural environment, and reverse to the controlled observation these observations involve no influence or guidance of any type of external force.

4.2.2 Recording Techniques of Observation

Many different techniques may be employed to study and document a subject's behaviour. The data collection techniques are all accurate but may be suitable for different purposes. While certain methods help gather detailed descriptions of behaviour, certain others facilitate documenting behaviour promptly and with bare minimum description.

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- **Anecdotal Records:** Anecdotal records refer to a few sentences jotted down in a notebook. These sentences pertain to what the subject is engaged in at a particular moment. Only those behaviours that can be seen or heard and that can be counted are documented while creating an anecdotal record.
- **Narrative Description:** Narrative description is also known as running behaviour record and specimen record, and is a formal method of observation. When following this technique, you are supposed to record continuously, as detailed as possible, what the subject is doing and saying when alone or when interacting with other people. In its methodology, it is similar to anecdotal record but is definitely more detailed. The researcher studies the context setting, the behaviour patterns, and the order in which they take place. The main aim of this technique is to gain a objective description of a subject's behaviour without conjecture, analysis, or assessment.
- **Checklists:** Checklists are usually standardized forms which list specific skills and behaviours based on standard levels, or are specifically compiled by the researcher for a particular research study.
- **Interviewing:** In this observation technique, the researching team tries to identify the subject's feelings or beliefs that are not visible through simple observation. During the process of interviewing, everything that the subject says must be recorded exactly as is. The interviewer should avoid any kind of editing of the interview subscript.
- **Time Sampling:** This method is distinct from others in two ways—it monitor and keeps account of a few chosen samples of subject's behaviour, and only during prearranged periods of time. When a behaviour pattern is seen during the specified time interval, it is recorded. This technique therefore helps to gather representative examples of behaviour.
- **Frequency Counts:** In some cases, a researcher may be more interested in studying the frequency of an occurrence or behaviour or another pattern, such as how often a consumer buys a particular product or how often an individual started a conversation with a colleague. To get this data, the researcher will need to keep a count of the frequency of the particular behaviour and study how long the behaviour lasts. This is usually done by simply marking an occurrence on a chart each time the behaviour is repeated.
- **Event Sampling:** This technique is focused on observing specific behaviours or events in a subject's behaviour pattern. However, it does not take into account the frequency or the length of the recording interval.

4.2.3 Advantages of Observation

The advantages of observation are as follows:

- This technique is employed to observe characteristics of various designs of school buildings and equipment.
- For coaching purposes, an observation of various skills in games and athletics is made.
- A study of the significant aspects of personality which express themselves in behaviours can be made.
- The behaviour of the children in a classroom situation can be effectively analysed.
- The behaviour of those who cannot read, write or speak can be observed.

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- Observation of skills in the workshop is made directly.
- Observation of pupils' behaviour as recorded in the cumulative records of pupils could serve as anecdotal evidence and supply data for research studied.

4.2.4 Characteristics of Observation for Research

The characteristics of observation for research are as follows:

- Observation schedule should be specific.
- Steps should be systematic.
- It should be quantitative.
- It should be recorded immediately.
- It should be made by experts.
- Schedule should be scientific. We should be able to check and substantiate the results.

Symonds gives a list of nine essential characteristics of good observation:

(i) Good eyesight
(ii) Alertness
(iii) The ability to estimate
(iv) The ability to discriminate
(v) Good physical condition
(vi) An immediate record
(vii) Good perception
(viii) Freedom from preconceptions
(ix) Emotional disinterest

Planning Administration Aspect of Observation

This includes:

(a) Securing an appropriate group of persons to observe.
(b) Deciding and arranging any special conditions for the group.
(c) Determining the length of each observation period, the interval between periods and the number of periods.

Points to be considered while defining the activities are:

(a) Inclusion of those activities which are true representatives of the general category which one is studying.
(b) Defining those activities very carefully.

While arranging for the record, the following points should receive attention:

(a) Deciding the form for recording so as to make note taking easy and rapid.
(b) Deciding the use of appropriate symbols, abbreviations and some use of shorthand.

One can train oneself to record by:

(a) Training oneself to observe others as perception improves with practice.
(b) Studying manuals that list observation techniques.

Planning effective observation include the following:

- Sampling to be observed should be adequate. There should be an appropriate group of subjects.

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- Units of behaviour should be defined as accurately as possible.
- Method of recording should be simplified.
- Detailed instructions may be given to observers to eliminate the difference in perspective of observers.
- Too many variables may not be observed simultaneously.
- Excessively long periods of observation without interspersed rest periods should be avoided.
- Observers should be fully trained.
- Observers should be well equipped.
- Conditions of observation should remain constant.
- Number of observations should be adequate.
- Records of observation must be comprehensive.
- Length of each observation period, interval between periods and number of periods should be clearly stated.
- Interpretations should be carefully made.

4.2.5 Disadvantages of Observation

The disadvantages of observation are as follows:

- It is very difficult to establish the validity of observations.
- Many items of observation cannot be defined.
- The problem of subjectivity is involved.
- Observation may give undue stress to aspects of limited significance simply because they can be recorded easily, accurately and objectively.
- Various observers observing the same event may concentrate on different aspects of a situation.
- The observer has little control over the physical situation.
- Children being observed become conscious and begin to behave in an unnatural manner.
- Many children try to pose and exhibit at the time of observation.
- There are certain situations which the observer is not allowed to observe, and he is helpless in that way to produce an accurate account.
- It may not be feasible to classify all the events to be observed.
- Observation is a slow and laborious process.
- There may be lack of agreement among the observers.
- The data to be observed may be unmanageable.
- Observation needs competent observers and it may be difficult to find them.
- Observation is a costly affair. It involves lot of expenses on travelling, staying at the places where the event is taking place and purchase of sophisticated equipment.

4.3 QUESTIONNAIRE

A **questionnaire** is a tool for research, comprising a list of questions whose answers provide information about the target group, individual or event. Although they are often designed for statistical analysis of the responses, this is not always the case. This method was the invention of Sir Francis Galton. Questionnaire is used when factual information is desired. When opinion rather than facts are desired, an opinionative or attitude scale is

Check Your Progress

1. What are two types of observation?
2. Mention three limitations of observation.
3. What is 'participant observation'?
4. Give one advantage of observation.
5. Mention one point for planning effective observation.

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used. Of course, these two purposes can be combined into one form that is usually referred to as 'questionnaire'.

Questionnaire may be regarded as a form of interview on paper. The procedure for the construction of a questionnaire follows a pattern similar to that of the interview schedule. However, because the questionnaire is impersonal, it is all the more important to take care of its construction.

A questionnaire is a list of questions arranged in a specific way or randomly, generally in print or typed and having spaces for recording answers to the questions. It is a form which is prepared and distributed for the purpose of securing responses. Thus a questionnaire relies heavily on the validity of the verbal reports.

According to Goode and Hatt, 'in general, the word questionnaire refers to a device for securing answers to questions by using a form which the respondent fills himself'.

Barr, Davis and Johnson define questionnaire as, 'a questionnaire is a systematic compilation of questions that are submitted to a sampling of population from which information is desired' and Lundberg says, 'fundamentally, questionnaire is a set of stimuli to which literate people are exposed in order to observe their verbal behaviour under these stimuli'.

4.3.1 Types of Questionnaire

Figure 4.1 depicts the types of questionnaires that are used by researchers.

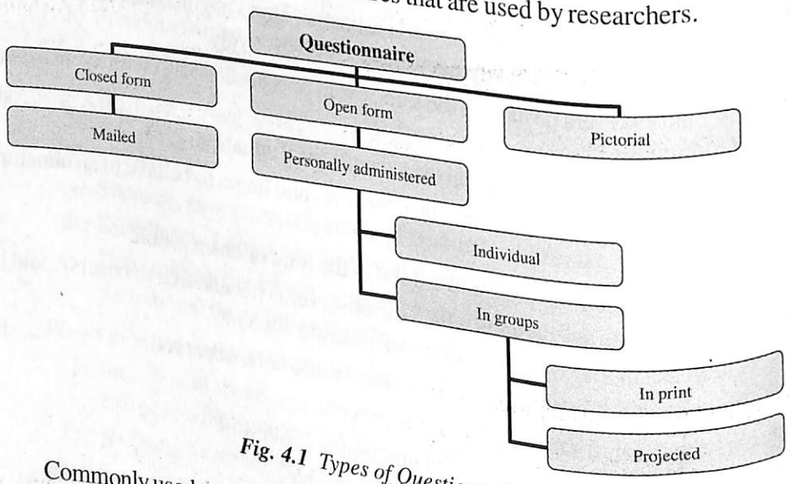

Fig. 4.1 Types of Questionnaires

Commonly used questionnaires are:

(i) Closed Form: Questionnaire that calls for short, check-mark responses are known as closed-form type or restricted type. They have highly structured answers like mark a yes or no, write a short response or check an item from a list of suggested responses. For certain type of information, the closed form questionnaire is entirely satisfactory. It is easy to fill out, takes little time, keeps the respondent on the subject, is relatively objective and is fairly easy to tabulate and analyse.

For example, How did you obtain your Bachelors' degree? (Put a tick mark against your answer)

(a) As a regular student

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(b) As a private student

(c) By distance mode

These types of questionnaires are very suitable for research purposes. It is easy to fill out, less time consuming for the respondents, relatively objective and fairly more convenient for tabulation and analysis. However, construction of such type of questionnaire requires a lot of labour and thought. It is generally lengthy as all possible alternative answers are given under each question.

(ii) Open Form: The open form, or unrestricted questionnaire, requires the respondent to answer the question in their own words. The responses have greater depth as the respondents have to give reasons for their choices. The drawback of this type of questionnaire is that not many people take the time to fill these out as they are more time consuming and require more effort, and it is also more difficult to analyse the information obtained.

Example: Why did you choose to obtain your graduation degree through correspondence?

No alternative or plausible answers are provided. The open form questionnaire is good for depth studies and gives freedom to the respondents to answer the questions without any restriction.

Limitations of open questionnaire are as follows:

- They are difficult to fill out.
- The respondents may never be aware of all the possible answers.
- They take longer to fill.
- Their returns are often few.
- The information is too unwieldy and unstructured and hence difficult to analyse, tabulate and interpret.

Some investigators combine the approaches and the questionnaires carry both the closed and open form items. In the close ended questions, the last alternative is kept open for the respondents to provide their optimum response. For example, 'Why did you prefer to join B.Ed. programme? (i) Interest in teaching (ii) Parents' wish (iii) For securing a government job (iv) Other friends opted for this (v) Any other.'

(iii) Pictorial Form: Pictorial questionnaires contain drawings, photographs or other such material rather than written statements and the respondents are to choose answers in terms of the pictorial material. Instructions or directions can be given orally. This form is useful for working with illiterate persons, young children and persons who do not know a specific language. It keeps up the interest of the respondent and decreases subjects' resistance to answer.

4.3.2 Questionnaire Administration Modes

Main modes of questionnaire administration are:

- **Through Mail:** Mailed questionnaires are the most widely used and also perhaps the most criticized tool of research. They have been referred to as a 'lazy person's way of gaining information'. The mailed questionnaire has a written and signed request as a covering letter and is accompanied by a self-addressed, written and stamped envelope for the return by post. The method of mailing out the

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questionnaire is less expensive in terms of time, funds required; it provides freedom to the respondent to work at his own convenience and enables coverage of a large population.

- **Personal Contact/Face-to-Face:** Personally administered questionnaires both in individual and group situations are also helpful in some cases and have the following advantages over the mailed questionnaire (i) the investigator can establish a rapport with the respondents; (ii) the purpose of the questionnaire can be explained; (iii) the meaning of the difficult terms and items can be explained to the respondents; (iv) group administration when the respondents are available at one place is more economical in time and expense; (v) the proportion of non-response is cut down to almost zero; and (vi) the proportion of usable responses becomes larger. However, it is more difficult to obtain respondents in groups and may involve administrative permission which may not be forthcoming.
- **Computerized Questionnaire:** It is the one where the questions need to be answered on the computer.
- **Adaptive Computerized Questionnaire:** It is the one presented on the computer where the next questions are adjusted automatically according to the responses given as the computer is able to gauge the respondent's ability or traits.

4.3.3 Appropriateness of Questionnaire

The qualities and features which make questionnaires an effective instrument of research and help to elicit maximum information are discussed below:

- **Type of Information Required:** The usefulness and effectiveness of a questionnaire is determined by the kind of information sought. Not every type of questionnaire can be elicited through it. A questionnaire which will consume more than 10-20 minutes is unlikely to be responded to well. Also, the questions should be explicit and capable of clear-cut replies.
- **Type of Respondent Reached:** A good deal depends upon the types of respondents covered by the questionnaire. All types of individuals cannot be good respondents. Only literate and socially conscious individuals would give any consideration to a questionnaire. Also, the respondent must be competent to answer the kind of questions contained in a particular questionnaire.
- **Accessibility of Respondents:** Questionnaires sent by mail can help to survey the opinion of the people living in far-flung places.
- **Precision of the hypothesis:** Appropriateness of the questionnaire also depends upon how realistic is the hypothesis in the mind of the researcher. The researcher must frame his questions in such a manner that they elicit responses needed to verify the hypothesis.

4.3.4 Types of Questions

There are many types of questions that can be asked, but the way to get to the correct answer is to know which the right question is. It requires knowledge and expertise to design the correct type of questionnaire.

The following is a list of the different types of questions which can be included in questionnaire design:

- **Open Format Questions:** Open format questions are those which give the respondent a chance to communicate their individual opinions. There are no set answers to choose from. Responses from open format questionnaires are insightful and even unexpected. Qualitative questions are an example of open format questions. An ideal questionnaire is one which ends with an open format question giving the respondent the chance to state their opinion or ask for their suggestions.

  Example: 'State your opinion about the grading system in education.'

  A respondent's answer to an open-ended question is coded into a response scale afterwards. An example of an open-ended question is a question where the testee has to complete a sentence (sentence completion item).
- **Closed Format Questions:** Multiple choice questions are the best example of closed format questions. Closed format questions generate responses that can be statistics or percentages in nature. Preliminary analysis can also be performed with ease. Closed format questions have the added advantage of being able to monitor opinions over a period of time as they can be put to different groups at different intervals.

  Example: 'Who is not an educationist among the following?'

  (i) Prof Yashpal, (ii) John Dewy, (iii) Milkha Singh, (iv) Rabindranath Tagore.
- **Leading Questions:** These types of questions force the audience to give a particular type of answer. Example of a leading question, 'How would you rate the grading system in India?'

  (i) Fair, (ii) Good, (iii) Excellent and (iv) Superb
- **Likert Questions:** Likert questions can help you ascertain how strongly your respondent agrees with a particular statement. Likert questions can also help to assess liking and disliking. Example: 'Are you punctual in attending your classes?'

  (i) Always, (ii) Mostly, (iii) Normally, (iv) Sometimes and (v) Never
- **Rating Scale Questions:** In rating scale questions, the respondent is asked to rate a particular issue on a scale that may range from poor to good. Rating scale questions usually have an even number of choices, so that respondents are not given the choice of a middle option. Example: 'How was the food at the restaurant?'

  (i) Good, (ii) Fair, (iii) Poor and (iv) Very Poor

Questions to be Avoided During a Questionnaire

The following questions should be avoided when preparing a questionnaire:

- **Embarrassing Questions:** Embarrassing questions are those that ask respondents about their personal and private life. Embarrassing questions are mostly avoided.
- **Positive/Negative Connotation Questions:** While defining a question, strong negative or positive overtones must be avoided. Depending on the positive or negative association of our question, we will get different data. Ideal questions should have neutral or subtle overtones.
- **Hypothetical Questions:** Hypothetical questions are questions that are based on assumption and hope. An example of a hypothetical question would be 'If you were a Director in the Education department what changes would you bring about?' These types of questions force the respondent to give his ideas on a particular subject. However, these kinds of questions do not give consistent or clear data.

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4.3.5 Steps Preparing and Administering the Questionnaire

The steps involved in preparing and administering the questionnaire are as follows:

(i) **Planning the Questionnaire:** One should get all the help possible in planning and constructing the questionnaire. Other questionnaires should be studied and items should be submitted for criticism to other members of the class or faculty.

(ii) **Modifying Questions:** Items can be refined, revised or replaced by better items. If a computer is not readily available for easily modifying questions and rearranging the items, it is advisable to use a separate card or slip for each item. This procedure also provides flexibility in arranging items in the most appropriate psychological order before the instrument is finalized.

(iii) **Validity and Reliability of Questionnaire:** Questionnaire designers rarely deal with the degree of validity of reliability of their instrument. There are ways to improve both validity and reliability of questionnaires. Basic to the validity of a questionnaire is asking questions in the least ambiguous way. The meaning of all terms must be clearly defined so that they have the same meaning to all respondents. The panel of experts may rate the instrument in terms of how effectively it samples significant aspects of content validity. The reliability of the questionnaire may be tested by a second administration of the instrument with a small sub-sample, comparing the responses with those of the first. Reliability may also be estimated by comparing the responses of an alternate form with the original from.

(iv) **Try Out or Pilot Testing:** The questionnaire should be tried on a few friends and acquaintances. What may seem perfectly clear to the researcher may be confusing to the other person who does not have the frame of reference that the researcher has gained from living with and thinking about an idea over a long period. It is also a good idea to pilot test the instrument with a small group of persons similar to those who will be used in the study. They may reveal defects that can be corrected before the final form is printed.

(v) **Information Level of Respondents:** It is important that the questionnaire be sent only to those who possess the desired information and are likely to be sufficiently interested to respond objectively and conscientiously. A preliminary card asking whether the individual would respond is recommended by some research authorities.

(vi) **Getting Permission:** If the questionnaire is to be used in a public school, it is essential that approval for the project is secured from the Principal. Students should be informed that participation is voluntary. If the desired information is delicate or intimate in nature, the possibility of providing for anonymous responses should be considered. The anonymous instrument is most likely to produce objective and honest responses.

(vii) **The Cover Letter:** A courteous, carefully constructed cover letter should be included to explain the purpose of the study. The cover letter should assure the respondent that all information will be held in strict confidence. The letter should promise some sort of inducement to the respondent for compliance with the request. In educational circles, a summary of questionnaire results is considered an appropriate reward, a promise that should be scrupulously honoured after the study has been completed.

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(viii) **Follow-Up Procedures:** Recipients are often slow to return completed questionnaires. To increase the numbers of returns, a vigorous follow-up procedure may be necessary. A courteous postcard reminding the recipient may bring in some additional responds. A further step in follow-up may involve a personal letter or reminder. In extreme cases, it may be appropriate to send the copy of questionnaire with a follow-up letter.

(ix) **Analysing and Interpreting Questionnaire Responder:** Data obtained by the questionnaire is generally achieved through calculation and counting. The totals are converted into proportion or percentages. Calculation of contingency coefficient of correlation is often made in order to suggest probability of relation among data. Computation of chi-square statistics in is also advisable.

4.3.6 Improving the Validity of a Questionnaire

The validity of the information collected through a questionnaire can be improved by using the following techniques:

- The questions should be relevant to the subject or problem.
- The questions should be perfectly clear and unambiguous.
- The questions should be retroactive and not repulsive.
- Check whether the information has been collected from a reasonably good proportion of respondents.
- The information should show a reasonable range of variety.
- The information should be consistent with what is already known or is expected.
- Use another external criterion like consultation of documents, or interview with a small group of respondents to cross check the truthfulness of the information given through the questionnaire.

Question sequence should be the following:

- Questions should flow logically from one to the next.
- The researcher must make sure that the answer to a specific question is not prejudiced by earlier questions.
- Questions should flow from the more general to the more specific.
- Questions should follow an order which goes from the least sensitive to the most sensitive.
- Questions should flow from factual and behavioural questions to attitudinal and opinion questions.
- Questions should flow from unaided to aided questions.

The three stages theory (also known as the sandwich theory) should be applied when sequencing questions. The order to be followed should be first, screening and rapport questions; second, the product specific questions; and third, demographic questions.

4.3.7 Questionnaire Construction Issues

The following are the various issues in construction of questionnaire:

- It is very important to know exactly how you are going to use the information received from the research conducted. If the research or information cannot be

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implemented or acted upon, then the research would just have been a waste of time, money and effort.

- Clear parameters regarding the research's aims and scope should be drawn before starting the research. This would include the questionnaire's time frame, budget, manpower, intrusion and privacy.
- The target audience selected will depend on how arbitrarily one has chosen the respondents and what the selection criteria are.
- The framework of expected responses should be clearly defined so that the responses received are not random.
- Only relevant questions should be included in the questionnaire as unrelated questions are a burden on the researcher and respondent.
- If you have formed a hypothesis which you want to research then you will know what questions need to be asked.
- The respondents' background and education should not influence the way they answer the questions.
- The type of scale, index, or typology to be used shall be determined.
- The way the data has been compiled will determine what information can be gathered, e.g., if the response option is yes/no then you will only know how many or what percent of your sample answered yes/no, we will not know how the average respondent answered.
- The questions asked (closed, multiple-choice, and open) should adhere to the statistical data analysis techniques available and your goals.
- Questions and prepared responses to choose from should not be biased. A biased question or questionnaire influences the responses given.
- The order in which the questions are presented or asked is also important as the earlier questions and their responses may influence the later ones.
- The wording should be kept simple to avoid ambiguity. Ambiguous words may cause misunderstanding, possibly invalidating questionnaire results. Double negatives should also be avoided.
- Questions should address only one issue at a time so that the respondent is not confused as to what response is required.
- The list of possible responses should be comprehensive so that respondents should not find themselves without a suitable response. A solution to this would be to add the category of 'other'.
- Categories in the questionnaire should be kept separate. For example, in both the 'married' category and the 'single' category — there may be a need for separate questions on marital status and living situation.
- Writing style should be informal yet to the point and suitable for the target audience.
- Personal questions about age, income, marital status, etc. should be placed at the end of the survey so that even if the respondent is hesitant to give out personal information, they would still have answered the other questions.
- Questions which try to trick the respondent may end in inaccurate responses.
- Presentation which is pleasing to the eye with the use of colours and images can end up distracting the respondent.
- Numbering the questions would be helpful.

- Whoever administers the questionnaire, be it research staff, volunteers or whether self-administered by the respondents, it should have clear, detailed instructions.

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Factors Affecting Reliability of Answers

The following are the various factors that affect the reliability of answers:

- **Confusing Questions:** If the questions are not easily intelligible or they are capable of being interpreted in more than one way, the answers are unreliable, because the answer may be the result of misinterpretation of the questions not intended by the researcher.
- **Prejudice Regarding Sample:** The responses received from the sample may not be true representations of the sample.
- **Lack of Coverage to Illiterates:** This method is inapplicable to illiterates and semi-illiterates as they will be unable to read the questions.
- **Response Selectivity:** The respondents of a questionnaire may belong to a selected group. Therefore the conclusions lack the kind of objectivity and representativeness essential for its validity.

4.3.8 Disadvantages of the Questionnaire Method

Like all other methods, the questionnaire is also limited in value and application. This means that it cannot be used in every situation and that its conclusions are not always reliable. Chief limitations of the method are:

- **Limited Response:** As noted earlier, this method cannot be used with illiterate or semi-illiterate groups. The number of persons who cooperate and respond to the questionnaire is very small.
- **Lack of Personal Contact:** There is very little scope of personal contact in this method. In the absence of personal contact, very little can be done to persuade the respondents to fill up the questionnaire.
- **Useless In-Depth Problems:** If a problem requires deep and long study, it is obvious that it cannot be studied by the questionnaire method.
- **Possibility of Wrong Answers:** A respondent may not really understand a question or may give the answer in a casual manner. In both cases, there is a strong likelihood of misleading information being given.
- **Illegibility:** Some persons write so badly that it is difficult to read their handwriting.
- **Incomplete Response:** There are people who give answers which are so brief that the full meaning is incomprehensible.

4.3.9 Importance of Questionnaire Method

As a matter of fact, this method can be applied in a very narrow field. It can be used only if the respondents are educated and willing to cooperate. However, it is still widely used, owing to the following merits:

- **Economical:** The questionnaire requires paper, printing and postage only. There is no need to visit the respondents personally or continue the study over a long period.
- **Time Saving:** Besides saving money, the questionnaire also saves time. Data can be collected from a large number of people within a small time frame.

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- **Most Reliable in Special Cases:** It is a perfect technique of research in some cases.
- **Research in Wide Area:** Mailed questionnaire comes very handy if the sample comprises of people living at great distances.
- **Suitable in Specific Type of Responses:** The information about certain problems can be best obtained through questionnaire method.

4.4 INTERVIEW

One of the main methods of data collection is conducting interviews. It takes place as a two-way conversation between the researcher and the respondent, whereby information is gathered by asking topic related questions.

We learn not only from the respondents' responses but also his gestures, facial expressions and pauses. Interviewing can be conducted either face-to-face or over the telephone by skilled personnel by using a structured schedule or an unstructured guide.

According to Rummel J. Francis: 'The interview method of collecting data requires the actual physical proximity of two or more persons, and generally requires that all the normal channels of communication be open to their use. It is necessary to see one another, to hear each other's voices, to understand one another's language, and to use all that is psychologically inherent in physical proximity. It usually entails a non-reciprocal relation between the individuals concerned. One party desires to get information from another — one party interviews the other — for a particular purpose.'

Theodore L. Torgerson has stated that the interview method of study extends certain aspects of the observational technique.

Thus, the interview method permits the gathering of development data to supplement the cross-sectional data obtained from observations. The interviewer can probe into casual factors, determine attitudes, discover when the problem started, enlist the interviewee in an analysis of his own problem and secure his support of the therapy to be applied.

4.4.1 Types of Interviews

The different types of interviews are as follows:

- **Group Interview:** A proper setting for group interviews requires a group of not more than 10 to 12 persons with some social, intellectual, and educational homogeneity, which ensures effective participation by all. For a full spontaneous participation of all, it is better to arrange a circular seating arrangement.
- **Diagnostic Interview:** Its purpose is to locate the possible causes of an individual's problems, getting information about his past history, family relations and personal adjustment problem.
- **Clinical Interview:** Such an interview follows after the diagnostic interview. It is a means of introducing the patient to therapy.
- **Research Interview**: Research interview is aimed at getting information required by the investigator to test his hypothesis or solve his problems of historical, experimental, survey or clinical type.

Check Your Progress

6. What are the modes of getting information through questionnaire?
7. List two steps to improve the validity of a questionnaire.
8. How can questionnaires be administered?

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- **Single Interviewer or Panel Interviews:** For the purpose of research, a single interviewer is usually present. In case of selection and treatment purposes, panel interviews are held.
- **Directed Interview:** It is structured, includes questions of the closed type and is conducted in a prepared manner.
- **Non-Directive:** It includes questions of the open-end form and allows much freedom to the interviewee to talk freely about the problem under-study.
- **Focused Interview:** It aims at finding out the responses of individuals to exact events or experiences rather than on general lines of enquiry.
- **Depth Interview:** It is an intensive and searching type of interview. It emphasizes certain psychological and social factors relating to attitudes, emotions or convictions.

It may be observed that on occasions several types are used to obtain the needed information.

Other classifications of interviews are:

- Intake interview, as the initial stage in clinic and guidance centres.
- Brief talk contacts as in schools and recreation centres.
- Single-hour interview.
- Clinical-psychological interview, stressing psychotherapeutic counselling and utilizing case history data and active participation by the counsellor in the re-education of the client.
- Psychiatric interviews, similar to psychological counselling, but varying with the personality and philosophical orientation of the individual worker and with the setting in which used.
- Psychoanalytic interviews.
- The interview form of test.
- Group interviews for selecting applicants for special course.
- Research interview.

4.4.2 Important Elements of Research Interview

The important elements of research interview are as follows:

(i) Preparation for Research Interview

- Decide the category and number of persons that you would like to interview.
- Have a clear conception of the purpose and the information required.
- Prepare a clear outline, a schedule or a check list of the best sequence of questions that will systematically bring out the desired information.
- Decide the type of interview that you are going to use, i.e., structured or non-structured interview.
- Have a well thought-out plan for recording responses.
- Fix up the time well in advance.
- Procure the tools to be used in recording responses.

(ii) Executing an Interview

- Be friendly and courteous and put the respondent at ease so that he talks freely.

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- Listen patiently to all opinions and never show surprise or disapproval of a respondent's answer.
- Assume an interested manner towards the respondent's opinion, and as far as possible do not divulge your own.
- Keep the direction of the interview in your own hands and avoid irrelevant conversation and try to keep the respondent on track.
- Repeat your questions slowly and with proper emphasis in case respondent shows signs of failing to understand a particular question.

(iii) Obtaining the Response

Perhaps the most difficult part of the job of an interviewer is to obtain a specific, complete response. People can often be evasive and answer 'do not know' if they do not want to make the effort of thinking. They can also misunderstand the question and answer incorrectly in which case the interviewer would have to probe more deeply.

An interviewer should be skilled in the technique as only then can he gauge whether the answers are incomplete or non-specific. Each interviewer must fully understand the motive behind the asking of the particular question and whether the answer is giving the information required. He should form the habit of asking himself, 'Does that completely answer the question that I just asked?'

Throughout, the interviewer must be extremely careful not to suggest a possible reply. The interviewer should always content himself with mere repetition (if the question is not understood to answer).

(iv) Reporting the Response

There are two chief means of recording opinion during the interview. If the question is preceded, the interviewer need only check a box or circle or code, or otherwise indicate which code comes closest to the respondent's opinion. If the question is not preceded, the interviewer is expected to record the response verbatim.

The following points may be kept in view in this respect:

- Quote the respondents directly, just as if the interviewers were newspaper reporters taking down the statement of an important official without paraphrasing the reply, summarizing it in the interviewer's own words, 'polishing up' any slang, or correcting bad grammar that distorts the respondent's meaning and emphasis.
- Ask the respondent to wait until the interviewer gets down 'that last thought'.
- Do not write as soon as you have asked the question and do not write while the respondent talks. Wait until the response is completed.
- Use common abbreviations.
- Do not record and evaluate the responses simultaneously.

(v) Closing the Interview

It should be accompanied by an expression of thanks in recognition of the respondent's generosity in sparing time and effort.

(vi) Use of Tape Recorder in Interview

- It reduces the tendency of the interviewer to make an unconscious selection of data favouring his biases.
- The tape recorded data can be played more than once, and thus it permits a thorough study of the data.
- Tape recorder speeds up the interview process.
- Tape recorder permits the recording of some gestures.
- The tape recorder permits the interviewer to devote full attention to the respondent.
- No verbal productions are lost in a tape recorded interview.
- Other things being equal, the interviewer who uses a tape recorder is able to obtain more interviews during a given time period than an interviewer who takes notes or attempts to reconstruct the interview from memory after the interview has been completed.

4.4.3 Indifferent Attitude of the Respondent and the Role of the Research Worker

It is observed that the research worker is likely to encounter several problems arising out of the apathy of the respondents. In such a situation the following points may be kept in view:

(i) When the respondent is really busy and has no time, the field worker may request for a more convenient time.

(ii) When the respondent simply wants to avoid the interview and is not inclined to be bothered about it, the field worker should try to explain to him the importance of the study, and how his own response is of material value in the case.

(iii) When the respondent is afraid to give the interview as it affects his boss or the party to which he belongs or any other cause which is likely to harm his interest, the field worker must assure the respondent that absolute secrecy would be maintained by the researcher and the organization.

(iv) When the respondent does not hold a high opinion about the outcome of such interviews in general, or has a poor opinion about the research organization or institution conducting it, it is the duty of the research worker at such times to explain to him the importance of the problem and convince him regarding the status of the research body.

(v) When the respondent is suspicious and he thinks that that the enquiry is either from the income tax department or some other secret agency, at such times he may generally ask such questions. Who are you? Who told you our name? Have you interviewed the neighbour?, etc. The research worker should try to eliminate his suspicion. A letter of authority, the letter head or the seal of the research body would prove to be useful on such occasions.

(vi) When the respondent is unsocial or otherwise confined to his own family (such a tendency is mostly found in the case of newly married couples), the research worker at such times will try to create his interest in the subject of investigation.

(vii) When the respondent is too haughty and thinks it below his dignity to grant an interview to petty research workers, the investigator should get a letter of introduction from an influential person.

4.4.4 Advantages and Disadvantages of the Interview Method

Advantages of Interview Over other Techniques

- A well-trained interviewer can obtain more data and greater clarity by altering the interview situation. This cannot be done in a questionnaire.

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- An interview permits the research worker to follow-up leads as contrasted with the questionnaire.
- Questionnaires are often shallow and they fail to dig deeply enough to provide a true picture of opinions and feelings. The interview situation usually permits much greater depth.
- It is possible for a skilled interviewer to obtain significant information through motivating the subject and maintaining rapport, other methods do not permit such a situation.
- The respondents when interviewed may reveal information of a confidential nature which they would not like to record in questionnaire.
- Interview techniques can be used in the case of children and illiterate persons who cannot express themselves in writing. This is not possible in a questionnaire.
- The percentage of response is much higher than in case of a mailed questionnaire.
- ***Removal of Misunderstanding:*** The field worker is personally present to remove any doubt or suspicion regarding the nature of enquiry or meaning of any question or term used. The answers are, therefore, not biased because of any misunderstanding.
- ***Creating a Friendly Atmosphere*:** The field worker may create a friendly atmosphere for proper response. He may start a discussion, and develop the interest of the respondent before showing the schedule. A right atmosphere is very conducive for getting correct replies.
- ***Possible to Secure Confidential Interview*:** The interviewee may disclose personal and confidential information which he would not ordinarily place in writing on paper. The interviewee may need the stimulation of personal contacts in order to be drawn out.
- ***Advantages of Clues*:** The interview enables the investigator to follow-up leads and to take advantage of small clues, in dealing with complex topics and questions.
- ***Permits Exchange of Ideas*:** The interview permits an exchange of ideas and information. It permits 'give and take'.
- ***Useful in the Case of Some Categories of Persons*:** The interview enables the interviewee to deal with young children, illiterates and those with limited intelligence or in who's state of mind is not quite normal.
- ***Useful Apart from Research Purposes*:** Interviews are also used for pupil counselling, for selection of candidates for instructional purposes, for employment, for psychiatric work, etc.
- ***Possibility of Asking Supplementary Questions:*** The respondent does not feel tired or bored. Supplementary questions may be put to enliven the whole discussion.
- ***Avoiding Handwriting*:** The difficulties of bad handwriting of the respondent, use of pencil, etc., are also avoided as every schedule is filled in by the interviewer.
- ***A Probe into Life Pattern is Possible*:** The personal contact with the respondent enables the field worker to probe more deeply into the character, living conditions and general life pattern of the respondent. These factors have a great bearing in understanding the background of any reply.
- ***Reliable Information*:** The information gathered through interviews has been found to be fairly reliable.
- ***Deeper Probe*:** It is possible for the interviewer to probe into attitudes, discover the origin of the problem, etc.
- ***Interview Technique is Very Close to the Teacher*:** It is generally accepted that no research technique is as close to the teacher's work as the interview.
- ***Possibility of Repetition*:** Sometimes interviews can be held at suitable intervals to trace the development of behaviour and attitudes.
- ***Useful for Several Purposes*:** Interviews can be used for student counselling, occupational adjustment, selection of candidates for educational courses, etc.
- ***Wide Applicability*:** Interviews can be used for all kinds of research methods — normative, historical, experimental, case studies and clinical studies.
- ***Cross Questioning*:** Interview techniques provide scope for cross questioning.
- ***Command of the Interviewer*:** This technique allows the interviewer to remain in command of the situation throughout the investigation.
- ***Wider Opportunities to Know the Interviewee*:** Through the respondent's incidental comments, facial expression, bodily movements, gestures, etc., an interviewer can acquire information that could not be obtained easily by other means.
- ***Useful for Judging Frankness, etc.*:** Cross questioning by the interviewer can enable him to judge the sincerity, frankness and insight of the interviewee.

Disadvantages of the Interview Method

The method of interview, in spite of its numerous advantages, also has the following limitations:

- **Very Costly:** It is a very costly affair. The cost per case is much higher in this method than in case of mailed questionnaires. Generally speaking, the cost per questionnaire is much less than the cost per interview.

  A large number of field workers may have to be engaged and trained in the work of collection of data. All this entails a lot of expenditure and a research worker with limited financial means finds it very difficult to adopt this method.
- **Biased Information:** The presence of the field worker while encouraging the respondent to reply, may also introduce a source of bias in the interview. At times the opinion of the respondent is influenced by the field worker and his replies may not be based on what he thinks to be correct but what he thinks the investigator wants.
- **Time Consuming:** It is a time consuming technique as there is no guarantee how much time each interview can take, since the questions have to be explained, interviewees have to assured and the information extracted.
- **Expertness Required:** It requires a high level of expertise to extract information from the interviewee who may be hesitant to part with this knowledge.

  Among the important qualities to be possessed by an interviewer are objectivity, insight and sensitivity.

4.5 SOCIOMETRY

Sociometry is 'a method for discovering, describing and evaluating social status, structure, and development through measuring the extent of acceptance or rejection between individuals in groups'. Franz defines sociometry as 'a method used for the discovery and

Check Your Progress

9. Define 'interview'.
10. Name the various types of interviews.
11. Give one advantage of using a tape recorder in an interview.
12. Name one advantage that the interview has over other methods of research like questionnaire.

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manipulation of social configurations by measuring the attractions and repulsions between individuals in a group'.

It is a technique to study the choices a person makes, the way he communicates and interacts with other people in his group. It is concerned with the dynamics between individuals in a group. In this method, a person is asked to select one or more persons from the group, given certain criteria and it is interesting to note who the person would choose.

William J. Goode and others state: 'These and other variants of sociometric techniques offer rather simple methods of ranking individuals on a continuum of 'acceptability' or 'outgoingness' on the part of group members. When their use is justified they may be powerful research tools since they meet the general problems of scaling very well.'

4.5.1 Sociometry Test

The key method used in this technique is the 'sociometric test'. Here, a member of a group is asked to select amongst the other members who they would choose for certain situations. The situation must be a real one to the group under study, e.g., 'group study', 'play', 'classroom seating', class monitor for students of a school, etc.

The person can be allowed to make two or three choices depending on the size of the group and each choice can be assigned a level of preference. For example, if asked to choose from a group of eight who they would like to work with for a group assignment, the person can choose three people stating his preference by numbering them 1, 2 and 3.

Another example would be, each member of a group consisting of 10 students is asked to write his first, second and sometimes third choices about some significant and pertinent type of social setting. He may be asked questions like:

- Whom would you choose to be the secretary of your debating society?
- Whom would you like to sit next to you in the class or in the bus while going for a picnic?
- With whom do you enjoy the most?
- With whom would you like to work in the science laboratory?
- With whom would you like to walk home?

All these questions are positive questions and hence show social acceptances. Negative questions may also be given to show social rejections.

In the above example, the individual has to name three persons in order of preference.

Data may be tabulated as under:

(i) Let the members of the group be numbered from A to J.

(ii) Write 'Choosers' in the vertical column and 'Chosen' in the horizontal column.

(iii) Total choices received by each member are shown at the bottom.

(iv) In the cells, check marks may be shown.

(v) Let 'f' stand for first, 's' for second and 't' for third choices respectively.

(vi) Add the number of each choice.

A similar table can be prepared for social rejections. In the vertical column will be listed the 'rejecters' and in the horizontal column 'rejectees'.

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Sociometric Matrix Showing who Chooses Whom

Q.: Whom would you like to be the secretary of your club?

Chosen / Choose	A	B	C	D	E	F	G	H	I	J
A			F		S	T				
B				F	S	T				
C	S	F			T					
D			T		S		F			
E			F						S	T
F							F	S		T
G	F			S	T					
H		F			S					T
I					T			S		F
J		F			S			T		
First Choice	1	3	2	1	-	-	2	-	-	1
Second Choice	1	-	-	1	5	-	-	2	1	-
Third Choice	-	-	1	-	3	2	-	1	-	3
Total	2	3	3	2	8	2	2	3	1	4

Guidelines for using this Technique and Interpretation of the Sociomatrix

- One person should be concentrated on at a time.
- A detailed study of the choices made and received should be made.
- The 'isolates' and the 'starts' may be looked for. An 'isolate' is one whom nobody chooses. Of course he is not rejected. A 'star' is a member of the group who receives most of the choices. Here 'E' is the 'star' with maximum choices in his favour. 'I' is an 'isolate' with one choice only.
- Attempts should be made to discover the causes for such selections.

  An individual may be isolated because:

  (a) She is a new member of the group.

  (b) She has a shy and withdrawing nature.

  (c) She does not try to be friends with others.

  (d) She may belong to a lower or upper socio-economic level and therefore is not acceptable to the group.
- Look for individuals who select each other. This might be due to factors like:

  (a) Close relations.

  (b) Neighbours.

  (c) Common interests and the like.
- A triangle shows three persons selecting each other. This may be an evidence of cliques, or sharp divisions in the group.

Simplest Form of Sociometric Matrix

For finding out the social structure of a group of 10 students, a one line questionnaire is given to the students asking 'Who would you like to be the secretary of your club'?

Students would be asked to mark against one of the roll members. Thereafter, the results are tabulated.

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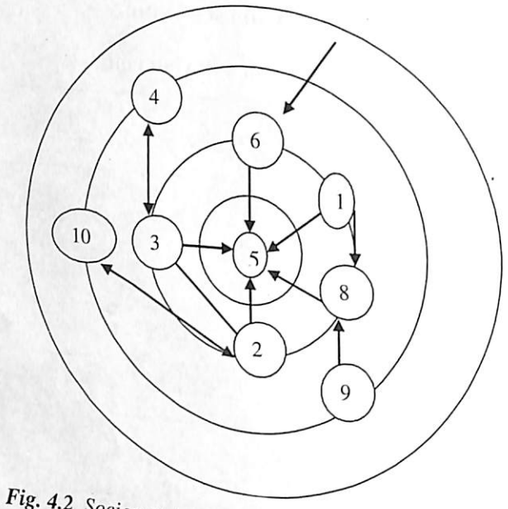

Fig. 4.2 Sociometric Matrix Showing Who Chooses Whom

Interpretation

- Student No. 5 is the 'star' as she has been chosen by the maximum number of students.
- Student Nos. 7 and 9 did not get any choice. This indicates that they tended to be isolated, i.e., not being social.
- Student No. 2 and 8 came next to roll No. 1.
- Student No. 5, i.e., the star has preferred roll No. 2.
- Mutual choices were: 2 and 5; 3 and 4; and 2 and 10.

Role of the Research Worker

In general, the research worker can work on three points:

(i) Providing opportunities for developing friendly relations.

(ii) Improving social skills of the group.

(iii) Building up competency for among the group members.

Reliable results can be achieved only when all the members constituting a group are fully acquainted with each other. The worker of the counsellor must establish friendly relations with the members of the group so that they may give their frank opinion about an individual or individuals.

4.5.2 Advantages of Sociometry

(i) This technique helps us get an idea of the group at a glance.

(ii) This enables us to form appropriate groups of students for carrying out various activities and projects.

(iii) Such tests at different times enable us to find out the changes taking place in the group structure.

(iv) It is useful in enabling us to understand the characteristics of an individual who is liked or disliked by the group. It gives an insight into the qualities of leadership which are appreciated by a particular group and helps us to compare one group with the other.

(v) It is very helpful for the guidance worker who studies the pupil relationships.

(vi) It helps us to compare one group with the other.

Check Your Progress

13. Define the term 'sociometry'.
14. Define the purpose of the sociometry test.
15. Write the advantages of the sociometric technique.

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4.6 SAMPLING TECHNIQUES IN EDUCATIONAL RESEARCH

A 'population' is any group of individuals/units that have one or more characteristics in common which are of interest to the researcher for a particular research. A population may include all the individuals of a particular type or a more restricted part of that group, e.g., a group of all the university teachers, or a group of male/female university teachers, or distance learners enrolled with NIOS, the National Institute of Open Schooling. For assessing the study habits of adolescent girls in a city, all the adolescent girls of that city who are studying in schools and colleges, make up the population for this study.

Usually, the population covers a very large group of people or objects making an accurate record of all the characteristics in the population are impossible. Researchers rarely survey the entire population for two reasons—the cost is too high and the population is dynamic in that the individuals making up the population may change over time. The population may be classified as real, artificial or hypothetical. A real population is one which actually exists.

An artificial population is created by the researcher in order to illustrate a principle or to make for more convenience and ease in carrying out the study of a problematic situation. Hypothetical population is an artificial population devised purely on theoretical basis. Population may also be categorized as 'known' (the frequency distribution and the parameters are known) and 'unknown' (a population for which no such estimates—mean, mode, median, etc., are available).

A 'sample' is a small percentage of the larger group who are selected for research. A sample can be statistically explained as being a subset of a population. The sample will be able to give an idea of the characteristics of the larger group from where it has been drawn. It is possible to make deductions about the larger population on the basis of the sample.

For selecting a sample, it is necessary to have a sampling frame. This is a complete, accurate and up-to-date list of units in the population. After defining a population and listing all the units, a researcher selects a sample of units from the sampling frame.

The term 'sampling' refers to the technique whereby a smaller group is selected from a larger one so that the more manageable smaller group can be observed and those observations can be applied to the larger group as well. This is only possible when the sample group shares the same characteristics as the larger group. Results deduced from sampling are particularly useful when inferences have to be made based on statistics.

Sampling is an important aspect of data collection. It is the selection of a certain percentage of a group of items according to predetermined plan. The benefits of sampling are that the cost is lower, data can be collected faster, and since the data set is smaller it is possible to monitor the accuracy and quality of the data closely. The limitations could be less accuracy, changeability of units, and misleading conclusions.

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4.6.1 Methods/Designs of Sampling

The sampling method was used in social sciences research in early 1754 by A.L. Bowley. Since then the method has been progressively used. The sampling methods are broadly classified into two types: (i) Probability Sampling and (ii) Non-Probability Sampling.

Criteria for Selecting Sampling

The type of sample has to be selected depending on the area of research. For this purpose, a variety of sampling methods can be employed, individually or in combination. Factors commonly influencing the choice of the sampling design include:

- Nature and quality of the frame.
- Availability of supporting information about units on the frame.
- Accuracy requirements and the need to measure accuracy.
- Whether detailed analysis of the sample is expected.
- Cost/operational concerns.

As there are various sampling methods, it becomes crucial to select an appropriate sampling method. Young has suggested the following three criteria to be considered while selecting a sampling method:

- A measurable or known probability sampling technique should be used to control the risk of errors in the sample estimate.
- Simple, straightforward and workable methods adapted to available facilities and personnel should be used.
- Achieving optimum balance between expenditure incurred and maximum of reliable information should be the guiding principle.

The decision whether a probability sampling or a non-probability sampling is to be applied rests on the constraints which are not very different from those stated earlier. These are: (i) Objectives of the study, (ii) Type of study, and (iii) Availability of the resources for the study.

(i) If the objective of the research is to apply the results of the study to a small local group then sampling may not be given as much consideration as in a study where the results are to be applied to a larger group. Action research generally does not require sampling from a larger group. Most of the time sampling is not very essential in historical research, whereas survey studies generally have a more rigorous sampling.

(ii) The availability of time, funds, manpower and equipment required is another important consideration in deciding about the size and technique of sampling.

(iii) If one is interested in obtaining an estimate of the sampling error, one may resort to probability sampling rather than to a non-probability one.

Probability Sampling

Probability sampling is a technique of sampling which gives the probability that a sample is representative of population. This kind of sample is selected in such a way that every element chosen has a known probability of being included.

Probability sampling is based on some statistical concepts, such as the *Law of Large Numbers, Central Limit Theorem* and the *Normal Distribution*, etc. In this type of sampling, the smaller groups are not selected based on the researcher's decision but by means of techniques which ensure that every member of the larger group has the likelihood of being included in the sample group. It is also called 'random sampling'.

The *Law of Large Number* states that as the sample size becomes larger, probability that the estimate differs from the parameter to a greater extent becomes small. Or in other words, a larger number provides a more precise measure of the parameter under consideration. However, one precaution must be taken. While increasing the size of the sample, care should be taken to maintain the representativeness of the sample, because a large sample does not automatically guarantee representativeness.

As per the second concept, sampling distribution approaches *normal distribution* provided more the irregular distribution in the population larger is the sample selected to avoid biases.

The following are different methods of probability sampling.

Random Sampling

In a Simple Random Sample (SRS) of a given size, all subsets in the group have the same odds of being selected. The frame is not sub-divided or partitioned. Besides this, any *pair* of units has the likelihood of being selected as any other such pair (this is applicable to triples as well, and so on). This reduces preconceptions and makes the analysis of results easier. However, SRS can be lead to sampling errors due to the randomness of the selection process which may not be an accurate representation of the entire population. For example, choosing 10 people randomly from any given country can produce a balanced group of men and women but can also have the likelihood of over representing or under representing one sex.

Thus, SRS can be cumbersome and tedious when sampling from an unusually large target population. In certain instances where research is very specific, for example, researchers might be interested in examining whether mechanical skills are equally applicable across racial groups, SRS cannot be used as random selection of a sub-group as it may not provide accurate findings.

Theoretically, this is a method of selecting '*n*' units from *N* units in such a way that everyone in the population of *N* units has an equal chance of being selected. This can be done through the following steps:

(i) Defining population by specifying its various limits.

(ii) Preparing the sampling frame.

(iii) Incorporating the names or serial numbers of individual units in the sampling frame (every unit is to be listed, order does not make any difference).

It is important to re-emphasize here that a random sample is not necessarily an identical representation of the population. After this, to get the required '*n*' units different techniques are available. These techniques are discussed below.

(a) **Lottery Method:** After numbering every unit in the population, they are well mixed. The required numbers of units are then drawn from all these well mixed units. The individuals' objects with these identification named numbers are then picked up for inclusion in the sample. However, this technique has some objections. When the population is very large and includes such individuals/objects which are of such nature that could not be mixed and further if 'well mixing' is not attained despite all efforts, the principle of randomness in the population may be violated.

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(b) **Random Table Method:** The use of random numbers or manual lot drawing will be too cumbersome to recommend in case of a large population. In such situations, computer generated random selection should be resorted, in order to save time and labour. Tables of random numbers have been generated by computers producing a random sequence of digits, e.g., random digit tables by Rand Corporation and prepared by Kendall & Smith, by Fisher & Yates and by Tippet are frequently used. The required numbers of units are selected from such a table in any convenient and systematic way. Now suppose we have to select 20 distance learners for interviews from 80 distance learners registered at a study centre. We may start with any column and any row. Because we want 20 numbers, i.e., two digit numbers, we have to select only the first two digits from each number. If we select the first column and start from first row then we will get following 22 digit numbers—23, 05, 14, 38, 97, 11, 43, ..., 61. You will notice that numbers greater than 80 will have to be deleted from this list and for the remaining numbers selecting any other column and the row the procedure will have to be repeated, till we get the required number, i.e., 20. If any number is repeated in this list, it is to be substituted by selecting the next number. Until a sample of desired size is obtained, the selection procedure is to be continued.

(c) **Selection of Sample:** In this method, names are arranged under the intended plan alphabetically, geographically or simply serially. Then, out of the list, every tenth or any other number of cases is taken up. If every tenth unit is to be selected, the selection begins as seventh, seventeenth, twenty-seventh, and so on, or fifth, fifteen, twenty-fifth, and so on.

(d) **Grid System:** In this method, selection of sample is made from a particular area. A map of the entire area is prepared, and then a screen of squares is placed on the map. The areas falling within the selected squares are taken as samples.

Advantages of Random Sampling

The advantages of random sampling are as follows:

(i) This method calls for no special expertise and training or even insight. It can be used mechanically by anybody.

(ii) Each individual of the population has an equal chance of being selected into the sample.

(iii) One individual does not affect the selection of the other.

(iv) It is free from subjective issue or personal error or bias and imagination of the investigator.

(v) It requires minimum knowledge of the population.

(vi) It provides appropriate data for research purposes.

(vii) Data can be used for inferential purposes.

Disadvantages of Random Sampling

The disadvantages of random sampling are as follows:

(i) Practically listing of all the units in the population may not be possible.

(ii) In case of heterogeneous population, the selected random sample may not truly represent the characteristics of the population.

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(iii) Representativeness cannot be assured.

(iv) This method does not use knowledge about population.

(v) Inferential accuracy of finding depends upon size of sample.

(vi) In case of population with infinite numbers, listing is out of the question.

(vii) It is difficult though not impossible as it involves high cost.

Systematic Sampling

Systematic sampling is a variant of the random process of sampling. In this technique of the requisite number of sample units are selected from the population. This sampling entails organizing the population in a predetermined order and then selecting from the list at regular intervals. One starts from a random number and then proceeds with the selection of every *k*th element from then onwards. In this case, k = (population size/ sample size). It should be noted that the starting point is not automatically the first in the list, but should be randomly chosen from the first to the *k*th element in the list, e.g., every 10th name from the telephone directory (an 'every 10th' sample, also referred to as 'sampling with a skip of 10').

As long as the starting point is randomized, systematic sampling is a type of probability sampling. It is easy to implement and the stratification induced can make it efficient, if the variable by which the list is ordered is correlated with the variable of interest. 'Every 10th' sampling is especially useful for efficient sampling from databases. However, systematic sampling is especially vulnerable to intervals in the list. If periodic intervals are present and the period is a multiple or factor of the interval used, the sample is not going to be an accurate representation of the target population, making the scheme less accurate than simple random sampling. Thus, we see that systematic sampling is an EPS (Equal Probability Sampling) method, as all elements share the same likelihood of being selected (in the example given, one in ten). It is not 'simple random sampling' because different subsets of the same size have different selection probabilities—e.g., the set {4, 14, 24, ..., 994} has a one-in-ten probability of selection, but the set {4, 13, 24, 34, ...} has zero probability of selection. Thus, it involves the following steps:

(i) Listing the population elements in some order, say alphabetically, merit-wise, etc.

(ii) Determining the desired number to be selected from the population, e.g., 10 per cent of 1000 means 100 out of 1000.

(iii) Starting with any number from among the numbers 1 to 10 (i.e., 1 to k, both inclusive), to select every '10' (or 'k') element from the list. If the number chosen from 1 to 10 is 4, then the selected numbers will be the 4, 14th, 24th . . . 994th elements making the sample with 100 elements.

(iv) As the elements are chosen from regular intervals, this technique is also known as 'sampling by regular intervals', sampling by fixed intervals or sampling by every k unit.

Advantages of Systematic Sampling

The advantages of systematic sampling are as follows:

(i) It is more practical in that it involves less labour.

(ii) As it is simpler to perform, it may reduce errors.

(iii) The procedure is speedy in comparison with simple random sampling.

(iv) Reduces the field cost.

(v) Inferential statistics may be used.

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Disadvantages of Systematic Sampling

The disadvantages of systematic sampling are as follows:

(i) Selection of every element, other than the first which is selected randomly, is linked with the first element. This makes the process different from the simple random method where selection of every element is independent of the other one.

(ii) When the list of elements has a periodic arrangement, there is a risk that the sample interval may coincide with the periodic interval in the list. Suppose, A, B, C, D and E are the 5 schools selected and then from each school 100 students are selected. The students from school 'A' are placed starting from 1, from school 'B' starting from 2, from school 'C' starting from 3, from school 'D' starting from 4 and from school 'E' starting from 5 with an interval of 5. Thus the school 'A' students will hold the numbers 1, 6, 11, 16, 21 . . . 496. The school 'B' students will hold the numbers 2, 7, 12, 17, 22, ... 497. Now in systematic sampling procedure suppose we decide to select 5 per cent of the total and randomly choose any number from 1 to 5 say '3' then starting from 3 we will have to select every 5th number. These numbers will be 3, 8, 13, 18... . . . . .498. Have you noticed that all these numbers belong to school 'C'? Why has it happened so? The answer is because every school is repeated in the list with an interval of '5' and elements are selected with an interval of '5'.

(iii) Another limitation of the systematic sampling method is the trend of the listed population. This is explained below—Suppose 100 students are listed in the decreasing order of academic merit. We want to draw a sample of 20 students from this using systematic sampling method. 20 out of 100 mean the size of interval is '5'. We can draw many samples from this listed population. If we randomly pick up a number from among 1 to 5, say 3 then the sample will comprise the elements '3', 8th, 13th, 18th . . . . . 98th.

(iv) This is not free from error, since bias may creep in due to different ways of making systematic list by differentiation.

(v) Knowledge of population is essential.

(vi) Information of each individual is essential.

(vii) There is risk in drawing conclusions from observation.

Stratified Sampling

To increase the precision, 'stratified sampling' can be one option. The term 'stratified' is very much self-explanatory. It involves dividing the population into such sub-populations (strata) that each one of them is homogeneous within itself. Population can be divided into different categories by using these 'strata' or layers. Each stratum or section is then treated independently as a separate sub-group.

There are many advantages of dividing into sub-sections. First, researchers can study these specific sub-groups closely, which may otherwise have got lost in a generalized random sample. Next, applying the stratified sampling method gives accurate statistical estimates provided that the categories or sub-divisions are chosen according to their relevance to the topic being researched rather than randomly and that the group size is proportionate to the entire population. Besides this, data is more readily available for an individual than for a large group. Finally, as each category or sub-division is treated as a separate population, different sampling techniques can be used on each, enabling researchers to apply the most appropriate approach.

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Certain limitations to using stratified sampling: *First*, is breaking down the population into so many sub-divisions can complicate the research and monitoring process. The researcher may also lose count of the size of the population. *Second*, many of the criteria may not apply to the sub-divisions, reducing the value of having so many strata. *Third*, in some cases such as designs with a large number of strata or those with a specified minimum sample size per group, stratified sampling can potentially require a larger sample than would other methods although in most cases, the required sample size would be no larger than would be required for simple random sampling. The steps to be followed in this method are as follows:

(i) Deciding upon one or more characteristics on the basis of which strata will be formed, e.g., location of schools—rural, urban, suburban, urban-slums, metropolitan, etc.

(ii) Dividing the population under consideration into strata on the basis of stratification characteristics/criteria.

(iii) Listing the units in each stratum separately.

(iv) Selecting requisite number of elements from each stratum using appropriate random selection technique.

Thus, all the elements selected from all the strata compose the required sample.

Important points to be noted while doing stratified sampling:

(i) The criteria for dividing the population into strata should be correlated with the variable being studied.

(ii) The criteria should be practical. It should not yield an unwieldy number of strata.

(iii) A good measure of the stratification criteria should be available, e.g., if a reliable and valid tool of determining socio-economic status is not available, stratification on this basis would lead to confounding of the results.

(iv) Selection of the elements at random from each stratum in the same proportion as that of the actual size of the stratum in the population improves the representativeness of the sample and helps in achieving higher efficiency at a reduced cost.

(v) In some studies (like census), stratification is not possible before the data have been collected. After collecting the data, stratification as per sex, age, educational level is carried out. Or a simple random sample of the required size is selected and the classification into strata is observed.

Stratified random sampling can be of following three types:

(a) ***Proportionate Sampling:*** It refers to the selection of a sample from each sampling unit that is proportionate to the size of unit. Its advantages include representativeness with respect to various variables used as basis of classifying categories and increased chances of comparison between strata.

(b) ***Disproportionate Sampling:*** It means that the size of the sample in each unit is not proportionate to the size of unit, but depends upon considerations involving personal judgment and convenience. This is more effective for comparing strata which have different error possibilities.

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(c) ***Optimum Allocation Stratified Sampling:*** It is representative as it selects units from each stratum in proportion to corresponding stratum in the population.

Advantages of Stratified Sampling

The advantages of stratified sampling are as follows:

(i) Stratified random sampling is very useful when a list of the elements in the population is not available.

(ii) It is the most applicable method of sampling when the population is heterogeneous.

Disadvantages of Stratified Sampling

The disadvantages of stratified sampling are as follows:

(i) It is costly and time consuming.

(ii) Criteria for stratification need to be decided carefully.

(iii) There is risk in generalization if not properly done.

Multiple/Double Sampling

Double sampling is a type of sampling which includes both questionnaire and interview methods for probing a research problem.

Multi-Stage and Multi-Phase Sampling

This is used in large-scale surveys for more comprehensive investigation. The researcher may have to use two, three or four stage sampling. In the multi-stage sampling, a selection of different types of sampling units, such as some *Districts* in a *State*, some *Talukas* in those *Districts* and then some *Schools*, is involved at different sampling stages.

Whereas in the multi-phase sampling, the researcher is concerned with the same type of sampling unit at each phase but some members are asked for more information than others, e.g., information regarding study habits of distance learners can be collected from 100 distance learners through a questionnaire and 20 out of them can be interviewed for more information. The main distinction between the multi-stage and the multi-phase sampling is the use of unit of sampling at different levels in multi-stage sampling but not in multi-phase sampling.

Advantages of Multi-Stage and Multi-Phase Sampling

The advantages of multi-stage and multi-phase sampling are as follows:

(i) In both the sampling methods, burden on respondents is reduced.

(ii) Relative cost also gets reduced.

(iii) Two-phase sampling is useful in studying rare cases.

(iv) In two-phase sampling, the resulting gain in precision is more due to possibility of getting more information in details.

(v) This kind of sampling gives a good reorientation of the population.

(vi) It is an objective procedure of sampling.

(vii) Observations thus derived can be used for inferential purposes.

Disadvantages of Multi-Stage and Multi-Phase Sampling

The disadvantages of multi-stage and multi-phase sampling are as follows:

(i) The disadvantage with this kind of sampling is that it is difficult and complex.

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(ii) Error may creep in at the primary or secondary stages.

(iii) It is subjective.

Cluster Sampling

In this type of sampling, the units of samples close to each other are chosen in clusters, for example, households in the same street or successive items of a production-line. The population is divided into clusters and some of them are chosen randomly. Then, the clustered units are selected using random sampling method.

In this method, the items to be studied are picked up at random at different stages. For example, if the idea is to study the problem of middle class working couples in a State, the first stage will be to pick up few districts in the State. The next stage will be to pick up at random few rural and urban areas for the study. The third stage will come when from each area few families belonging to the middle class will be picked up, and the last stage will be when working couples out of these families will be chosen for study. Thus the stages would be:

State—Districts—Rural and Urban Areas— Middle Classes—Working Couples

Advantages of Cluster Sampling

The advantages of cluster sampling are as follows:

(i) This method of sampling is very economical, especially when the cost of measuring a unit is relatively small.

(ii) It is easier to administer.

(iii) Large number of units can be sampled for a given cost.

(iv) Practical when the population is large.

Disadvantages of Cluster Sampling

When the sampling unit is to be an individual element/unit or number in the population, this method is not applicable. It may not be comprehensive.

Non-Probability Sampling

The non-probability sampling methods are based on the judgments of the investigator as the most important elements of control. The guiding principles in non-probability methods are— availability of the subjects, the personal judgment of the investigator, and convenience in carrying out the research.

Such samples use human judgment in selecting units and have no theoretical base for estimating population characteristics. The non-probability sampling methods are of following types:

- Incidental or accidental sampling
- Judgment sampling
- Purposive sampling
- Quota sampling
- Snowball sampling

A non-probability sample is termed as 'non-random sample' due to the very fact that it is selected through non-random method. The main feature of such a sample is the lack of control of the sampling error on account of which this method of sampling is referred to as 'uncontrolled sampling' method. This description of the non-probability

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sampling should not be taken in negative sense. In spite of all this, many a times it is the demand of the situation to go for non-probability sampling method. Let us now study the different non-probability sampling methods one by one.

• Incidental or Accidental Sampling

Incidental sampling is also known as accidental or convenience sampling. When a readily or easily available group is selected as a sample, it is termed as an 'incidental sample'. Samples are taken because they are more frequently available. It refers to groups which are used as samples of population because they are readily available. Incidental sampling is an easy method but parametrical tests cannot be used for it. A teacher-educator, e.g., may select the students from a school situated in the same campus which serves as a practising school for the concerned college of education, find the effectiveness of concept attainment model to teach a mathematical concept say, a quadrilateral.

Advantages of Incidental Sampling

The administrative convenience of obtaining samples for the study, the ease of testing, saving in time and completeness of the data collected are some of the merits of this method.

Disadvantages of Incidental Sampling

Since there is no well-defined population and no random sampling method is applied to select the sample, the standard error formulae are applied with a high degree of approximation. Hence, no valid generalization can be drawn. Any attempt at generalization based on such data and conclusion thereof will be misleading.

• Judgment Sampling

It involves selection of groups from the population on the basis of available information. The groups should be representative of the population. It has good evidence and is based on experience. It is an economical method.

• Purposive Sampling

Another non-probability sampling method is 'purposive sampling'. The sample is selected by some arbitrary method because it is known to be representative of the total population, or it is known that it will produce well matched groups. The idea is to pick out the sample in relation to some criterion which is considered important for the particular study.

In this method, samples are chosen because they resemble some larger group with respect to one or more characteristics. The controls of criteria for categorization in such samples are usually identified as representative areas, such as a state, a district, a city, etc., or representative characteristics of individuals, such as age, sex, socio-economic status, etc., or representative types of groups, such as elementary school teachers, secondary school teachers, college teachers, university teachers, etc. These controls criteria may be further sub-divided, e.g., the group of college teachers can be divided into male and female teachers or teachers in science/arts/commerce colleges, etc.

It has to be noticed here that up to this stage the controls are somewhat similar to stratification criteria. After deciding upon the category required for the research, the researcher has to select the sample. Actual selection of the units for inclusion in the sample is done purposively and not randomly, e.g., in order to tackle the problem of indiscipline only the undisciplined students are selected as the sample, excluding others on the basis of past experience.

Advantages of Purposive Sampling

This method of sampling is useful where a small sample is required. It is focused on solving problems of particular groups.

Disadvantages of Purposive Sampling

This method is applicable only for the selection of samples, such as special cases like 'best teacher award winners' from the population of teachers or 'meritorious past students of the school' from the population of the past students.

• Quota Sampling

This is another method of non-probability sampling. It involves the selection of the sample units within each stratum on the basis of the judgment of the researcher. What distinguishes it from probability sampling is that once the strength of the sample (e.g., how many women teachers from among college teachers) is decided it forms the 'quota'. The choice of the actual units to fit into this framework is left to the researcher.

Quota sampling is thus a method of stratification sampling in which the selection of sample units within the stratum is non-random. These quotas are determined by the proportion of the groups, e.g., in order to study the attitude of school teachers towards environment education, first the school teachers will be stratified into men and women teachers, quotas for these strata will be fixed and then the teachers will be selected (not randomly).

• Snowball Sampling

It refers to the many techniques which use the probability method to select the first respondents. Additional respondents are added on the basis of referrals by the first respondents. This technique is used to locate members of rare population by referrals.

4.6.2 Sampling Design

Sampling design refers to a definite plan for obtaining a sample from the sampling frame. It refers to the technique or procedure, which a researcher adopts in selecting some sampling units from where inferences about population are drawn. Sampling data is obtained before collecting the final data.

Need for Sampling

We can define sampling as the process of obtaining information about an entire population by examining only a part of it. Sampling is required for the following reasons:

- It saves time and money. A sample study is usually less expensive than a census study.
- It produces results at a faster speed.
- It enables more accurate measurement for a sample study as it is conducted by experienced investigators.
- It is the only method for an infinitely large population.

It usually enables a researcher to estimate sampling errors and thus assists him/her in obtaining information concerning some characteristics of the population, such as age group or gender.

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Advantages of Sampling

The various advantages of sampling are as follows:

- The solution to know the true or actual values of the various parameters of the population would be to take into account the entire population. This is not feasible due to the cost and time involved; therefore, sampling seems more economical.
- As the magnitude of operation involved in a sample survey is small, the execution of the fieldwork and the analysis of results can be carried out at a faster rate and in a lesser time.
- Very small staff is required for gathering and analysing information and preparing reports; therefore, sampling is a very cheap process.
- A researcher can collect detailed information in a lesser time than is possible in a census survey.
- As the scale of operation involved in a sample survey is small, the quality of interviewing, supervision and other related activities is better than the census survey.
- Sampling provides adequate information needed for the purpose and is sufficiently reliable for surveys.

Sampling Characteristics

Usually, sampling involves determining a property or attribute to adhere for the purpose of differentiating between items of a given population. These attributes which are the objects of study are called 'characteristics'.

The process of distinguishing the items is usually of two types, quantitative or qualitative. In quantitative sampling, characteristics pertaining to variables are dealt with. On the other hand, qualitative sampling is concerned with the characteristics related to attributes.

The basic idea behind sampling is to use common characteristics of average items as samples for a larger entity. Thus, it involves choosing a subset of population elements for study. Thus, for example, if the population to be dealt is, say of roads, then the characteristics could be length, duration, roughness, carriage capacity, etc.

Sampling proves to be a much cheaper and quicker mode of estimation where the population is absolutely huge. However, it is absolutely necessary to take ample care while determining which characteristics should be sampled. Those characteristics, which are rare, should be avoided. Similarly, even if there are certain most common characteristics, but those, which do not contribute in any way to draw reliable estimates, then such characteristics should not be sampled.

Steps in Sampling

The sampling process involves the following seven tasks:

1. **Defining the Population:** It involves completely defining the population by specifying the following terms:
   (i) Elements
   (ii) Sampling units
   (iii) Extent
   (iv) Time

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2. **Selecting the Sampling Frame:** The sampling frame should be selected in such a way that it consists of almost all the sampling units. A sample should be selected in such a way that it has all the characteristics of the population. Some of the popular sampling frames are census reports, and electoral registers.
3. **Specifying the Sampling Unit:** Sampling unit is the basic unit that contains elements of the target population.
4. **Specifying the Sampling Method:** This method depicts how the sample units are selected. The most important decision in this method is to determine, which of the two—probability and non-probability—samples is to be chosen.
5. **Determining the Sample Size:** This method includes decision-making about the number of elements to be chosen.
6. **Specifying the Sampling Plan:** This method dictates that one should indicate how decisions made so far are to be implemented. All the expected issues in relation to the sampling survey must be answered by the sample plan.
7. **Selecting the Sample:** This is the final step in the sample process, which includes a good deal of fieldwork and office work. This is introduced in the actual selection of the sample elements. It mainly depends on the sampling plan and the sample size required.

Characteristic of Good Sampling

Samples can be of different types. The following are the characteristics of a good sample:

- **True Representative:** A good sample is a true representative of the population corresponding to its properties.
- **Free from Bias:** A good sample does not permit prejudices, pre-conceptions and imagination to influence its choice.
- **Comprehensive:** Comprehensiveness is a quality of a sample which is controlled by the specific purpose of the investigation. A sample may be have all the traits required, but still not be a good representative of population.
- **Economical:** A sample should be economical from energy, time and money viewpoint.
- **Approachable:** A sample should be easily approachable. The research tools can be easily administered on them.
- **Good Size:** Size of a sample should be such that it yields an accurate result. The probability of error can be estimated.
- **Feasible:** A good sample makes the research more feasible.
- **Practical:** A good sample has the practicability for research situations.
- **Objective:** This refers to objectivity in selecting a sampling procedure or absence of subjective elements from situation.
- **Accurate:** A good sample yields accurate estimates of statistics and does not allow for errors.

4.6.3 Representative Sample

When a researcher is performing the research study, he/she may select a comparatively small number of subjects from the entire population. Thus, for example, the researcher

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may choose students of BCom First Year from all universities in a country. Literally, in this case, the researcher is using a representative sample. Thus, we can define representative sample as the sample, which possesses the same characteristics as that of its parent population or variable. Thus, it factually represents the variation that exists in the parent variable on the general level.

The significance of a representative sample lies in the fact that it represents the population more accurately. For this, it is absolutely necessary that the sampling process is kept free from errors. Errors may occur when representative sampling is based on surveys that may be hampered with non-response errors or self-selection errors. By non-response errors, we mean that a survey is conducted in such a way that the researcher has targeted a large number of subjects, but only a few per cent have responded.

This can be explained with an example. Suppose that a supermarket tries to conduct a survey of its customers by offering feedback forms to every consumer and instructing them to put the filled-in form in a drop box. In this case, it is possible that some customers may fill the form, whereas some may just carry it and discard outside. Suppose that the number of customers visiting every day is around 400. In case, just 75 of these have put in completely filled forms, the management cannot infer these 75 customers as representing the total 400. As such, if these 75 are used for the process of sampling, inaccurate generalizations are bound to occur.

In the case of self-selection error, there may be, for example, a few customers, who may have chosen to fill only half of the form. As can be seen, in this case too, the sampling is not possible on such a self-selected partial feedback of the customers. Another possibility is that the researcher may be tempted to conduct sampling by personal standards, which can greatly obstruct the generalization purpose of sampling. In this case, there is a possibility of the measurement being distorted or miscalculated as a result of subjective influence on the part of the surveyor.

Sampling involves application of a number of predefined concepts and types for conducting the survey meant for research. It is also necessary for the researcher to get acquainted with the various terms involved in the usage of sampling, for effective application of the sampling method.

- **Precision:** It is the range within which the population average lies in accordance with reliability specified in the confidence level as a percentage of the estimate or as a numerical quantity. For example, if the estimate is ₹ 5,000 and the precision desired is ± 4 per cent, then the true values would be not less than ₹ 4,800 or not more than ₹ 5,200. This is the range within which the answer should lie. But, the estimate should not go beyond this level.
- **Confidence Level and Significance Level:** The confidence level is the expected percentage of times that the actual value will fall within the stated precision limits. Therefore, if we say that the confidence level is 90 per cent, then we mean that there are 90 chances in 100 that the sample result gives the true condition of the population within a specified range, as against the other 10 chances when the condition is not true. While precision is the range within which the answer may vary, confidence level indicates the likelihood that the answer will fall within that range, and the significance level indicates the likelihood that the answer will fall outside that range.

4.6.4 Sampling Distribution

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Sampling distribution is often required in sampling analysis. If we take samples and for each sample we compute several statistical measures, such as mean and standard deviation, we can find that each sample may give its own value for the statistics under consideration. We can have the sampling distribution of mean or standard deviation or of any other statistical measure as well.

There are the following types of sampling distribution that are commonly used:

- **Sampling Distribution of Mean:** It refers to the probability distribution of all possible means of random samples of a given size. If the samples are taken from a normal population, N (μ, σ_p), the sampling distribution of mean would be normal with mean $\mu_{\bar{x}} = \mu$ and the standard deviation = $\sigma_p\sqrt{n}$, where μ is the mean of population, σ_p is the standard deviation of the population and '*n*' means the number of items in a sample. When the sampling is from a population that is not normal, which may be positive or negative, as per central limit theorem, the sampling distribution of mean tends to be quite closer to the normal distribution, provided that the number of sample items is larger than 30. If we want to reduce the sampling distribution of mean to unit normal distribution, we can write as normal variants Z for the sampling distribution of mean. This characteristic of sampling distribution of mean is very useful in several decision situations for acceptance or rejection of hypotheses.
- **Sampling Distribution of Proportion:** This type of distribution is useful in statistics. For example, if we have worked out the proportion of defective parts in a large number of samples, such as 100 items that have been taken from an infinite population, by plotting a probability distribution of the said proportions you obtain the sampling distribution. If 'p' is the proportion of defectives and 'q' is the proportion of non- defectives and if 'p' is treated as a random variable, then the sampling distribution of proportion of success has a mean = p with standard deviation as:

  $= \sqrt{\frac{p.q}{n}}$, where 'n' is the sample size.

- **Student's Distribution:** When the population standard deviation (σ_p) is not known and the sample is of small size, 't' distribution is used for the sampling distribution of mean and workout *t* variable as:

  $t = \left(\overline{X-\mu}\right) / \left(\sigma_s/\sqrt{n}\right)$

  Where, $\sigma_s = \sqrt{\frac{\sum\left(X_i - \overline{X}\right)^2}{n}}$-1

  i.e., the sample standard deviation. 't'-distribution is also symmetrical and is close to the distribution of standard normal variant, 'z', except for small values of 'n'. The variable 't' differs from 'z' when we use sample standard deviation (σ_s) in the calculation of 't', whereas we use standard deviation of population (σ_p) in the calculation of 'z'. There is a different 't' for every sample and the degree of freedom for a sample of size 'n' is n – 1. The shape of the 't' distribution becomes approximately equal to the normal distribution as the sample size becomes larger.

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But, when 'n' is small, the 't' distribution is far from normal and when $n \to \alpha$, t distribution is identical to the normal distribution.

- **F Distribution:** If $(\sigma_{s1})^2$ and $(\sigma_{s2})^2$ are the variances of two independent samples of n_1 and n_2 respectively taken from two independent populations, having the same variance, $(\sigma_{p1})^2 = (\sigma_{p2})^2$, the ratio $F = (\sigma_{s1})^2 / (\sigma_{s2})^2$, where $(\sigma_{s1})^2 = \sum(\overline{X_{1i}} - \overline{X_1})^2 / n_1 - 1$ has an F distribution with $n_1 - 1$ and $n_2 - 1$ degrees of freedom.

- **Chi-Square (χ^2) Distribution:** This type of distribution method is used when you are dealing with the collection of values that include adding up squares. Variance of samples requires adding a collection of squared quantities, and thus having distribution that is related to chi-square distribution. Collection of samples variance is divided by the known population variance and then the quotients are multiplied by (n – 1), where 'n' means the number of items in the sample. This way, you obtain the chi-square distribution. Chi-square distribution is not symmetrical and all the values are positive.

You need to know the degrees of freedom for using the chi-square distribution. This method is used for judging the significance of difference between observed and expected frequencies. The generalized shape of χ^2 distribution depends on the degree of freedom and χ^2 is written as:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Here, O = Observed Frequency

E = Expected Frequency .

Chi-square test is applicable in large number of problems. The test is, in fact, a technique through the use of which it is possible for us to (a) Test the goodness of fit, (b) Test the homogeneity of a number of frequency distributions and (c) Test the significance of association between two attributes. In other words, chi-square test is a test of independence, goodness of fit and homogeneity. At times chi-square test is also used to test the significance of population variance through confidence intervals, especially in case of small samples.

χ^2 test helps us in stating whether different samples come from the same universe. Through this test, we can also explain whether the results worked out on the basis of sample/samples are in conformity with well defined hypothesis or the results fail to support the given hypothesis. As such the test can be taken as an important decision-making technique.

4.6.5 Sampling Errors

Even if utmost care has been taken in selecting a sample, the results derived from a sample study may not be exactly equal to the true value in the population. The reason is that estimate is based on a part and not on the whole and samples are seldom, if ever, perfect miniature of the population. Hence, sampling gives rise to certain errors known as 'sampling errors' or sampling fluctuations.

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In other words, a sample survey requires study in small portions of population as there can be certain amount of inaccuracy in the information collected during sampling analysis. This inaccuracy is called sampling error or error variance. Sampling errors are those errors, which arise on account of sampling and generally happen to be random variations in the sample estimates of the actual population values. Figure 4.3 shows sampling error.

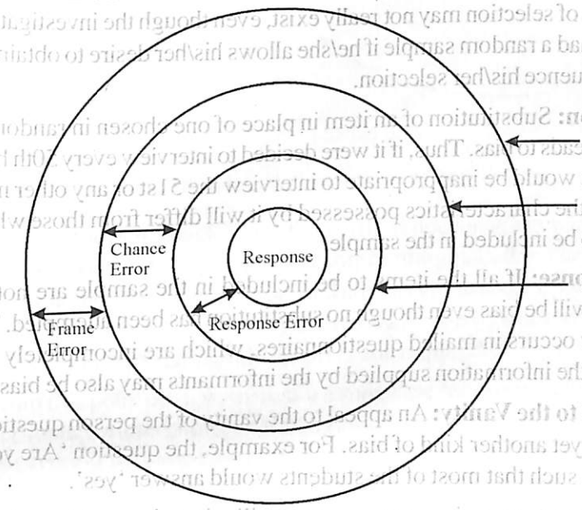

Fig. 4.3 Sampling Error

Sampling errors occur randomly and are equally likely to be in either direction and the magnitude of sampling error depends on the nature of the universe. The more uniform the universe is, the smaller is the sampling error. Sampling error is inversely proportional to the size of the sample and vice-versa. In addition, sampling error is the product of the critical value at a certain level of significance and the standard error.

Sampling Error = Frame Error + Chance Error + Response Error

Sampling errors would not be present in a complete enumeration survey. However, the errors can be controlled. The modern sampling theory helps in designing the survey in such a manner that the sampling errors can be made insignificant. Sampling errors are of two types: (i) Biased and (ii) Unbiased.

These errors arise from any bias in selection, estimation, etc. For example, if in place of simple random sampling, if deliberate sampling has been used in a particular case; some bias is introduced in the result, and hence such errors are called 'biased sampling errors'.

An error in statistics is the difference between the value of a statistic and that of the corresponding parameter. These errors arise due to chance differences between the members of population included in the sample and those not included.

Thus, the total sampling error is made up of errors due to bias, if any, and the random sampling error. The essence of bias is that it forms a constant component of error that does not decrease in a large population as the number in the sample increases. Such error is, therefore, also known as 'cumulative/non-compensating error'. The random sampling error, on the other hand, decrease as an average as the size of the sample increases. Such error is, therefore, also known as 'non-cumulative/compensating error'.

Bias may arise due to: (i) Faulty process of selection, (ii) Faulty work during the collection, and (iii) Faulty methods of analysis.

Faulty selection of the sample may give rise to bias in a number of ways. Some of which are discussed below:

(a) **Deliberate Selection:** The deliberate selection of a 'representative' sample.

(b) **Conscious/Unconscious Bias in the Selection of 'Random' Sample:** The randomness of selection may not really exist, even though the investigator claims that he/she had a random sample if he/she allows his/her desire to obtain a certain result to influence his/her selection.

(c) **Substitution:** Substitution of an item in place of one chosen in random sample sometimes leads to bias. Thus, if it were decided to interview every 50th household in a street, it would be inappropriate to interview the 51st or any other number in its place as the characteristics possessed by it will differ from those which were originally to be included in the sample.

(d) **Non-Response:** If all the items to be included in the sample are not covered then, there will be bias even though no substitution has been attempted. This fault particularly occurs in mailed questionnaires, which are incompletely returned. Moreover, the information supplied by the informants may also be biased.

(e) **An Appeal to the Vanity:** An appeal to the vanity of the person questioned may give rise to yet another kind of bias. For example, the question 'Are you a good student?' is such that most of the students would answer 'yes'.

Any consistent error in measurement will give rise to bias whether the measurements are carried out on a sample or on all the units of the population. The danger of error is, however, likely to be greater in sampling work, since the units measured are usually smaller.

Bias may arise due to improper formulation of the decision problem or wrongly defining the population, specifying the wrong decision, securing an inadequate frame, and so on. Biased observations may result from a poorly designed questionnaire, an ill-trained interviewer, failure of a respondent's memory, etc. Bias in the flow of data may be due to unorganized collection procedure, faulty editing or coding of responses.

In addition to bias which arises from faulty process of selection and faulty collection of information, faulty methods of analysis may also introduce bias. Such bias can be avoided by adopting the proper methods of analysis.

If possibilities of bias exist, fully objective conclusions cannot be drawn. The first essential of any sampling or census procedure must, therefore, be the elimination of all sources of bias. The simplest and the only certain way of avoiding bias in the selection process is for the sample to be drawn either entirely at random or subject to restrictions which while improving the accuracy are of such a nature that they do not introduce bias in the results. In certain cases, systematic selection may also be permissible.

Once the absence of bias has been ensured, attention should be given to the random sampling errors. Such errors must be reduced to the minimum so as to attain the desired accuracy.

Apart from reducing errors of bias, the simplest way of increasing the accuracy of a sample is to increase its size. The sampling error usually decreases with increase in sample size and in fact in many situations the decrease is inversely proportional to the square root of the sample size. Figure 4.4 illustrates the increase and decrease proportion between sampling error ad sample size.

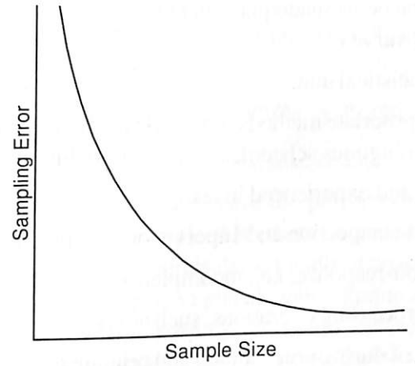

Fig. 4.4 *Sampling Error and Sample Size*

From Figure 4.4, it is clear that though the reduction in sampling error is substantial for initial increases in sample size, it becomes marginal after a certain stage. In other words, considerably great effort is needed after a certain stage to decrease the sampling error than in the initial instances. Hence after that stage sizable reduction in cost can be achieved by lowering even slightly the precision required.

From this point of view, there is a strong case for resorting to a sample survey to provide estimates within permissible margins of error instead of a complete enumeration survey, as in the latter the effort and the cost needed will be substantially higher due to the attempt to reduce the sampling error to zero.

As regards non-sampling error, they are likely to be more in case of complete enumeration survey than in case of a sample survey, since it is possible to reduce the non-sampling errors to a greater extent by using better organization and suitably trained personnel at the field and tabulation stages.

The behaviour of the non-sampling errors with increase in sample size is likely to be opposite of that of sampling error, that is, the non-sampling error is likely to increase with increase in sample size. In many situations, it is quite possible that the non-sampling error in a complete enumeration survey is greater than both the sampling and non-sampling errors taken together in a sample survey, and naturally in such situations the latter is preferred to the former.

When a complete enumeration of units in the universe is made, one would expect that it would give rise to data free from errors. However, in practice it is not so. For example, it is difficult to completely avoid errors of observation or ascertainment. So also in the processing of data tabulation errors may be committed affecting the find results. Errors arising in this manner are termed as non-sampling errors, as they are due to factors other than the inductive process of inferring about the population from a sample.

Thus, the data obtained in an investigation by complete enumeration, although free from sampling error, would still be subject to non-sampling error, whereas the results of a sample survey would be subject to sampling error as well as non-sampling error.

Non-sampling errors can occur at every stage of planning and execution of the census or survey. Such errors can arise due to a number of causes, such as defective methods of data collection and tabulation, faulty definition, incomplete coverage of the population or sample, etc. More specifically, non-sampling errors may arise from one or more of the following factors:

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(i) Data specification being inadequate and inconsistent with respect to the objective of the census or survey.

(ii) Inappropriate statistical unit.

(iii) Inaccurate/Inappropriate methods of interview, observation or measurement with inadequate or ambiguous schedules, definitions or instructions.

(iv) Lack of trained and experienced investigators.

(v) Lack of adequate inspection and supervision of primary staff.

(vi) Errors due to non-response, i.e., incomplete coverage in respect of units.

(vii) Errors in data processing operations, such as coding, punching, verification, etc.

(viii) Errors committed during presentation and printing of tabulated results.

These sources are not exhaustive, but are given to indicate some of the possible sources of error. In a sample survey, non-sampling errors may also arise due to defective frame and faulty selection of sampling units.

In some situations, the non-sampling errors may be large and deserve greater attention than the sampling errors. While, in general sampling errors decrease with increase in sample size, non-sampling errors tend to increase with the sample size.

In the case of complete enumeration, non-sampling errors and in the case of sample surveys, both sampling and non-sampling errors require to be controlled and reduced to a level at which their presence does not vitiate the use of final results.

The reliability of samples can be tested in the following ways:

(i) More samples of the same size should be taken from the same universe and their results be compared. If results are similar, the sample will be reliable.

(ii) If the measurements of the universe are known then they should be compared with the measurements of the sample. In case of similarity of measurement, the sample will be reliable.

(iii) Sub-samples should be taken from the samples and studied. If the results of sample and sub-sample study show similarity, the sample should be considered reliable.

Check Your Progress

16. Define the terms 'sample' and 'sampling'.
17. Define the term sampling design.
18. How the sampling frame is selected? Name some popular sampling frames.
19. What is sampling error? How it occurs?
20. How the reliability of samples can be tested?

4.7 INQUIRY FORM AND RATING SCALES

An inquiry form is a specific type of form that is prepared by an individual who has an enquiry or questions to be answered. Most research processes use these forms to simplify the process of answering inquiries. An inquiry is any process that has the aim of enhancing knowledge, resolving suspicion or solving a problem. Inquiry typically means to seek information about something or to conduct a formal investigation.

Rating scale is one of the significant techniques used in context with inquiry form. Form here is a specific term used to define expression or judgment regarding some situation, object or character. Basically, these opinions are expressed or rated on a scale of values. The values are rated using rating techniques, a strategy by which such judgments can be quantified. Rating scale thus is extremely useful method to assess quality, especially when it is difficult to measure quality objectively. For example, 'How good was the performance?' is such a question which cannot be answered objectively. Rating scales record the judgment or opinions and then specifies the degree or amount of different degrees of quality. All these degrees are typically arranged along a line as the rating scale as shown below.

Excellent Very Good Good Average Below Average Poor Very Poor

These rating scales are used to measure the performance degree of answers given by an individual selected as a sample. Also these degrees of rating scale are used by organizations/offices for making appraisals. Typically, it provides a scale for rating/assigning values to each trait being rated a scale value giving a valid estimate of its status and then comparing the separate ratings into an overall score. The traits are the abilities, behavioural characteristics and performance of individual that can be judged and evaluated through suitable rating scales. These are, in general, of two types — qualitative scales and frequency scales. Qualitative scales rate the quality or characteristics of the behaviour or performance using terms like 'excellent' and 'very poor', as illustrated below (for a biology class):

The trait or characteristic to be rated: Laboratory skill/handling of the apparatus/taking readings and reporting observations.

5	4	3	2	1
Excellent	Good	Average	Poor	V. Poor

In frequency scales, the frequency of a behaviour is to be rated by using terms ranging from 'always' to 'never' as shown below.

The skill or behaviour to be rated: Solves numerical problems with proper steps and relevant diagrams.

5	4	3	2	1
Always	Frequently	Sometimes	Rarely	Never

4.8 ORGANIZATION AND STATISTICAL ANALYSIS OF DATA

4.8.1 Meaning, Importance and Steps Involved in Processing Data

Research does not merely consist of data that is collected. Research is incomplete without proper analysis of the collected data. Processing of data involves analysis and manipulation of the collected data by performing various functions. The data has to be processed in accordance with the outline laid down at the time of developing the research plan. Processing of data is essential for ensuring that all relevant data has been collected to perform comparisons and analyses. The functions that can be performed on data are as follows:

- Editing
- Coding
- Tabulation
- Classification

Usually, experts are of the opinion that the exercise of processing and analysing of data is inter-related. Therefore, the two should be thought as one and the same thing. It is argued that analysis of data generally involves a number of closely-related operations,

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which are carried out with the objective of summarizing the collected data and organizing it in such a way that they are able to answer the research questions associated with it.

However, in technical terms, processing of data involves data representation in a way that it is open to analysis. Similarly, analysis of data is defined as the computation of certain measures along with searching for the patterns of relationship that may exist among data groups.

Editing of Data

Editing of data involves the testing of data collection instruments in order to ensure maximum accuracy. This includes checking the legibility, consistency and completeness of the data. The editing process aims at avoiding equivocation and ambiguity. The collected raw data is also examined to detect errors and omissions, if any. A careful scrutiny is performed on the completed questionnaires and schedules to assure that the data has the following features:

- Accuracy
- Consistency
- Unity
- Uniformity
- Effective arrangement

The stages at which editing should be performed can be classified as follows:

- **Field Editing**: This involves reviewing the reporting forms, by the investigator, that are written in an abbreviated or illegible form by the informant at the time of recording the respondent's responses. Such type of editing must be done immediately after the interview. If performed after some time, such editing becomes complicated for the researcher, as it is difficult to decipher any particular individual's writing style. The investigator needs to be careful while field editing and restrain the researcher from correcting errors or omission by guesswork.
- **Central Editing**: This kind of editing involves a thorough editing of the entire data by a single editor or a team of editors. It takes place when all the schedules created according to the research plan have been completed and returned to the researcher. Editors correct the errors such as data recorded in the wrong place or the data recorded in months when it should be recorded in weeks. They can provide an appropriate answer to incorrect or missing replies by reviewing the other information in the schedule. At times, the respondent can be contacted for clarification. In some cases, if the answer is inappropriate or incomplete and an accurate answer cannot be determined on any basis, then the editor should delete or remove that answer from the collected data. He/She can put a note as 'no answer' in this case. The answers that can be easily deciphered as wrong should be dropped from the final results.

Besides using the above-stated methods according to the data source, the researcher should also keep in mind the following points while editing:

- Familiarity with the instructions given to interviewers and coders
- Know-how of editing instructions
- Single line striking for deleting of an original entry
- Standardized and distinctive editing of data
- Initialization of all answers that are changed

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Coding of Data

Coding of data can be defined as representing the data symbolically using some predefined rules. Once data is coded and summarized, the researcher can analyse it and relationships can be found among its various categories.

Checklist for Coding

This enables the researcher to classify the responses of the individuals according to a limited number of categories or classes. Such classes should possess the following important characteristics:

- Classes should be appropriate and in accordance to the research problem under consideration.
- They must include a class for every data element.
- There should be a mutual exclusivity, which means that a specific answer can be placed in one and only one cell of a given category set.
- The classes should be one-dimensional. This means that every class is defined in terms of only one concept.

Significance of Coding

Coding of data is necessary for its efficient analysis. Codiing facilitates reduction of data from a variety to a small number of classes. Thus, only that information which is important and critical for analysis is retained in the research. Coding decisions are usually taken at the designing stage of the questionnaire. This makes it possible to pre-code the questionnaire choices, which in turn, is helpful for computer tabulation.

However, in case of hand coding, some standard method should be used. One such method is to code in the margin with a coloured pencil. The other method is to transcribe data from the questionnaire to a coding sheet. Whatever method is adopted, you should ensure that coding errors are altogether eliminated or reduced to a minimum level.

Classification of Data

Research studies involve extensive collection of raw data and usage of the data to implement the research plan. To make the research plan easier, the data needs to be classified in different groups for understanding the relationship among the different phases of the research plan. Classification of data involves arrangement of data in groups or classes on the basis of some common characteristics. The methods of classification can be divided under the following two headings:

- Classification according to attributes
- Classification according to class intervals

Figure 4.5 shows the categories of data.

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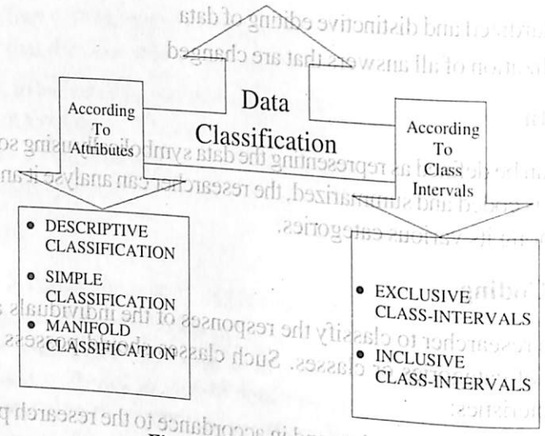

Fig. 4.5 Data Classification

Classification of Data According to Attributes

Data is classified on the basis of similar features as follows:

- **Descriptive Classification:** This classification is performed according to the qualitative features and attributes which cannot be measured quantitatively. These features can be either present or absent in an individual or an element. The features related to descriptive classification of attributes can be literacy, sex, honesty, solidarity, etc.
- **Simple Classification:** In this classification the elements of data are categorized on the basis of those that possess the concerned attribute and those that do not.
- **Manifold Classification:** In this classification two or more attributes are considered simultaneously and the data is categorized into a number of classes on the basis of those attributes. The total number of classes of final order is given by 2^n, where n = number of attributes considered.

Classification of Data According to Class Intervals

Classifying data according to the class intervals is a quantitative phenomenon. Class intervals help categorize the data with similar numerical characteristics, such as income, production, age, weight, etc. Data can be measured through some statistical tools like mean, mode, median, etc. The different categories of data according to class intervals are as follows:

- **Statistics of Variables:** This term refers to the measurable attributes, as these typically vary over time or between individuals. The variables can be discrete, i.e., taking values from a countable or finite set, continuous, i.e., having a continuous distribution function, or neither. This concept of variable is widely utilized in the social, natural and medical sciences.
- **Class Intervals:** They refer to a range of values of a variable. This interval is used to break up the scale of the variable in order to tabulate the frequency distribution of a sample. A suitable example of such data classification can be given by means of categorizing the birth rate of a country. In this case, babies aged zero to one year will form a group; those aged two to five years will form another group, and so on. The entire data is thus categorized into several numbers of groups or classes or in other words, class intervals. Each class interval has an upper limit as well as a lower limit, which is defined as 'the class limit.' The difference between two class limits is known as class magnitude. Classes can have equal or unequal class magnitudes.

The number of elements, which come under a given class, is called the frequency of the given class interval. All class intervals, with their respective frequencies, are taken together and described in a tabular form called the frequency distribution.

Problems Related to Classification of Data

The problems related to classification of data on the basis of class intervals are divided into the following three categories:

(i) **Number of Classes and their Magnitude:** There are differences regarding the number of classes into which data can be classified. As such, there are no pre-defined rules for the classification of data. It all depends upon the skill and experience of the researcher. The researcher should display the data in such a way that it should be clear and meaningful to the analyst.

As regards the magnitude of classes, it is usually held that class intervals should be of equal magnitude, but in some cases unequal magnitudes may result in a better classification. It is the researcher's objective and judgement that plays a significant role in this regard. In general, multiples of two, five and ten are preferred while determining class magnitudes. H.A. Sturges suggested the following formula to determine the size of class interval:

$$i = R/(1+3.3 \log N)$$

where,

i = size of class interval.

R = Range (difference between the values of the largest element and smallest element among the given elements).

N = Number of items to be grouped.

Sometimes, data may contain one or two or very few elements with very high or very low values. In such cases, the researcher can use an open-ended interval in the overall frequency distribution. Such intervals can be expressed below two years; or twelve years and above. However, such intervals are not desirable, yet cannot be avoided.

(ii) **Choice of Class Limits:** While choosing class limits, the researcher must determine the mid-point of a class interval. A mid-point is, generally, derived by taking the sum of the upper and lower limit of a class and then dividing it by two. The actual average of elements of that class interval should remain as close to each other as possible. In accordance with this principle, the class limits should be located at multiples of two, five, ten, twenty and hundred and such other figures. The class limits can generally be stated in any of the following forms:

- **Exclusive Type Class Intervals**: These intervals are usually stated as follows:
  - 10–20
  - 20–30

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- 30–40
- 40–50

These intervals should be read in the following way:

- 10 and under 20
- 20 and under 30
- 30 and under 40
- 40 and under 50

In the exclusive type of class intervals, the elements whose values are equal to the upper limit of a class are grouped in the next higher class. For example, an item whose value is exactly thirty would be put in 30–40-class interval and not in 20–30-class interval. In other words, an exclusive type of class interval is that in which the upper limit of a class interval is excluded and items with values less than the upper limit, but not less than the lower limit, are put in the given class interval.

o **Inclusive Type Class Intervals**: These intervals are normally stated as follows:

- 11–20
- 21–30
- 31–40
- 41–50

This should be read as follows:

- 11 and under 21
- 21 and under 31
- 31 and under 41
- 41 and under 51

In this method, the upper limit of a class interval is also included in the concerning class interval. Thus, an element whose value is twenty will be put in 11–20-class interval. The stated upper limit of the class interval 11–20 is twenty but the real upper limit is 20.999999 and as such 11–20 class interval really means eleven and under twenty-one. When data to be classified happens to be a discrete one, then the inclusive type of classification should be applied. But when data happens to be a continuous one, the exclusive type of class intervals can be used.

(iii) **Determining the Frequency of Each Class**: The frequency of each class can be determined using tally sheets or mechanical aids. In tally sheets, the class groups are written on a sheet of paper and for each item a stroke (a small vertical line) is marked against the class group in which it falls. The general practice is that after every four small vertical lines in a class group, the fifth line for the element falling in the same group is indicated as a diagonal line through the above said four lines. This enables the researcher to perform the counting of elements in each one of the class groups. Table 4.1 displays a hypothetical tally sheet.

Table 4.1 A Tally Sheet

Income groups (Rupees)	Tally mark	Number of families (Class frequency)
Below 600	𝍸 𝍸 𝍸 III	15
601-900	𝍸 𝍸 I	9
901-1300	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	25
1301-1500	𝍸 𝍸 𝍸 IIII	16
1501 and above	𝍸 𝍸 II	10
Total		75

In case of large inquiries and surveys, class frequencies can be determined by means of mechanical aids, i.e., with the help of machines. Such machines function, either manually or automatically and run on electricity. These machines can sort out cards at a speed of around 25,000 cards per hour. Although this method increases the speed, it is an expensive method.

Tabulation of Data

In simple terms, tabulation means placing the results and data collected from research in a tabular form.

Methods of Tabulation

Tabulation can be done either manually or mechanically using various electronic devices. Several factors like the size and type of study, cost considerations, time pressures and availability of tabulating machines decide the choice of tabulation. Relatively large data requires computer tabulation. Manual tabulation is preferred in case of small inquiries, when the number of questionnaires is small and they are of relatively short length. The different methods used in hand tabulation are as follows:

- **Direct Tally Method**: This method involves simple codes, which the researcher can use to directly tally data with the questionnaire. The codes are written on a sheet of paper called tally sheet and for each response, a stroke is marked against the code in which it falls. Usually, after every four strokes against a particular code, the fifth response is indicated by drawing a diagonal or horizontal line through the strokes. These groups are easy to count and the data is sorted against each code conveniently.
- **List and Tally Method**: In this method, code responses may be transcribed into a large worksheet, allowing a line for each questionnaire. This facilitates listing of a large number of questionnaires in one worksheet. Tallies are then made for each question.
- **Card Sort Method**: This is the most flexible hand tabulation method, where the data is recorded on special cards that are of convenient sizes and shapes and have a series of holes. Each hole in the card stands for a code. When the cards are stacked, a needle passes through a particular hole representing a particular code. These cards are then separated and counted. In this way, frequencies of various codes can be found out by the repetition of this technique.

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Significance of Tabulation

Tabulation enables the researcher to arrange data in a concise and logical order. It summarizes the raw data and displays the same in a compact form for further analysis. It helps in the orderly arrangement of data in rows and columns. The various advantages of tabulation of data are as follows:

- A table saves space and reduces descriptive and explanatory statements to the minimum.
- It facilitates and eases the comparison process.
- Summation of elements and detection of omissions and errors becomes easy in a tabular description.
- A table provides a basis for various statistical computations.

Checklist for Tables

A table should communicate the required information to the reader in such a way that it becomes easy for him/her to read, comprehend and recall information when required. Certain conventions have to be followed during tabulation of data. These are as follows:

- All tables should have a clear, precise and adequate title to make them intelligible enough without any reference to the text.
- Tables should be featured with clarity and readability.
- Every table should be given a distinct number to facilitate an easy reference.
- The table should be of an appropriate size and tally with the required information.
- Headings for columns and rows should be in bold font letters. It is a general rule to include an independent variable in the left column or the first row. The dependent variable is contained in the bottom row or the right column.
- Numbers should be displayed such that they are neat and readable.
- Explanatory footnotes, if any, regarding the table should be placed directly beneath the table, along with the reference symbols used in the table.
- The source of the table should be indicated just below the table.
- The table should contain thick lines to separate data under one class from the data under another class and thin lines to separate the different subdivisions of the classes.
- All column figures should be properly aligned.
- Abbreviations should be avoided in a table to the best possible extent.
- If data happens to be large, then it should not be crowded in a single table. It makes the table unwieldy and inconvenient.

Tabulation can also be classified as complex and simple. The former type of tabulation gives information about one or more groups of independent variables, whereas, the latter shows the division of data in two or more categories.

4.8.2 Use of Statistical Tools for Analysis

A researcher needs to be familiar with different statistical methods so as to be able to use the appropriate method in his research study. There are certain basic statistical methods that can be classified into the following three groups:

- Descriptive statistics
- Inferential statistics
- Measures of central tendency and dispersion

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Descriptive Statistics

According to Smith, descriptive statistics is the formulation of rules and procedures where data can be placed in a useful and significant order. The foundation of applicability of descriptive statistics is the need for complete data presentation. The most important and general methods used in descriptive statistics are as follows:

- **Ratio**: This indicates the relative frequency of the various variables to one another.
- **Percentage**: Percentages (%) can be derived by multiplying a ratio with 100. It is thus a ratio representing a standard unit of 100.
- **Frequency Table**: It is a means to tabulate the rate of recurrence of data. Data arranged in such a manner is known as distribution. In case of a large distribution tendency, larger class intervals are used. This facilitates the researcher to acquire a more orderly system.
- **Histogram**: It is the graphical representation of a frequency distribution table. The main advantage of graphical representation of data in the form of histogram is that data can be interpreted immediately.
- **Frequency Polygon**: It is used for the representation of data in the form of a polygon. In this method, a dot that represents the highest score is placed in the middle of the class interval. A frequency polygon is derived by linking these dots. An additional class is sometimes added in the end of the line with the purpose of creating an anchor.
- **Cumulative Frequency Curve**: The procedure of frequency involves adding frequency by starting from the bottom of the class interval, and adding class by class. This facilitates the representation of the number of persons that perform below the class interval. The researcher can derive a curve from the cumulative frequency tables with the purpose of reflecting data in a graphical manner.

Inferential Statistics

Inferential statistics enable researchers to explore unknown data. Researchers can make deductions or statements using inferential statistics with regard to the broad population from which samples of known data has been drawn. These methods are called inferential or inductive statistics. These methods include the following common techniques:

- **Estimation**: It is the calculated approximation of a result, which is usable, even if the input data may be incomplete or uncertain. It involves deriving the approximate calculation of a quantity or a degree. For example, drawing an estimate of cost of a project; or deriving a rough idea of how long the project will take.
- **Prediction**: It is a statement or claim that a particular event will surely occur in future. It is based on observation, experience and a scientific reasoning of what will happen in the given circumstances or situations.
- **Hypothesis Testing**: Hypothesis is a proposed explanation whose validity can be tested. Hypothesis testing attempts to validate or disprove preconceived ideas. In creating hypothesis, one thinks of a possible explanation for a remarked behaviour. The hypothesis dictates that the data selected should be analysed for further interpretations.

There are also two chief statistical methods based on the tendency of data to cluster or scatter. These methods are known as measures of central tendency and measures of dispersion.

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Measures of Central Tendency and Dispersion

Central tendency refers to the central point around which data revolves. The measures of central tendency help know the point about which items have a tendency to cluster. This measure is supposed to be the representative figure for the entire mass of data. The measure of central tendency is also called statistical average. The most common techniques used in central tendency can be listed as follows:

- **Mode**: It refers to the score, value or category of the variable which is observed more frequently. It is also known as the arithmetic average. The mode in data distribution is that element around which there is maximum concentration. In general, mode is the size of the element, which has the maximum frequency. Mode is mostly useful in the study of popular sizes like that of shoes and shirts.
- **Median**: It refers to the middle value of a series of data arranged in an increasing order. It divides the series into two halves. In one half, all the values are less than the median, whereas in the other half all the items have values more than the median. Thus, for example, if the values of the items arranged in the ascending order are 50, 66, 75, 80, 90, 95, 100, then the value of the fourth element namely, 80 is the value of the median. This is because the median divides the frequencies into two equal parts. It can also be described as the fiftieth percentile. The formula for median can be written as follows:

$$\text{Median (M)} = \text{Value of } \left(\frac{n+1}{2}\right)^{\text{th}} \text{ item}$$

Median is a positional average and is used only in the context of qualitative phenomena, for example, in estimating intelligence, etc., which are often encountered in sociological fields.

- **Mean**: It refers to the measure of central tendencies that are derived by addition of all scores and dividing them by the number of scores. It is the simplest tool of measuring the central tendency and is the most widely used measure. It is chiefly used in summarizing the essential features of a series and in facilitating data comparison. It is used in further statistical calculations and is better in comparison to simple averages, especially in economics and social studies, where direct quantitative measurements are possible. The formula for mean can be stated as follows:

$$\text{Mean (or } \bar{X})^* = \frac{\sum X_i}{n} = \frac{X_1 + X_2 + \ldots + X_n}{n}$$

where $\bar{X}$ = The symbol we use for mean (pronounced as $\bar{X}$ bar)

$\sum$ = Symbol for summation

X_i = Value of the jth item X, i=1, 2,...., n

n = Total number of items

*If we use assumed average A, then mean would be worked out as:

$$\bar{X} = A + \frac{\sum (X_i - A)}{n} \quad \text{or} \quad \bar{X} = A + \frac{\sum f_i (X_i - A)}{\sum f_i}$$

in case of frequency distribution.

This is also known as the short cut method of finding $\bar{X}$.

In case of frequency distribution, we can workout the mean in the following way:

$$\bar{X} = \frac{\sum f_i X_i}{\sum f_i} = \frac{f_1 X_1 + f_2 X_2 + \cdots + f_n X_n}{f_1 + f_2 + \cdots + f_n} = n$$

Sometimes, instead of calculating the simple mean, we may calculate the weighted mean for a realistic average. The weighted mean can be worked out as follows:

$$\bar{X}_w = \frac{\sum w_i X_i}{\sum w_i}$$

Where $\bar{X}_w$ = Weighted mean

w_i = Weight of ith item X

X_i = Value of the ith item X

The measure of dispersion, on the other hand, defines how much the scores in a sample vary from one another. Although, an average can indicate a series only as best as a single figure can, it cannot show the scatter of values of elements of a variable in the series around the true value of average. The measures of dispersion are used to calculate this scatter value of different variables. The most commonly used devices in measures of dispersion are as follows:

- **Range**: It refers to the simplest possible measure of dispersion and is defined as the difference between the values of the extreme items of a series. Thus, the formula for range can be stated as follows:

$$\text{Range} = \begin{pmatrix}\text{Highest value of}\\ \text{an item in a series}\end{pmatrix} - \begin{pmatrix}\text{Lowest value of}\\ \text{an item in a series}\end{pmatrix}$$

Range is useful in terms that it provides an idea of variability very quickly.

- **Mean Deviation**: The average of the difference of the values of elements from the average value of the series is known as mean deviation. Such a difference is technically described as deviation. While calculating mean deviation, the minus sign of deviations is ignored while taking their total for obtaining the sum of the mean deviation. The formula for deriving mean deviation is as follows:

$$\text{Mean deviation from mean } (\delta_{\bar{X}}) = \frac{\sum | X_i - \bar{X} |}{n},$$

if deviations, $| X_i - \bar{X} |$, are obtained from arithmetic average

$$\text{Mean deviation from median } (\delta_m) = \frac{\sum | X_i - M |}{n},$$

if deviations, $| X_i - M |$ are obtained from median

$$\text{Mean deviation from mode } (\delta_z) = \frac{\sum | X_i - Z |}{n},$$

if deviations $\sum | X_i - Z |$ are obtained from mode

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where δ = symbol for mean deviation (pronounced as delta)

X_i = ith values of the variable X

n = number of elements

$\bar{X}$ = arithmetic average

M = median

Z = mode

- **Standard Deviation**: Standard deviation is the calculation of the dispersion of distribution of scores. It is the most widely used measure of dispersion of a series and is commonly denoted by the symbol σ (sigma). Standard deviation can be defined as the square root of the average of squares of deviations, when such deviations for the values of individual items in a series are obtained from the arithmetic average. It is derived using the following formula:

$$\text{Standard deviation}(\sigma) = \sqrt{\frac{\sum f_i (X_i - \bar{X})^2}{\sum f_i}}$$

in case of frequency distribution

where f_i means the frequency of the ith item.

Standard deviation is used mostly in research studies and is regarded as the most satisfactory measure of dispersion in a series. It is less affected by fluctuations of sampling. It is commonly used in the context of estimation and testing of hypotheses.

To conclude, we can infer that statistical methods play the role of enabling the researcher to accurately utilize the gathered information. This in turn helps him to be more specific in describing his findings.

4.9 INTERPRETATION OF DATA

Analysis of data is the process of transformation of data for the purpose of extracting some useful information, which in turn facilitates the discovery of some useful conclusions. Finding conclusions from the analysed data is known as interpretation of data. However, if the analysis is done, in the case of experimental data or survey, then the value of the unknown parameters of the population and hypothesis testing is estimated.

Analysis of data can be either descriptive or inferential. Inferential analysis is also known as statistical analysis. The descriptive analysis is used to describe the basic features of the data in a study, such as persons, work groups and organizations. The inferential analysis is used to make inferences from the data, which means that we are trying to understand some process and make some possible predictions based on this understanding.

Types of Analysis

The various types of analyses are as follows:

- **Multiple Regression Analysis**: This analysis is used to predict the single dependent variable by a set of independent variables. In multiple regression analysis, independent variables are not correlated.

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- **Multiple Discriminant Analysis**: In multiple discriminant analysis, there is one single dependent variable, which is very difficult to measure. One of the main objectives of this type of analysis is to understand the group differences and predict the likelihood that an entity, i.e., an individual or an object belongs to a particular class or group based on several metric independent variables.
- **Canonical Analysis**: It is a method for assessing the relationship between variables. This analysis also allows you to investigate the relationship between the two sets of variables.

Univariate, Bivariate and Multivariate Analyse

There are also many types of analyses performed according to the variance that exists in data. They are carried out to check if the differences among three or more variables are so significant that data has to be evaluated statistically. There are three types of such analyses; namely, univariate, bivariate and multivariate analyses. These types are discussed as follows:

- **Univariate Analysis**: In this analysis, only a single variable is taken into consideration. It is usually the first activity pursued while analysing data. It is performed with the purpose of describing each variable in terms of mean, median or mode, and variability. Examples of such analysis are averages or a set of cases that may come under a specific category amidst a whole sample.
- **Bivariate Analysis**: This type of analysis examines the relationship between two variables. It tries to find the extent of association that exists between these variables. Thus, a bivariate analysis may help you, for example, to find whether the variables of irregular meals and migraine headaches are associated. It may also help to find the extent to which they may be associated. Here, two variables are thus statistically measured simultaneously.
- **Multivariate Analysis**: This type of analysis involves observation and analysis of three or more than three statistical variables at a time. Such an analysis is performed using statistical tests or even a tabular format. Thus, for example, using the multivariate analysis method, you can study the variables of age, educational qualification and annual income of a given set of population at the same time.

Usually, these types of analyses are more convenient when performed in a tabular format. Multivariate analysis involves, using a cross-classification or contingency table. Such a table is made of two columns and two rows, showing the frequencies of two variables that are displayed in rows and columns. This is more popularly known as constructing the bivariate table. Traditionally, the independent variable is displayed in columns and the dependent one in rows. A multivariate table, if related to the same data, is the result of combining the bivariate tables. In this case, each bivariate table is known as a partial table. Usually, a multivariate table is created with the purpose of explaining or replicating the primary relationship that is found in the bivariate table. Table 4.2(a) and (b) shows an example of a bivariate table and a multivariate table.

Table 4.2 (a) Bivariate Table

	1991	1992	1993
Percentage of students failed	33 per cent	38 per cent	42 per cent
Percentage of students passed	67 per cent	62 per cent	58 per cent

Table 4.2 (b) Multivariate Table

	1991	1992	1993
	First Attempt	Second Attempt	Third Attempt
Percentage of students who passed in Maths	27 per cent	35 per cent	–
Percentage of students who passed in English	53 per cent	60 per cent	44 per cent

Although the data in both tables is related, as the variable of attempts is distinct, the multivariate table has been displayed separately in this example. However, you should note that the tables have dealt simultaneously with two or more variables of the data.

Data Interpretation

Data interpretation refers to the identification of trends in different variables. The researcher uses statistics for this purpose. The researcher is required to be familiar with the knowledge of the scales of measurement. This enables him/her to choose the appropriate statistical method for his/her research project. The scales of measurement facilitate the allotment of numerical values to characteristics adhering to any specific rules. This measurement is also related to such levels of measurement of data like nominal, ordinal and internal and ratio levels. These levels can be explained as follows:

- **Nominal Measurement**: The nominal measurement assigns a numerical value to a specific characteristic. It is the fundamental form of measurement. The nominal measurement calculates the lowest level of data available for measurement.
- **Ordinal Measurement**: This type of measurement involves allotting a specific feature to a numerical value in terms of a specific order. The ordinal scale displays the way in which the entity is measured. The ordinal scale of measurement is used to calculate and derive data pertaining to the median, percentage, rank order, correlations and percentile.
- **Interval Measurement**: A researcher can depict the difference between the first aspect of a data and its another aspect using this level of measurement. The interval scale of measurement is useful for the researcher in several ways. It can be applied to calculate arithmetic mean, averages, standard deviations and to determine the correlation between different variables.
- **Ratio Measurement**: In this method, there are fixed proportions (ratios) between the numerical and the amount of the characteristics that it represents. A researcher should remember while measuring the ratio levels that, there exists a fixed zero point. The ratio level of measurement facilitates researchers in determining if the aspects of ratio measurement possess any certain characteristic. Almost any type of arithmetical calculations can be executed using this scale of measurement.

The most important feature of any measuring scale is its reliability and validity, which is explained as follows:

- **Reliability**: This term deals with accuracy. A scale measurement can be said to be reliable, when it exactly measures only that which it is supposed to measure. In other words, when the same researcher repeats a test, i.e., with a different group but resembling the original group, he/she should get the same results as the former.
- **Validity**: According to Leedy, validity is the assessment of the soundness and the effectiveness of the measuring instrument. There are several types of validity, which include the following:
  - o **Content Validity**: It deals with the accuracy with which an instrument measures the factors or content of the course or situations of the research study.
  - o **Prognostic Validity**: It depends on the possibility to make judgements from results obtained by the concerned measuring instrument. The judgement is future oriented.
  - o **Simultaneous Validity**: This involves comparison of one measuring instrument with another; one that measures the same characteristic and is available immediately.

4.10 WRITING OF RESEARCH PROPOSAL AND REPORT

The following are the various methods used in writing research proposal and report.

4.10.1 Research Proposal

In simple terms, a research proposal means a written application that proposes to pursue or conduct a research study. It aims at presenting the idea around which the research study revolves. A research proposal should be able to communicate that the researcher has applied deep thought to the subject of research, and has put considerable effort in collecting the required information, scrutinizing the available data and contemplated a well-organized plan for the research. It tries to emphasize the need of conducting the research, and thus, necessarily involves formulation of a good research question. The basic components of a research proposal are as follows:

- Title
- Abstract
- Background
- Objective
- Technical approach
- Bibliography

Table 4.2 (a) Bivariate Table

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- Abstract
- Background
- Objective
- Technical approach
- Bibliography

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Based on this format, the desired form and features of the contents contained in a research proposal can be enumerated as follows:

- A research proposal starts with a foreword that contains the core question, which the researcher aims to answer. Thus, it is written with the purpose of explaining something.
- It also contains a concise review of the prevailing literature related to the researcher's subject. Here, the researcher aims at reviewing the major works related to his/her topic and specify the arguments that have been formulated.
- The research proposal includes a statement regarding the argument or explanation that the researcher aims to present.
- The proposal should also indicate the way in which the researcher's argument is going to be different from the arguments made by other authors. In other words, it should emphasize the aspects in which the argument is unique.
- The proposal should also include a short summary of the different parts of the research.
- A short bibliography containing the important sources being used should also be written in the proposal. This can also mean including databases, websites and interviews.
- The researcher should opt for quality rather than quantity in writing his/her proposal. Thus, a proposal need not be long and an approximately 3–4 pages of reasearch proposal is quite sufficient.

The research proposal is supposed to communicate the researcher's overall effort that is involved in conducting the research. As such, a researcher should take ample care while writing the research proposal. You should keep in mind while writing such a proposal that the ideas involved in the research study need to be presented in a comprehensive and reliable format. The reader should get a clear-cut idea of what the research is all about and what argument it aims to convey. It should also emphasize the researcher's thought process and the depth of his/her knowledge of the concerned subject matter of research.

4.10.2 Research Process

The process of research can be implemented as a series of actions or steps that are essential to be performed in a specific order. These actions or activities usually overlap each other rather than pursuing a specific sequence. A brief description of the steps is given as follows:

- **Selecting the Topic:** The first step of a researcher is to select a topic of research. While doing so, he/she should restrict it to the most potential topic that is open for extensive research out of several alternatives. The factors to be considered for topic selection are:
  - o Relevance
  - o Scope for research, i.e., the required data should be available and accessible
  - o Contribution to knowledge in the specific field
  - o Required cooperation from the research guide
- **Define the Research Problem:** The research problems can be related to either the state of nature or to the relationship of variables. In defining the research problem, the researcher should study the existing literature including books and journals available in the field with an interdisciplinary perspective to base his/her research topic on some reliable background. He/She should also concentrate on the relevance of the present research with the past works.
- **Mention the Objective of Research:** After selecting the topic and defining the research problem, the researcher should mention the objective of research. This means that he/she should explain what he/she aims to achieve through the research. His/Her objective should also include an explanation of the extent to which the research work is related to the specific field.
- **Survey Existing Literature:** To understand the basis of research, it is important for the researcher to review the existing literature. This involves:
  - o Surveying the existing books available in the field.
  - o Reviewing other published materials like articles, journals, reports and conference proceedings.

  The researcher should then prepare his/her own index for a period, in chronological order, in addition to his/her consultation of various indices.
- **Determine the Sample Design:** Often, we select only a few items for universal study purposes, for example, blood testing on sample basis to perform census inquiry. The item selected is technically known as a sample. The researcher must decide the way of selecting a sample or decide about the sample design. A sample design is a definite plan determined for data collection to obtain a sample from a given population. The various types of sample designs are as follows:
  - o Deliberate sampling
  - o Simple random sampling
  - o Systematic sampling
  - o Stratified sampling
  - o Quota sampling
  - o Cluster sampling
  - o Multi-stage sampling
  - o Sequential sampling

  The researcher should decide the sample design after considering the nature of inquiry and other related factors. Sometimes, several of these methods of sampling are used in the same study, which in turn is called 'mixed sampling'.
- **Data Collection:** There are a variety of ways to collect data. Primary data can be collected through experiments or through surveys. If the researcher performs an experiment, he/she observes some quantitative measurements. This helps him/her to examine the truth in his/her hypothesis. In the case of survey, however, the researcher can adopt one or more of the following ways to collect data:
  - o By observation
  - o Through personal interview
  - o Through telephone interview
  - o By mailing of questionnaires
  - o Through schedules
- **Execute the Project:** This is the most important step in the research process. The researcher should ensure that the project is performed in a logical way and in time. If a survey is to be carried out, steps should be taken to ensure that it is

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under statistical control so that the collected data is in accordance with the predetermined standard of accuracy.

- **Analysis of Data:** After data collection, the researcher turns to the task of analysing them. The bulk data should be compressed into a few manageable groups and tables for further analysis. The researcher can analyse the collected data by using various statistical measures.
- **Hypothesis Testing:** After analysing the data, the researcher should test the hypothesis, if any. He/She should check if the facts support the hypothesis or are contrary to the hypothesis. Statisticians have developed tests like Chi square test, *t*-test and F-test, for hypothesis testing. This testing further results in either acceptance or rejection of the hypothesis.
- **Generalizations and Interpretations:** The real value of research lies in its ability to arrive at certain generalizations. If the researcher cannot find a hypothesis to start with, he/she might seek to explain his/her findings on the basis of some theory. This is called 'interpretation'. This may give rise to new questions and further lead to more research.
- **Preparation of Report or Thesis:** This is the concluding step of research, where the researcher has to prepare the report of what has been done by him/her. Generally, the report should be designed in accordance with the following layout:
  - o **The Preliminary Pages:** Here the title, date, acknowledgements and foreword with the table of contents should be mentioned.
  - o **The Main Text:** This should be divided into introduction, summary, main report and conclusion.
  - o **End Matter:** This should contain appendices, bibliography and index.

  A report should be written in a precise and objective style in simple language. Charts and illustrations should be included to lay emphasis on the study of research.

4.10.3 Introduction to Report Writing

A report can be defined as a written document, which presents information in a specialized and concise manner. For example, a list of employees prepared by the HR department for salary distribution can be termed as a report. In other words, a report is information presented in a logical and concise manner.

There is a difference between report writing and other compositions because a report is written in a very short and conventional form. A report should cover all mandatory matters but nothing extra should be written. For writing a report, at first the relevant data is collected and then it is presented in a concise and objective manner. Then after successfully establishing the structure of the report, the formatting features that improve the look and readability of the report are added.

Reports can be divided into different categories. The two main types of reports are:

- Informational report
- Interpretive report

Informational Report

The report that consists of a collection of data or facts and is written in an orderly way is called an informational report. The main purpose of this type of report is to present the information in its original form without any conclusion and recommendation. Informational reports are further divided into four parts, which are:

- **Inspection Report:** The report which shows the outcome of a product or equipment to assure its proper functioning or to describe its quality is called an inspection report. This type of report is mainly used in manufacturing organizations.
- **Inventory Report:** The report which is made to keep the stock of various things like furniture, equipment, stationery, utensils and other accessories is called an inventory report.
- **Assessment Report:** These reports are made to maintain the database of the employees in an organization. Generally, these reports are useful for the HR department.
- **Performance Report:** The report which is made to measure the performance of the employees in an organization for purposes like appraisal or promotion are called performance reports.

Interpretive Report

Interpretive reports are those reports which contain a collection of data with its interpretation or any recommendation explicitly specified by the writer. This type of report also includes data analysis and conclusions made by the report writer. Writing interpretive reports is different from writing an informational report because it contains different elements. The possible elements that can be used in the interpretive reports are:

- Cover
- Frontpiece
- Title page
- Copyright notice
- Forwarding letter
- Preface
- Acknowledgements
- Table of contents
- List of illustrations
- Abstract and summary
- Introduction
- Discussion
- Conclusions
- Recommendations
- Appendices
- List of references
- Bibliography
- Glossary
- Index

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Characteristics of a Good Report

Reports are used for various purposes by various departments of an organization. Industries, governments, businesses and scientific projects, all resort to report writing to collect information and keep track of their performance and progress. The most important aspect of a report is to convey the information in clear-cut terms. It should provide facts in a direct, straightforward and accurate style. In this light, the characteristics of a good report can be classified under four heads, which are as follows:

- Language and style of the report
- Structure of the report
- Presentation of the report
- References in the report

Each of these aspects of report writing needs to be given due attention, as they are interrelated to each other. A report given with a lucid style but with very less and hypothetical information is of no use to the reader. Similarly, the report writer needs to avoid overcrowding of information that may make the reader feel confused and lost in reading data, thereby losing its charm. A systematic scrutiny of each of these aspects of a report is, therefore, necessary.

Language and Style of Report

A report must have a clear logical structure with clear indication of where the ideas are leading. It should be able to make a good first impression. The presentation of the report is very important. All reports must be written in good language, using short sentences and correct grammar and spellings. The main points to be kept in mind in this light are as follows:

- **Context and Style**
  - o Appropriate, informative title for the content of report
  - o Crisp, specific, unbiased writing with minimal jargon
  - o Adequate analysis of prior relevant research
- **Questions/Hypotheses**
  - o Clearly stated questions or hypotheses
  - o Thorough operational definitions of key concepts along with exact wording or measurement of key variables
- **Research Procedures**
  - o Full and clear description of the research design
  - o Demographic profile of the participants/subjects
  - o Specific data gathering procedures
- **Data Analysis**
  - o Appropriate inferential statistics for sample or experimental data and appropriate use of descriptive statistics
  - o Clear and reasonable interpretation of the statistical findings, accompanied by effective tables and figures
- **Summary**
  - o Fair assessment of the implications and limitations of the findings
  - o Effective commentary on the overall implications of the findings for theory and/or policy

Structure of Report

Before you write a report, you should define the high level structure of the report. Defining a clear logical structure will make the report easier to write and to read. There are two types of report structures, which are listed as follows:

- **Report Structure I:** In general, the report writing structure comprises the following sub-headings:
  - o Title Page
  - o Abstract
  - o Table of Contents
  - o Introduction
  - o Technical Detail and Results
  - o Discussion and Conclusions
  - o References
  - o Appendices
- **Report Structure II:** There is also a specific structure of report writing pertaining to technical or scientific reports which is as follows:
  - o Introduction
  - o Background and Context
  - o Technical Details
  - o Results
  - o Discussion and Conclusion
- **Order of Writing:** The following is correct order of writing:
  - o Start with the technical chapters/sections.
  - o Follow with the discussion.
  - o Finally, write the conclusions, introduction and abstract, if you are including any.
- **Appendix:** The appendix should contain the following:
  - o Material that suits or goes well with the flow of the main report but cannot be included in the main text of the report either because it is too long or is not essential reading. For example, lists of parameter values, etc.
  - o Bibliography, i.e., list of all the sources of material, you referred to in your report.

Presentation of Report

As stated earlier, mere data overloading or just a lucid style of writing may not be a plus point to good report writing. Both the aspects need to be given due consideration, so that they interact to give a simple, easy-to-read and comprehensive type of report. Same goes with the presentation of the contents of the report. Printing mistakes, informal use of font size and style can distract the attention of the reader. On the other hand, effective use of tables and figures for better understanding of data and writing its conclusions facilitate easy comprehension. The main points of focus, where due attention is required on part of the report writer are as follows:

- **Capitals:** This requires taking care of the following aspects:
  - o Using capitals only for proper nouns, place names, organization names, etc.

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- Defining acronyms at the first point of usage. For example, Incorporated (Inc).
- Using bold, italics or underlines for emphasis, instead of capitals.

- **Headings:** The basic points to be kept in mind for headings are as follows:
  - Differentiate headings from the rest of the text using different fonts, bold, italics or underlines.
  - Maintain consistency in formatting headings using predefined styles.
  - Avoid headings beyond three levels.
- **Tables, Figures and Equations:** In general, certain formatting standards are pursued while giving tables and figures that are as follows:
  - Descriptive labelling of all tables at the top with reference in the text.
  - All figures must be labelled descriptively at the top and must be referenced in the text.
  - All equations must be numbered consecutively.
- **General Presentation:** The following points must be considered for preparing any general presentation:
  - Sheets should be plain like white A4 size, printed in one side only.
  - Text should be justified on both sides and leave a blank line between paragraphs.
  - A staple in the top right hand corner is sufficient for most of the reports.
- **References in the Report:** Several report types like scientific, engineering, technical and census reports contain either original writing or text adopted from previous work. As such, a report writer should be careful and avoid the violation of copyright laws and plagiarism. The necessary rule of thumb in this regard can be stated as follows:
  - ***Citations and Referencing***
    - A citation is the acknowledgement in your writing of the work of other authors and includes paraphrasing and making direct quotes.
    - Unless citation is very necessary, you should write the material in your own words. This shows that you understand what you have read and know how to apply it, to your own context.
    - Direct quotes should be used sparingly.
  - ***Direct Quotes***
    - *Short Direct Quotes:* These need to be placed between quotation marks. For example, Rosenfield defines a cluster as a 'geographically bounded concentration of similar, related or complementary businesses, with active channels for business transactions, communications and dialogue that share specialized infrastructure, common opportunities and threats'. This shows clearly that the words being used are not your own words.
    - *Longer Direct Quotes:* There are occasions when it is useful to include longer direct quotes. If you are quoting more than about 40 words, you should again use quotation marks but also indent the text. For example, the sustainability of higher value added industry is grounded in the diminishing significance of cost structures. At the level of the European Union, a weak capacity to innovate has been identified as an innovation, in the sense of product, process, and organizational innovation, accounts for a very large amount, perhaps 80–90 per cent of the growth in productivity in advanced economies.

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Mechanics of Writing a Report

There are several mechanics of writing a report, which are strictly followed for preparing technical reports. The following points should be considered for writing a technical report:

- **Size and Physical Design:** The manuscript, if handwritten, should be in black or blue ink and on unruled paper of 8½" × 11" size. A margin of at least one-and-half inches is set at the left side and half inch at the right side of the paper. The top and bottom margins should be of one inch each. If the manuscript is to be typed, then all typing should be double spaced and on one side of the paper, except for the insertion of long quotations.
- **Layout:** According to the objective and nature of the research, the layout of the report should be decided and followed in a proper manner.
- **Quotations:** Quotations should be punctuated with quotation marks and double spaces, forming an immediate part of the text. However, if a quotation is too lengthy, then it should be single spaced and indented at least half an inch to the right of the normal text margin.
- **Footnotes:** Footnotes are meant for cross-references. They are placed at the bottom of the page, separated from the textual material by a space of half an inch as a line that is around one-and-a-half inches long. Footnotes are always typed in a single space, though they are divided from one another by double space.
- **Documentation Style:** The first footnote reference to any given work should be complete, giving all essential facts about the edition used. Such footnotes follow a general sequence and order:
  - In case of the single volume reference:
    - Author's name in normal order
    - Title of work, underlined to indicate italics
    - Place and date of publication
    - Page number reference

    For example:

    John Gassner, *Masters of the Drama*. New York: Dover Publications, Inc.1954, p.315.
  - In case of a multi-volume reference:
    - Author's name in the normal order
    - Title of work, underlined to indicate italics
    - Place and date of publication
    - Number of the volume
    - Page number reference

    For example:

    George Birkbeck Hill, *Life Of Johnson*. Whitefish, June 2004, Volume 2, p.124.

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- In case of works arranged alphabetically:
  - For works arranged alphabetically such as encyclopaedias and dictionaries, page reference is usually not needed. In such cases, order is illustrated according to the names of the topics.
  - Name of the encyclopaedia
  - Number of editions

  For example:

  'Salamanca', *Encyclopaedia Britannica*, 14th Edition.
- In case of periodicals reference:
  - Name of the author in normal order
  - Title of article, in quotation marks
  - Name of the periodical, underlined to indicate italics
  - Volume number
  - Date of issuance
  - Pagination

  For example:

  P.V. Shahad, 'Rajesh Jain's Ecosystem', in *Business Today*, Vol.14, 18 December 2005, p. 28.
- In case of multiple authorship:

  If there are more than two authors or editors, then in the documentation the name of only the first is given and the multiple authorship is indicated by 'et al' or 'and others'.
  - Author's name in normal order
  - Title of work, underlined to indicate italics
  - Place and date of publication
  - Pagination

  For example:

  Alexandra K. Wigdor, *Ability Testing: Uses Consequences and Controversies*, 1981, p.23.

Subsequent references to the same work need not be detailed. If the work is cited again without any other work intervening, it may be indicated as *ibid*, followed by a comma and the page number.

- **Punctuations and Abbreviations in Footnotes:** Punctuation concerning the book and author names has already been discussed. They are general rules to be strictly adhered. Some English and Latin abbreviations are often used in bibliographies and footnotes to eliminate any repetition.

Table 4.3 shows the various English and Latin abbreviations used in bibliographies and footnotes.

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Table 4.3 *English and Latin Abbreviations used in Bibliographies and Footnotes*

Abbreviations	Meaning
Anon.,	Anonymous
Ante.,	Before
Art.,	Article
Aug.,	Augmented
bk.,	Book
bull.,	Bulletin
cf.,	Compare
ch.,	Chapter
col.,	Column
diss.,	Dissertation
ed.,	editor, edition, edited
ed. cit.,	edition cited
e.g.	exempli gratia: for example
eng.,	Enlarged
et.al.,	and others
et seq., ex.,	et sequens: and the following Example
f.,ff.,	figure(s)
fn.,	Footnote
ibid.,ibidem	in the same place
id.,idem.,	the same
ill.,illus., or	
illust(s)	illustrated, illustration(s)
Intro., intro.,	introduction
l., ll.,	line(s)
loc. cit.,	in the place cited; used as op.cit.,
MS., MSS.,	Manuscript(s)
N.B. nota bene	note well
n.d.,	no date
n.p.,	no place
no pub.,	no publisher
no(s) .,	number(s)
o.p.,	out of print
op.cit:	in the work cited
p.pp	page(s)
passim:	here and there
Post:	After
rev.,	Revised
tr., trans.,	translator, translated, translation
vid or vide:	see, refer to
viz.,	Namely
vol. Or vol(s) .,	volume(s)
vs., versus.,	Against

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- **Use of Statistics, Charts and Graphs:** Statistics contribute to clarity and simplicity in a report. They are usually presented in the form of tables, charts, bars, line-graphs and pictograms.
- **Final Draft:** It requires careful scrutiny with regard to grammatical errors, logical sequence and coherence in the sentences of the report.
- **Index:** An index acts as a good guide to the reader. It can be prepared both as subject index and author index giving names of subjects and names of authors, respectively. The names are followed by the page numbers of the report, where they have appeared or been discussed.

4.10.4 Research Report: An Overview

In simple terms, a research report means a written document, which describes the findings of some individual or a group of individuals. It gives an account of something seen, heard, done, etc. The findings may comprise such information like data, surveys, resolutions, or policies, on which the concerned individual or individuals have to submit their reports about the proceedings along with the relevant conclusions.

The preparation and presentation of a research report is the most important part of the research process. No matter how well designed the research study is, it is of little value, unless communicated effectively to others in the form of a research report. Moreover, if the report is confusing or poorly written, then the time and effort spent on gathering and analysing data would be wasted. It is therefore, essential to summarize and communicate the result to the management of an organization with the help of an understandable and logical research report.

Research reports are helpful during the research study, in the sense that they facilitate maintenance of vast data in a logical way. Thus, in case the researcher experiences any difficulty during the course of the study, it becomes easier to refer to the contents of the report to get the relevant data. Research report writing essentially involves systematic arrangement of data. This helps in discovering flaws in reasoning, which may have been missed earlier while conducting a research.

Format of Research Report

The layout of the research report is of utmost importance because the reader should be able to grasp logically, what has been said and not feel lost in the bulk findings mentioned in the research. This requires preparing a proper layout of the report. Report layout means allotting the research findings in a comprehensible format. The layout should contain the following points:

- **Preliminary Pages:** In the preliminary pages, the report should carry a 'title' and a 'date', followed by acknowledgements in the form of 'Preface' or 'Foreword'. The 'Table of Contents' should come next, followed by a 'list of tables and illustrations'. This entails the reader to an easy reading and quick location of the required information.
- **Main Text:** The main text comprises the complete outline of the research report with all the details. The title of the research study is repeated at the top of the first page of the main text, and then followed with the other details on the pages numbered consecutively, beginning with the second page. The main text can be classified into the following sections:

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- **Introduction:** The purpose of introduction is to introduce the research projects to the readers. It should clearly state the objectives of research, i.e., it should make clear, why the problem was considered worth investigating. A brief summary of other relevant research can be included as well, to enable the reader to see the present study in that context.
- **Methodology used for Performing the Study:** The introduction should contain answers to questions like how was the study carried out, what was the basic design, what were the experimental directions, what questions were asked in the questionnaires used, etc. Besides this, the scope and limitations of the study must be marked out.
- **Statement of Findings and Recommendations:** The research report should comprise a statement of findings and recommendations in a non-technical language so that it is easily comprehensible.
- **Results:** A detailed presentation of the findings of the study, with supporting data in tabular forms along with the validation of results, should be given. This section should contain statistical summaries and deductions of the data rather than the raw data. There should be a logical sequence and sectional presentation of the results.
- **Implications of the Result:** The researcher should write down his/her results clearly and precisely, again at the end of the main text. The implications derived from the results of the research study should be stated in the research plan. The report should also mention the conclusion drawn from the study, which should be clearly related to the hypothesis stated in the introductory section.
- **Summary:** The next step is to conclude the report with a short summary, mentioning in brief the research problem, the methodology, the major findings and the major conclusions drawn from the research results.
- **End Matter:** The end of the research report should consist of appendices, listed in respect of all technical data such as questionnaires, sample information and mathematical derivations. The bibliography of the referred sources and an index should also be given.

Precautions for Writing Research Reports

A research report is the means of conveying the research study to a specific target audience. The following precautions should be taken while preparing the research report:

- It should be long enough to cover the subject and short enough to preserve interest.
- It should not be dull and complicated.
- It should be simple, without the usage of abstract terms and technical jargons.
- It should offer ready availability of findings with the help of charts, tables and graphs, as readers prefer quick knowledge of main findings.
- The layout of the report should be in accordance with the objective of the research study.
- There should be no grammatical errors and writing should adhere to techniques of report writing in case of quotations, footnotes and documentations.

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- It should be original, intellectual and contribute to the solution of a problem or add knowledge to the concerned field.
- Appendices should be listed with respect to all the technical data in the report.
- It should be attractive, neat and clean, whether handwritten or typed.
- The report writer should be careful about the possessive form of the word 'it is' with 'it's'. The correct possessive form of 'it's' is 'its'. The use of 'it is' is the contractive form of 'it is'.
- A report should not have contractions. Examples are 'didn't' or 'it's'. In report writing, it is best to use the non-contractive form. Hence, the examples would be replaced by 'did not' and 'it is'. Using 'Figure' instead of 'Fig.' and 'Table' instead of 'Tab.' will spare the reader of having to translate the abbreviations, while reading. If abbreviations are used, use them consistently throughout the report. For example, do not switch between 'versus' and 'vs'.
- It is advisable to avoid using the word 'very' and other such words that try to embellish a description. They do not add any extra meaning and, therefore, should be dropped.
- Repetition hampers lucidity. The report writer must avoid repeating the same word more than once within a sentence.
- When using the words 'this' or 'these', it must be clear to the reader as to what is being referred to. This reduces ambiguity in the writing and helps to tie sentences together.
- Do not use the word 'they' to refer to a singular person. You can either rewrite the sentence to avoid needing such a reference or use the singular 'he or she.'

4.10.5 Written and Oral Reports

A written report plays a vital role in every business operation. The manner in which an organization writes business letters and business reports creates an impression of its standard. Therefore, the organization should emphasize on the improvement of writing skills of the employees in order to maintain effective relations with their customers.

Writing effective written report requires a lot of hard work. Therefore, before you begin writing, it is important to know the objective, i.e., the purpose of writing, collection and organization of required data.

Guidelines for an Effective Written Report

Writing a report is the best way to communicate, and often the only way to convey one's ideas to others. Thus, it is necessary that the writing should be effective. To improve the effectiveness of writing a report, following are the important points that should be kept in mind:

- Take breaks in between writing, since this gives you the time to incubate the ideas.
- Start writing a short manuscript first, and later on, the detailed one. Create an outline and organize the complete work.
- Make a checklist of the important points that are necessary to be covered in the manuscript.

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- Focus on one objective at a time.
- Use dictionary and relevant reference materials as and when required.

Principles of Writing a Report

To write a useful report, it is necessary to follow certain principles. The following are the principles that must be followed while writing a report:

- **Principle of Purpose:** A report must have a clear and meaningful purpose that can be converted into an effective management. A clear statement of the purpose helps prepare a well-focussed report on which the management can work. Specification of the purpose is important because:
  - o Reports are the analysis of facts and proposals.
  - o Reports are the record of a particular business activity.
- **Principle of Organization:** A report that is written should be well designed and well ordered. The managerial plan of a report must include the following:
  - o Purpose of report
  - o Information required to be included in the report
  - o Method used to collect report data
  - o Summary of the report
  - o Problems and solutions of the subject mentioned in the report
  - o An appendix that describes and confirms the content and conclusion of the report
- **Principle of Brevity:** Reports should be concise. It is essential because:
  - o Long reports are costly.
  - o Long reports are difficult to examine.
  - o Long reports are prone to disapproval, as they seem insufficient.
  - o Long reports focus on irrelevant minor details that may lead to ignorance of major points.
- **Principle of Clarity:** Reports should be clear. Clarity can be maintained by using simple language for writing the report. New terms, if any in the report, should be properly explained to avoid confusion.
- **Principle of Scheduling:** Reports should be prepared at that time when there is no undue burden on the staff or when the staff has sufficient time to prepare reports. However, the time period between the gathering of data and generating finished reports should not be long; otherwise, the report may become outdated and useless if it is not completed in time.
- **Principle of Cost:** While preparing reports, it is necessary that the cost-benefit analysis of the report should be done. A report should be minimum at costs and maximum at benefits. If the cost of preparation of the report is high but its benefit is low, then it is not advisable to prepare that report.

Different Formats of Written Reports

A written report can be written in various formats, which are as follows:

- **Straight-Line Format:** This format is used when the information is to be presented in alphabetical, sequential or numerical orders. This format is used to generate descriptive reports.

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- **Building Blocks Format:** This format is used when the information presented leads to some conclusion. The report in this format starts with a brief introduction, contains some logical facts and finally the conclusions and recommendations.
- **Inverted Pyramid Format:** The report in this format has the most important item at the top, and the least important item at the bottom of the report. That is, items are listed in the descending order with the most important item at the top. This style of writing or format is also known as journalistic style or format.

Oral Report

At times, oral presentation of the results that are drawn out of research is considered effective, particularly in cases where policy recommendations are to be made. This approach proves beneficial because it provides a medium of interaction between a listener and a speaker. This leads to a better understanding of the findings and their implications. However, the main drawback of oral presentation is lack of any permanent records related to the research. Oral presentation of the report is also effective when it is supported by various visual devices such as slides, wall charts and white boards that help in better understanding of the research reports.

Advantages of Oral Reports

Oral reports help in direct communication without any delay. The following are some of the advantages of an oral report:

- It provides immediate feedback to the participants of the oral report. Moreover, participants can also ask for further clarification, elaboration and justifications.
- It is time saving.
- It helps develop relationship among employees by building healthy atmosphere in an organization.
- It is an effective tool of persuasion in business.
- It is economical as it saves large amount of money spent on stationery.
- It provides the speaker with the opportunity to correct and make himself/herself clear on the spot.
- It helps speakers to immediately understand the reaction of the group that they are addressing.

Disadvantages of Oral Reports

There are many disadvantages of oral reports, which are:

- Oral reports may not always be time saving. Sometimes, the meeting between the speaker and the listener can continue for a very long time without any satisfactory conclusion.
- A listener of the oral report cannot always retain the entire message.
- The messages in the oral reports do not have any legal validity as they are not documented.
- Oral reports may sometimes be misleading, if the thoughts of the speaker are not organized carefully.
- Lengthy oral messages may sometimes cause problems.

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Principles of Oral Reports

Oral reports should follow some principles in order to make communication of the oral report between the speaker and the listener effective. The following are the basic principles of oral reports:

- It is the responsibility of a manager to inform his subordinates about the tasks that they have to perform.
- To obtain full commitment of employees for achieving their objectives, all important information that directly or indirectly affects the objective should be communicated to the employees. Also, employees should be aware of the matters that are relevant to their circumstances.
- It is the duty of a manager to see that the information of the report communicated to the subordinates is clear to them and is complete.
- Proper planning for information flow should be done.
- The information in the oral report should provide proper feedback that helps maintain effective industrial relation.

ACTIVITY

1. Make up a questionnaire for a group of college students to assess the quality of the new canteen management. Describe by what mode the questionnaire would be administered.
2. Prepare an interview following the steps in 4.5.2. State what type of interview it is, how many participants and what were the questions asked. Submit a detailed report.

DID YOU KNOW

A report or account is any informational work (usually of writing, speech, etc.) made with the specific intention of relaying information or recounting certain events in a widely presentable form. Written reports are documents which present focused, salient content to a specific audience. Reports are often used to display the result of an experiment, investigation, or inquiry. The audience may be public or private, an individual or the public in general. Reports are used in government, business, education, science, and other fields. Reports use features, such as graphics, images, voice, or specialized vocabulary in order to persuade that specific audience to undertake an action.

4.11 SUMMARY

- Data collection is an important process of the research process. A researcher uses various tools to gather data like research, observation, questionnaires, interviews and sociometry.
- Observation is a process in which one or more persons observe some real-life situation and record pertinent occurrences. It is used to evaluate the overt behaviour of the individuals in controlled and uncontrolled situations.

Check Your Progress

21. What is a report?
22. Cite a few characteristics of a good report.
23. What is the usage of footnotes?
24. List the four types of informational reports.
25. List the various formats of written reports.
26. List the principles of report writing.

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- Observation can either be participant or non-participant.
- Questionnaire is a list of questions arranged in a specific way or randomly, generally in print or typed and having spaces for recording answers to the questions.
- Questionnaires can be either closed or open, with each form having its limitations and advantages. Questionnaires can be administered via mail, face to face or via the computer.
- Interview is a two-way systematic conversation between an investigator and an informant, initiated for obtaining information relevant to as a specific study.
- Sociometry is a method for discovering, describing and evaluating social status, structure, and development through measuring the extent of acceptance or rejection between individuals in a group.
- An artificial population is created by the researcher in order to illustrate a principle, or to make for more convenience and ease in carrying out the study of a problematic situation.
- The term 'sampling' refers to the technique whereby a smaller group is selected from a larger one so that the more manageable smaller group can be observed and those observations can be applied to the larger group as well.
- The sampling methods are broadly classified into two types: (i) Probability sampling and (ii) Non-probability sampling.
- Probability sampling is a technique of sampling which gives the probability that a sample is representative of population. This kind of sample is selected in such a way that every element chosen has a known probability of being included.
- Theoretically, random sampling is a method of selecting 'n' units from N units in such a way that everyone in the population of N units has an equal chance of being selected.
- Systematic sampling is a variant of the random process of sampling. In this technique, the requisite number of sample units are selected from the population. This sampling entails organizing the population in a predetermined order and then selecting from the list at regular intervals.
- Double sampling is a type of sampling which includes both questionnaire and interview methods for probing a research problem.
- The main distinction between the multi-stage and the multi-phase sampling is the use of unit of sampling at different levels in multi-stage sampling but not in multi-phase sampling.
- The guiding principles in non-probability methods are— availability of the subjects, the personal judgment of the investigator, and convenience in carrying out the research.
- Incidental sampling is also known as accidental or convenience sampling. When a readily or easily available group is selected as a sample, it is termed as an 'incidental sample'.
- Sampling design refers to a definite plan for obtaining a sample from the sampling frame. It refers to the technique or procedure, which a researcher adopts in selecting some sampling units from where inferences about population are drawn. Sampling data is obtained before collecting the final data.

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- The sampling frame should be selected in such a way that it consists of almost all the sampling units. A sample should be selected in such a way that it has all the characteristics of the population.
- Sampling unit is the basic unit that contains elements of the target population.
- A good sample is a true representative of the population corresponding to its properties. A good sample does not permit prejudices, pre-conceptions and imagination to influence its choice.
- Size of a sample should be such that it yields an accurate result. The probability of error can be estimated.
- Representative sample is the sample which possesses the same characteristics as that of its parent population or variable. Thus, it factually represents the variation that exists in the parent variable on the general level. The significance of a representative sample lies in the fact that it represents the population more accurately.
- Sampling distribution is often required in sampling analysis. Sampling distribution of mean refers to the probability distribution of all possible means of random samples of a given size.
- Sampling may give rise to certain errors known as 'sampling errors' or sampling fluctuations. Sampling errors are those errors, which arise on account of sampling and generally happen to be random variations in the sample estimates of the actual population values.
- A sample survey requires study in small portions of population, as there can be certain amount of inaccuracy in the information collected during sampling analysis. This inaccuracy is called sampling error or error variance.
- Sampling errors occur randomly and are equally likely to be in either direction and the magnitude of sampling error depends on the nature of the universe. The more uniform the universe is, the smaller is the sampling error.
- Sampling errors are of two types: (i) Biased and (ii) Unbiased. Biased errors arise from any bias in selection, estimation, etc.
- A research proposal should be able to communicate that the researcher has applied deep thought to the subject of research, and has put considerable effort in collecting the required information, scrutinizing the available data and contemplated a well-organized plan for the research.
- The research process involves preparation of a research report to explain the hypothesis. This is done by logical analysis of the subject, preparation of the rough draft and then making the final draft of the hypothesis.
- A report must have a clear and meaningful purpose that can be converted into an effective management. A clear statement of the purpose helps prepare a well-focussed report on which the management can work.
- According to the objective and nature of the research, the layout of the report should be decided and followed in a proper manner.
- A written report should be clear, specific and convincing in order to be effective. While writing, you should avoid ambiguity and follow a friendly, lively, pleasant and sophisticated style of writing.
- An oral report facilitates direct communication without any delay. It is time saving and also provides immediate feedback to the participants.

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- Writing interpretive reports is different from writing an informational report because it contains different elements.

4.12 KEY TERMS

- **Observation:** A process in which one or more persons monitor some real-life situation and record pertinent occurrences
- **Questionnaire:** A tool for research comprising a list of questions whose answers provide information about the target group, individual or event
- **Sample:** It is a small percentage of the larger group selected for research
- **Random error:** This is any form of error that can occur while performing activities like data collection, conducting surveys, coding, performing transfer and analysis
- **Sampling design:** Refers to the technique or procedure adopted by a researcher in selecting some sampling units from where inferences about population are drawn
- **Sampling frame:** A list containing all elementary units or a group of such units that may form the basis of a sampling process
- **Informational report:** The report that consists of a collection of data or facts and is written in an orderly way
- **Interpretive report:** These reports which contain a collection of data along with the interpretation or any recommendation explicitly specified by the writer
- **Inventory report:** The report made to keep the stock of various things like furniture, equipment, stationery, utensils and other accessories
- **Performance report:** The reports made to measure the performance of the employees in an organization for different purposes like appraisal or promotion
- **Report:** A report is a written document which presents information in a specialized and concise manner
- **Research layout:** Allotting the research findings in a comprehensible format.
- **Research proposal:** A written application that proposes to pursue or conduct a research study
- **Research report:** A written document, which describes the findings of some individual or a group of individuals
- **Sample design:** A definite plan determined for data collection to obtain a sample from a given population

4.13 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Participatory and non-participatory observations are two types of observation.
2. The following are the limitations of observation:
   (i) Observation may give undue stress to aspects of limited significance simply because they can be recorded easily, accurately and objectively.
   (ii) Various observers observing the same event may concentrate on different aspects of a situation.
   (iii) Try to pose and exhibit at the time of observation.

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3. In the process of 'participant observation', the observer becomes more or less one of the group members and may actually participate in some activity or the other of the group as a listener, a visitor or any other role.
4. Observation technique is employed to observe characteristics of various designs of school buildings and equipment.
5. Too many variables may not be observed simultaneously to ensure effective observation.
6. Mail or post and face-to-face interview are the modes of getting information through questionnaire.
7. The questions should be relevant to the subject or problem and should be perfectly clear and unambiguous.
8. Questionnaires can be administered through mail, via personal contact and via the Internet.
9. The interview is, in a sense, the foundation upon which all other elements rest, for it is the data-gathering phase.
10. The various types of interview are: (i) Group Interview, (ii) Diagnostic Interview, (iii) Clinical Interview, (iv) Research Interview, (v) Single Interviewer or Panel Interviews, (vi) Directed Interview, (vii) Non-directive, (viii) Focused Interview, (ix) Depth Interview.
11. The tape recorder permits the interviewer to devote full attention to the respondent.
12. An interview permits the research worker to follow-up leads as contrasted with the questionnaire.
13. 'Sociometry' is a technique to study the choices a person makes, the way he communicates and interacts with other people in his group.
14. Sociometry is a test under which each member of a group is asked to choose from all other members those with whom she prefers to associate in a specific situation. The situation must be a real one to the group under study, e.g., 'group study', 'play', 'classroom seating', class monitor for students of a school.
15. Sociometry technique helps us to have an idea of the group at a glance. This enables us to form appropriate groups of students for carrying out various activities and projects. Such tests at different times enable us to find out the changes taking place in the group structure.
16. A 'sample' is a small percentage of the larger group who are selected for research. A sample can be statistically explained as being a subset of a population. The sample will be able to give an idea of the characteristics of the larger group from where it has been drawn. It is possible to make deductions about the larger population on the basis of the sample.

    The term 'sampling' refers to the technique whereby a smaller group is selected from a larger one so that the more manageable smaller group can be observed and those observations can be applied to the larger group as well. This is only possible when the sample group shares the same characteristics as the larger group.
17. Sampling design refers to a definite plan for obtaining a sample from the sampling frame. It refers to the technique or procedure, which a researcher adopts in selecting some sampling units from where inferences about population are drawn. Sampling data is obtained before collecting the final data.

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18. The sampling frame should be selected in such a way that it consists of almost all the sampling units. A sample should be selected in such a way that it has all the characteristics of the population. Some of the popular sampling frames are census reports and electoral registers.
19. Sampling gives rise to certain errors known as 'sampling errors' or sampling fluctuations. Sampling errors are those errors, which arise on account of sampling and generally happen to be random variations in the sample estimates of the actual population values. These errors arise from any bias in selection, estimation, etc. Typically, a sampling error occurs when a sample survey requires study in small portions of population as there can be certain amount of inaccuracy in the information collected during sampling analysis. This inaccuracy is called sampling error or error variance.
20. The reliability of samples can be tested in the following ways:
    - More samples of the same size should be taken from the same universe and their results be compared. If results are similar, the sample will be reliable.
    - If the measurements of the universe are known then they should be compared with the measurements of the sample. In case of similarity of measurement, the sample will be reliable.
    - Sub-samples should be taken from the samples and studied. If the results of sample and sub-sample study show similarity, the sample should be considered reliable.
21. A 'report' is a written document which presents information in a specialized and concise manner. Research report is a written document, which describes the findings of some individual or a group of individuals.
22. A few characteristics of a good report include: (i) language and style of the report, (ii) structure of the report, (iii) presentation of the report, and (iv) references in the report.
23. Footnotes are meant for cross-references. They are placed at the bottom of the page, separated from the textual material by a space of half an inch as a line that is around one-and-a-half inches long. Footnotes are always typed in a single space, though they are divided from one another by double space.
24. The four types of informational report are: (i) inspection report, (ii) inventory report, (iii) assessment report, and (iv) performance report.
25. A written report can be written in various formats, that include: (i) straight-line format, (ii) building blocks format, and (iii) inverted pyramid format.
26. The principles of writing a report are: (i) Principle of purpose, (ii) Principle of organization, (iii) Principle of brevity, (iv) Principle of clarity, (v) Principle of scheduling, and (vi) Principle of cost.

4.14 QUESTIONS AND EXERCISES

Short-Answer Questions

1. What are the characteristics of 'observation'?
2. What are the major problems arising with in connection with observation technique?
3. List the steps involved in preparing and administering a questionnaire.
4. Name five steps which can be taken to improve a questionnaire.
5. What are the steps involved in preparing for a research interview?
6. List the various methods of sampling.
7. What criteria are required for selecting a sample?
8. What is sampling design?
9. List the steps required in sampling process.
10. What is representative sample?
11. What are sampling errors? What are its different types?
12. Mention the steps involved in the report writing process.
13. Write a note on oral presentation.
14. What do you mean by informational report?
15. Discuss the precautions that need to be taken while writing a research report.

Long-Answer Questions

1. Discuss the various types of observation.
2. Elaborate on the steps taken to ensure that observation is effective.
3. Discuss the types of commonly used questionnaires.
4. What are the different types of questions? Give examples to support your answer.
5. Describe the various types of interviews.
6. Explain the various methods of sampling. Discuss the criteria for selecting each sample type with the help of examples.
7. Discuss the need, characteristics features and steps involved in sampling.
8. Elaborate in brief the various types of sampling distributions.
9. What are sampling errors? How they occur and what impact they have on research process?
10. 'Apart from reducing errors of bias, the simplest way of increasing the accuracy of a sample is to increase its size.' Justify the statement with the help of examples.
11. Explain the significance of a research report.
12. Explain the layout of a research report.
13. Give a detailed account on the mechanics of report writing.
14. Discuss the guidelines that must be followed while preparing a written report.

4.15 FURTHER READING

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UNIT 5 MEASURES OF CENTRAL TENDENCY AND VARIABILITY

Structure

5.0 INTRODUCTION

In this unit, you will learn about the measures of central tendency and dispersion. There are several commonly used measures of central tendency, such as arithmetic mean, mode and median. These values are very useful not only in presenting the overall picture of the entire data but also for the purpose of making comparisons among two or more sets of data. In addition, you will learn about the geometric mean and harmonic mean. If α, β, γ are in GP, then β is called a geometric mean between α and γ written as GM. If a, b, c are in HP, then b is called a Harmonic Mean between a and c, written as HM. Moreover, you will also learn about the measures of dispersion. A measure of dispersion or simply dispersion may be defined as statistics signifying the extent of the scatteredness of items around a measure of central tendency. Finally, you will learn about the coefficient of variation.

5.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Explain the significance of various measures of central tendency
- Explain how to measure central tendency

- Discuss the various types of measures of dispersion
- Understand about the coefficient of variation

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5.2 MEASURES OF CENTRAL TENDENCY

There are several commonly used measures of central tendency, such as arithmetic mean, mode and median. These values are very useful not only in presenting the overall picture of the entire data but also for the purpose of making comparisons among two or more sets of data.

As an example, questions like 'How hot is the month of June in Delhi?' can be answered, generally by a single figure of the average for that month. Similarly, suppose we want to find out if boys and girls at age 10 years differ in height for the purpose of making comparisons. Then, by taking the average height of boys of that age and average height of girls of the same age, we can compare and record the differences.

While arithmetic mean is the most commonly used measure of central location, mode and median are more suitable measures under certain set of conditions and for certain types of data. However, each measure of central tendency should meet the following requisites.

1. It should be easy to calculate and understand.
2. It should be rigidly defined. It should have only one interpretation so that the personal prejudice or bias of the investigator does not affect its usefulness.
3. It should be representative of the data. If it is calculated from a sample, then the sample should be random enough to be accurately representing the population.
4. It should have sampling stability. It should not be affected by sampling fluctuations. This means that if we pick 10 different groups of college students at random and compute the average of each group, then we should expect to get approximately the same value from each of these groups.
5. It should not be affected much by extreme values. If few very small or very large items are present in the data, they will unduly influence the value of the average by shifting it to one side or other, so that the average would not be really typical of the entire series. Hence, the average chosen should be such that it is not unduly affected by such extreme values.

Meaning of the Measures of Central Tendency

If the progress scores of the students of a class are taken and they are arranged in a frequency distribution, we may sometime find that there are very few students who either score very high or very low. The marks of most of the student will lie somewhere between the highest and the lowest scores of the whole class. This tendency of a group about distribution is named as central tendency and typical score that lies in between the extremes and shared by most of the students is referred to as a measure of central tendency. Tate in 1955 defines the measures of central tendency as, *"A sort of average or typical value of the items in the series and its function is to summarize the series in terms of this average value"*.

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The most common measures of central tendency are:

1. Arithmetic Mean or Mean
2. Median
3. Mode

Let us consider the three measures of central tendency.

(a) Arithmetic Mean: This is also commonly known as simply the mean. Even though average, in general, means any measure of central location, when we use the word average in our daily routine, we always mean the arithmetic average. The term is widely used by almost every one in daily communication. We speak of an individual being an average student or of average intelligence. We always talk about average family size or average family income or Grade Point Average (GPA) for students, and so on.

Calculating Arithmetic Mean (*M*): The simplest but most useful measure of central tendency is the arithmetic mean. It can be defined as the sum of all the values of the items in a series divided by the number of items. It is represented by the symbol *M*.

Calculation of Mean in the Case of Ungrouped Data

Let $X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9$ and X_{10} be the scores obtained by 10 students on an achievement test. Then the arithmetic mean or mean score of the group of these ten students can be calculated as:

$$M = X_1 + X_2 + X_3 + X_4 + X_5 + \ldots + X_{10}/10$$

The formula for calculating the mean of an ungrouped data is as follows:

$$M = \sum X/N$$

Where, $\sum X$ stands for the sum of sores or values of the items and N for the total number in a series or group.

Calculation of Mean in the Case of Grouped Data (Data in the form of Frequency Distribution)

General Method: In a frequency distribution where all the frequencies are greater than one, the mean is calculated by the formula:

$$M = \sum f X / N$$

Where, X represents the mid-point of the class interval, f its respective frequency and N the total of all frequencies.

Short-Cut Method: Mean for the grouped data can be computed easily with the help of following formula:

$$M = A + \sum f x' / N \times i$$

Where,

A = Assumed mean

i = Class interval

f = Respective frequency of the mid-values of the class intervals

N = Total Frequency

x' = $X - A / i$

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Combined Mean: If the arithmetic averages and the number of items in two or more related groups are known, the combined (or composite) mean of the entire group can be obtained by the following formula:

$$\overline{\overline{X}} = \left[\frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}\right]$$

The advantage of combined arithmetic mean is that, one can determine the over, all mean of the combined data without having to going back to the original data.

An Example:

Find the combined mean for the data given below

$n_1 = 10, x_1 = 2, n_2 = 15, x_2 = 3$

Solution:

$$\overline{\overline{X}} = \left[\frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}\right]$$

$$= \left[\frac{10\times 2 + 15\times 3}{10+15}\right]$$

$$= \frac{20+45}{25}$$

$$= 2.6$$

For discussion purposes, let us assume a variable X which stands for some scores, such as the ages of students. Let the ages of 5 students be 19, 20, 22, 22 and 17 years. Then variable X would represent these ages as follows:

X: 19, 20, 22, 22, 17

Placing the Greek symbol σ(Sigma) before $\overline{X}$ would indicate a command that all values of X are to be added together. Thus:

$\sigma\overline{X} = 19 + 20 + 22 + 22 + 17$

The mean is computed by adding all the data values and dividing it by the number of such values. The symbol used for sample average is $\overline{X}$ so that:

$$\overline{X} = \frac{19+20+22+22+17}{5}$$

In general, if there are n values in the sample, then

$$\overline{X} = \frac{X_1 + X_2 + \ldots\ldots + X_n}{n}$$

In other words,

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}, \qquad i = 1, 2 \ldots n.$$

The above formula states, add up all the values of X_i where the value of i starts at 1 and ends at n with unit increments so that $i = 1, 2, 3, \ldots n$.

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If instead of taking a sample, we take the entire population in our calculations of the mean, then the symbol for the mean of the population is μ (mu) and the size of the population is N, so that:

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}, \qquad i = 1, 2 \ldots N.$$

If we have the data in grouped discrete form with frequencies, then the sample mean is given by:

$$\overline{X} = \frac{\Sigma f(X)}{\Sigma f}$$

Where, Σf = Summation of all frequencies' n

$\Sigma f(X)$ = Summation of each value of X multiplied by its corresponding frequency (f).

Example 1: Let us take the ages of 10 students as follows:

19, 20, 22, 22, 17, 22, 20, 23, 17, 18

This data can be arranged in a frequency distribution as follows:

(X)	(f)	$f(X)$
17	2	34
18	1	18
19	1	19
20	2	40
22	3	66
23	1	23
	Total = 10	200

In the above case we have $\Sigma f = 10$ and $\Sigma f(X) = 200$, so that:

$$\overline{X} = \frac{\Sigma f(X)}{\Sigma f}$$

$$= 200/10 = 20$$

Characteristics of the Mean

The arithmetic mean has three interesting properties. These are:

1. The sum of the deviations of individual values of X from the mean will always add up to zero. This means that if we subtract all the individual values from their mean, then some values will be negative and some will be positive, but if all these differences are added together then the total sum will be zero. In other words, the positive deviations must balance the negative deviations. Or symbolically:

$$\sum_{i=1}^{n}(X_i - \overline{X}) = 0, i = 1, 2, \ldots n.$$

2. The second important characteristic of the mean is that it is very sensitive to extreme values. Since the computation of the mean is based upon inclusion of all values in the data, an extreme value in the data would shift the mean towards it, thus making the mean unrepresentative of the data.

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3. The third property of the mean is that the sum of squares of the deviations about the mean is minimum. This means that if we take differences between individual values and the mean and square these differences individually and then add these squared differences, then the final figure will be less than the sum of the squared deviations around any other number other than the mean. Symbolically, it means that:

$$\sum_{i=1}^{n}(X_i - \overline{X})^2 = \text{Minimum}, i = 1, 2, \ldots n.$$

Advantages of Mean

The following are the various advantages of mean:

1. Its concept is familiar to most people and is intuitively clear.
2. Every data set has a mean, which is unique and describes the entire data to some degree. For instance, when we say that the average salary of a professor is ₹ 25,000 per month, it gives us a reasonable idea about the salaries of professors.
3. It is a measure that can be easily calculated.
4. It includes all values of the data set in its calculation.
5. Its value varies very little from sample to sample taken from the same population.
6. It is useful for performing statistical procedures, such as computing and comparing the means of several data sets.

Disadvantages of Mean

The following are the various disadvantages of mean:

1. It is affected by extreme values, and hence, not very reliable when the data set has extreme values especially when these extreme values are on one side of the ordered data. Thus, a mean of such data is not truly a representative of such data. For instance, the average age of three persons of ages 4, 6 and 80 years gives us an average of 30.
2. It is tedious to compute for a large data set as every point in the data set is to be used in computations.
3. We are unable to compute the mean for a data set that has open-ended classes either at the high or at the low end of the scale.
4. The mean cannot be calculated for qualitative characteristics, such as beauty or intelligence, unless these can be converted into quantitative figures such as intelligence into IQs.

(b) Median: The median is a measure of central tendency and it appears in the centre of an ordered data. It divides the list of ordered values in the data into two equal parts so that half of the data will have values less than the median and half will have values greater than the median.

If the total number of values is odd, then we simply take the middle value as the median. For instance, if there are 5 numbers arranged in order, such as 2, 3, 3, 5, 7, then 3 is the middle number and this will be the median. However, if the total number of values in the data is even, then we take the average of the middle two values. For instance, let there be 6 numbers in the ordered data such as 2, 3, 3, 5, 7, 8, then the average of middle two numbers which are 3 and 5 would be the median, which is

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$$\text{Median} = \frac{(3+5)}{2} = 4$$

In general, the median is $\frac{n+1}{2}$ th observation in the ordered data.

The median is a useful measure in the sense that it is not unduly affected by extreme values and is specially useful in open ended frequencies.

Calculating Median (M_d): If the items of a series are arranged in ascending or descending order of magnitude, the measure or value of the central item in the series is termed as median. The median of a distribution can thus be said as the point on the score scale below which half (or 50 per cent) of the scores fall. Thus median is the score or the value of that central item which divides the series into two equal parts. Therefore, it should be understood that the central item itself is not the median. It is only the measure or value of the central item that is known as the median. For example, if we arrange in ascending or descending order the marks of 5 students, then the marks obtained y the third student from either side will be termed as median of the scores of the group of students under consideration.

Computation of Median for Ungrouped Data

The following two situations could arise:

1. **When N (No. of Items in a Series) is Odd:** In this case where N is odd (not divisible by 2), the median can be computed by the following formula:

   M_d = The measure or value of the $(N + 1)/2$ th item.

2. **When N (No. of Items in a Series) is Even:** In this case where N is even (divisible by 2), the median can be determined by the following formula:

   M_d = The value of the $(N/2)$ th item + The value of $[(N/2) + 1]$ th item/2

Calculation of Median for Grouped Data (In the Form of Frequency Distribution)

If the data is available in the form of a frequency distribution like below, then calculation of median first requires the location of median class.

Scores	*f*
65-69	1
60-64	3
55-59	4
50-54	7
45-49	9
40-44	11
35-39	8
30-34	4
25-29	2
20-24	1
	$N = 50$

NOTES

Actually, median is the measure or score of the central item. Therefore, it is needed to locate the central item. It may be done through the formulae given earlier in case of ungrouped data for the odd and even values of N (total frequencies). Here, in the present distribution, N (= 50) is even. Therefore, median will fall somewhere between the score of 25th and 26th items in the given distribution. In the given frequency distribution table, if we add frequencies either above or below we may see that the class interval designated as 40-44 is to be labeled as the class where the score representing median will fall.

After estimating the median class, the median of the distribution may be interpolated with the help of following formula:

$$M_d = L + [\,(N/2) - F\,/f] \times i$$

Where,

L = Exact lower limit of the median class

F = Total of all frequencies before in the median class

f = Frequency of the median class

i = Class interval

N = Total of all the frequency

By applying the above formula, we can compute the median of the given distribution in the following way:

$$M_d = 39.5 + (50/2) - 15 \,/\, 11 \times 5 = 39.5 + 10/11 \times 5$$
$$= 39.5 + 50/11 = 39.5 + 4.55 = 44.05$$

Advantages of Median

The following are the advantages of median:

1. Median is a positional average and hence the extreme values in the data set do not affect it as much as they do to the mean.
2. Median is easy to understand and can be calculated from any kind of data, even for grouped data with open-ended classes.
3. We can find the median even when our data set is qualitative and can be arranged in the ascending or the descending order, such as average beauty or average intelligence.
4. Similar to mean, median is also unique meaning that there is only one median in a given set of data.
5. Median can be located visually when the data is in the form of ordered data.
6. The sum of absolute differences of all values in the data set from the median value is minimum meaning that it is less than any other value of central tendency in the data set, which makes it more central in certain situations.

Disadvantages of Median

The following are the disadvantages of median:

1. The data must be arranged in order to find the median. This can be very time consuming for a large number of elements in the data set.
2. The value of the median is affected more by sampling variations. Different samples from the same population may give significantly different values of the median.

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3. The calculation of median in case of grouped data is based on the assumption that the values of observations are evenly spaced over the entire class interval and this is usually not so.
4. Median is comparatively less stable than the mean, particularly for small samples, due to fluctuations in sampling.
5. Median is not suitable for further mathematical treatment. For instance, we cannot compute the median of the combined group from the median values of different groups.

(c) **Mode:** The mode is another form of average and can be defined as the most frequently occurring value in the data. The mode is not affected by extreme values in the data and can easily be obtained from an ordered set of data. It can be useful and more representative of the data under certain conditions and is the only measure of central tendency that can be used for qualitative data. For instance, when a researcher quotes the opinion of an average person, he is probably referring to the most frequently expressed opinion which is the modal opinion. In our example of ages of 10 students as:

19, 20, 22, 22, 17, 22, 20, 23, 17 and 18

The mode is 22, since it occurs more often than any other value in this data.

Calculating Mode (M_0): Mode is defined as the size of a variable which occurs most frequently. It is the point on the score sale that corresponds to the maximum frequency of the distribution. In any series, it is the value of the item which is most characteristics or common and is usually repeated the maximum number of times.

Computation of Mode for Ungrouped Data

Mode can easily be computed merely by looking at the data. All that one has to do is to find out the score which is repeated maximum number of times.

For example, suppose we have to find out the value of mode from the following scores of students:

25, 29, 24, 25, 27, 25, 28, 25, 29

Here the score 25 is repeated maximum number of times and thus, value of the mode in this case is 25.

Computation of Mode for Grouped Data

When data is available in the form of frequency distribution, the mode is computed from the following formula:

$$\text{Mode } (M_0) = 3\,M_d - 2M$$

Where, M_d is the median and M is the mean of the given distribution. The mean as well as the median of the distribution are first computed and then, with the help of the above formula, mode is computed.

Another Method for Grouped Data

Mode can be computed directly from the frequency distribution table without calculating mean and median. For this purpose, we can use the following formula:

$$M_0 = L + f_1 / (f_1 + f_{-1}) \times i$$

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Where,

L = Lower limit of the model class (the class in which mode maybe supposed to lie)

i = Class interval

f_1 = Frequency of the class adjacent to the modal class for which lower limit is greater than that for the modal class.

f_{-1} = Frequency of the class adjacent to the modal class for which the lower limit is less than that for the modal class.

Advantages of Mode

The following are the advantages of mode:

1. Similar to median, the mode is not affected by extreme values in the data.
2. Its value can be obtained in open-ended distributions without ascertaining the class limits.
3. It can be easily used to describe qualitative phenomenon. For instance, if most people prefer a certain brand of tea then this will become the modal point.
4. Mode is easy to calculate and understand. In some cases it can be located simply by observation or inspection.

Disadvantages of Mode

The following are the disadvantages of mode:

1. Quite often, there is no modal value.
2. It can be bi-modal or multi-modal or it can have all modal values making its significance more difficult to measure.
3. If there is more than one modal value, the data is difficult to interpret.
4. A mode is not suitable for algebraic manipulations.
5. Since the mode is the value of maximum frequency in the data set, it cannot be rigidly defined if such frequency occurs at the beginning or at the end of the distribution.
6. It does not include all observations in the data set, and hence, less reliable in most of the situations.

5.2.1 Weighted Arithmetic Mean

In the computation of arithmetic mean we had given equal importance to each observation in the series. This equal importance may be misleading if the individual values constituting the series have different importance as in the following example:

The Raja Toy shop sells

Toy Cars at	₹ 3 each
Toy Locomotives at	₹ 5 each
Toy Aeroplanes at	₹ 7 each
Toy Double Decker at	₹ 9 each

What shall be the average price of the toys sold, if the shop sells 4 toys, one of each kind?

Mean Price, i.e., $\bar{x} = \frac{\sum x}{4} = \text{Rs}\frac{24}{4} = ₹ 6$

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In this case the importance of each observation (Price quotation) is equal in as much as one toy of each variety has been sold. In the above computation of the arithmetic mean this fact has been taken care of by including 'once only' the price of each toy.

But if the shop sells 100 toys: 50 cars, 25 locomotives, 15 aeroplanes and 10 double deckers, the importance of the four price quotations to the dealer is **not equal** as a source of earning revenue. In fact their respective importance is equal to the number of units of each toy sold, i.e.,

The importance of Toy Car	50
The importance of Locomotive	25
The importance of Aeroplane	15
The importance of Double Decker	10

It may be noted that 50, 25, 15, 10 are the quantities of the various classes of toys sold. It is for these quantities that the term 'weights' is used in statistical language. Weight is represented by symbol 'w', and Σw represents the sum of weights.

While determining the 'average price of toy sold' these weights are of great importance and are taken into account in the manner illustrated below:

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3 + w_4x_4}{w_1 + w_2 + w_3 + w_4} = \frac{\sum wx}{\sum w}$$

When w_1, w_2, w_3, w_4 are the respective weights of x_1, x_2, x_3, x_4 which in turn represent the price of four varieties of toys, viz., car, locomotive, aeroplane and double decker, respectively.

$$\bar{x} = \frac{(50\times3)+(25\times5)+(15\times7)+(10\times9)}{50+25+15+10}$$

$$= \frac{(150)+(125)+(105)+(90)}{100} = \frac{470}{100} = ₹ 4.70$$

The table below summarizes the steps taken in the computation of the weighted arithmetic mean.

$$\Sigma w = 100; \quad \Sigma wx = 470$$

$$\bar{x} = \frac{\sum wx}{\sum w} = \frac{470}{100} = 4.70$$

The weighted arithmetic mean is particularly useful where we have to compute the *mean of means*. If we are given two arithmetic means, one for each of two different series, in respect of the *same variable*, and are required to find the arithmetic mean of the combined series, the weighted arithmetic mean is the only suitable method of its determination.

Weighted Arithmetic Mean of Toys Sold by the Raja Toy Shop

Toys	*Price Per Toy* ₹x	*Number Sold* w	*Price × Weight* xw
Car	3	50	150
Locomotive	5	25	125
Aeroplane	7	15	105
Double Decker	9	10	90
		$\Sigma w = 100$	$\Sigma xw = 470$

Example 2: The arithmetic mean of daily wages of two manufacturing concerns A Ltd. and B Ltd. is ₹ 5 and ₹ 7, respectively. Determine the average daily wages of

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both concerns if the number of workers employed were 2,000 and 4,000, respectively.

Solution: (*a*) Multiply each average (viz., 5 and 7) by the number of workers in the concern it represents.

(*b*) Add up the two products obtained in (*a*) above

(*c*) Divide the total obtained in (*b*) by the total number of workers.

Weighted Mean of Mean Wages of A Ltd. and B Ltd.

Manufacturing Concern	*Mean Wages* x	*Workers Employed* w	*Mean Wages × Workers Employed* wx
A Ltd.	5	2,000	10,000
B Ltd.	7	4,000	28,000
		$\Sigma w = 6{,}000$	$\Sigma wx = 38{,}000$

$$\bar{x} = \frac{\Sigma wx}{\Sigma w}$$

$$= \frac{38{,}000}{6{,}000}$$

$$= ₹\ 6.33$$

The above mentioned examples explain that 'Arithmetic Means and Percentage' are not original data. They are derived figures and their importance is relative to the original data from which they are obtained. This relative importance must be taken into account by weighting while averaging them (means and percentage).

5.2.2 Different Positional Numbers

The position of value in statistics is determined using specific methods for a given set of data or observations. The following are the popular common measures of positions:

- **Percentiles:** Percentiles are those values which divide a given data set into hundred equal parts. It is the value of a variable below which certain per cent of observations fall. For instance, the 25th percentile is the value below which 25 per cent of the observations occur. The 25th percentile is also referred as the first quartile, the 50th percentile as the median or second quartile and the 75th percentile as the third quartile.
- **Quartiles:** It segments the data in four regions and is commonly used to measure the position of value in statistics. It is a number and not a range of values.
- **Standard Scores:** It is also termed as Z-values, Z-scores, normal scores and standardized variables. It is a dimensionless quantity and can be calculated using the following formula:

$$Z = (X - \mu)/\sigma$$

Measures of Position Values

We have defined the median as the value of the item which is located at the centre of the array, we can define other measures which are located at other specidied points. Thus, the *N*th *percentile* of an array is the value of the item such that *N* per cent items lie *below* it. Clearly then the *N*th percentile P_n of grouped data is given by,

$$P_n = l + \frac{\frac{nN}{100} - C}{f} \times i$$

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where, l is the lower limit of the class in which $nN/100$th item lies, i its width, f its frequency, C the cumulative frequency upto (but not including) this class, and N is the total number of items.

We similarly define the *N*th *decile* as the value of the item below which $(nN/10)$ items of the array lie. Clearly,

$$D_n = P_{10n} = l + \frac{\frac{nN}{10} - C}{f} \times i \quad (1)$$

The other most commonly referred to measures of location are the quartiles. Thus, *n*th quartile is the value of the item which lie at the $n(N/4)$th item. Clearly Q_2, the second quartile is the median. For grouped data,

$$Q_n = P_{25n} = l + \frac{\frac{nN}{4} - C}{f} \times i \quad (2)$$

Some measures other than measures of central tendency are often employed when summarizing or describing a set of data where it is necessary to divide the data into equal parts. These are positional measures and are called quantiles and consist of quartiles, deciles and percentiles. The quartiles divide the data into four equal parts. The deciles divide the total ordered data into ten equal parts and percentiles divide the data into 100 equal parts. Consequently, there are three quartiles, nine deciles and 99 percentiles. The quartiles are denoted by the symbol Q so that Q_1 will be such point in the ordered data which has 25 per cent of the data below and 75 per cent of the data above it. In other words, Q_1 is the value corresponding to $\left(\frac{n+1}{4}\right)$ th ordered observation. Similarly, Q_2 divides the data in the middle, and is also equal to the median and its value Q_2 is given by:

Q_2 = The value of $2\left(\frac{n+1}{4}\right)$ th ordered observation in the data.

Similarly, we can calculate the values of various deciles. For instance,

$D_1 = \left(\frac{n+1}{10}\right)$ th observaton in the data

$D_7 = 7\left(\frac{n+1}{10}\right)$ th observation in the ordered data.

Percentiles are generally used in the research area of education where people are given standard tests and it is desirable to compare the relative position of the subject's performance on the test. Percentiles are similarly calculated as:

$P_7 = 7\left(\frac{n+1}{100}\right)$ th observation in the ordered data.

$P_{69} = 69\left(\frac{n+1}{100}\right)$ th observation in the ordered data.

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Quartiles

The formula for calculating the values of quartiles for grouped data is given as follows.

$$Q = L + (j/f)C$$

Where,

Q = The quartile under consideration.

L = Lower limit of the class interval which contains the value of Q.

j = The number of units we lack from the class interval which contains the value of Q, in reaching the value of Q.

f = Frequency of the class interval containing Q.

C = Size of the class interval.

Let us assume we took the data of the ages of 100 students and a frequency distribution for this data has been constructed as shown.

The frequency distribution is as follows:

Ages (CI)	*Mid-point (X)*	*(f)*	*f(X)*	$f(X)^2$
16 and upto 17	16.5	4	66	1089.0
17 and upto 18	17.5	14	245	4287.5
18 and upto 19	18.5	18	333	6160.5
19 and upto 20	19.5	28	546	10647.0
20 and upto 21	20.5	20	410	8405.0
21 and upto 22	21.5	12	258	5547.0
22 and upto 23	22.5	4	90	2025.0
		Totals = 100	1948	38161

In our case, in order to find Q_1, where Q_1 is the cut-off point so that 25 per cent of the data is below this point and 75 per cent of the data is above, we see that the first group has 4 students and the second group has 14 students making a total of 18 students. Since Q_1 cuts off at 25 students, it is the third class interval which contains Q_1. This means that the value of L in our formula is 18.

Since we already have 18 students in the first two groups, we need 7 more students from the third group to make it a total of 25 students, which is the value of Q_1. Hence, the value of (j) is 7. Also, since the frequency of this third class interval which contains Q_1 is 18, the value of (f) in our formula is 18. The size of the class interval C is given as 1. Substituting these values in the formula for Q, we get

$$Q_1 = 18 + (7/18)1$$
$$= 18 + 0.38 = 18.38$$

This means that 25 per cent of the students are below 18.38 years of age and 75 per cent are above this age.

Similarly, we can calculate the value of Q_2, using the same formula. Hence,

$$Q_2 = L + (j/f)C$$
$$= 19 + (14/28)1$$
$$= 19.5$$

This also happens to be the median.

By using the same formula and the same logic we can calculate the values of all deciles as well as percentiles.

We have defined the median as the value of the item which is located at the centre of the array. We can define other measures which are located at other specified

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points. Thus, the Nth percentile of an array is the value of the item such that N per cent items lie below it. Clearly then, the Nth percentile Pn of grouped data is given by

$$P_n = l + \frac{\frac{nN}{100} - C}{f} \times i$$

Here, l is the lower limit of the class in which $nN/100$th item lies, i its width, f its frequency, C the cumulative frequency upto (but not including) this class, and N is the total number of items.

We can similarly define the Nth decile as the value of the item below which $(nN/10)$ items of the array lie. Clearly,

$$D_n = P_{10n} = l + \frac{\frac{nN}{10} - C}{f} \times i$$

where the symbols have the obvious meanings.

The other most commonly referred to measures of location are the quartiles. Thus, nth quartile is the value of the item which lies at the $n(N/5)$th item. Clearly, Q_2, the second quartile is the median for grouped data.

$$Q_n = P_{25n} = l + \frac{\frac{nN}{4} - C}{f} \times i$$

5.2.3 Geometric Mean

If α, β, γ are in GP, then β is called a *geometric mean* between α and γ, written as GM.

If $a_1, a_2, \ldots, a_n$ are in GP, then $a_2, \ldots, a_{n-1}$ are called *geometric means* between a_1 and a_n.

Thus, 3, 9, 27 are three geometric means between 1 and 81.

Non-zero quantities $a_1, a_2, a_3, \ldots, a_n, \ldots$, each term of which is equal to the product of preceding term and a constant number, form a *Geometrical Progression* (written as G.P.).

Thus, all the following quantities are in G.P.

(a) 1, 2, 4, 8, 16,...

(b) $3, -1, \frac{1}{3}, \frac{-1}{9}, \frac{1}{27}, \ldots$

(c) $1, \sqrt{2}, 2, 2\sqrt{2}, \ldots$

(d) $a, \frac{a}{b}, \frac{a}{b^2}, \frac{a}{b^3}, \ldots$, where $a \neq 0, b \neq 0$.

(e) $1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \ldots$

The constant number is termed as the *common ratio* of the G.P.

The *n*th Term of a G.P.

Let first term be a and r, the common ratio, By definition the G.P. is $a, ar, ar^2, \ldots$

$$1\text{st term} = a = ar^0 = ar^{1-1}$$
$$2\text{nd term} = ar = ar^1 = ar^{2-1}$$
$$\ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots\ \ldots$$

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In general, nth term = ar^{n-1}.

In examples of the preceding section, we compute 5th, 7th, 3rd, 11th and 8th term of (a), (b), (c), (d) and (e) respectively.

In (a) Ist term is 1 and common ratio = 2.

Hence, 5th term = $ar^4 = 1.2^4 = 16$.

In (b) $a = 3$, $r = \frac{-1}{3}$, hence, 7th term = $ar^6 = 3\left(\frac{-1}{3}\right)^6 = \frac{1}{243}$.

In (c) $a = 1$, $r = \sqrt{2}$, hence, 3rd term = $ar^2 = 2$.

In (d) Ist term = a, $r = \frac{1}{b}$, hence, 11th term = $ar^{10} = \frac{a}{b^{10}}$.

In (e) $a = 1$, $r = \frac{1}{5}$, hence, 8th term = $ar^2 = \frac{1}{5^7} = \frac{1}{78125}$.

Sum of First *n* Terms of a G.P.

Let $a, ar, ar^2, \ldots$ be a given G.P. and let S_n be the sum of its first n terms.

Then, $S_n = a + ar + ar^2 + \ldots + ar^{n-1}$.

This gives that $rS_n = ar + ar^2 + \ldots + ar^{n-1} + ar^n$

Subtracting, we get, $S_n - rS_n = a - ar^n = a(1 - r^n)$

In case $r \neq 1$, $S_n = \frac{a(1-r^n)}{(1-r)}$

In case $r = 1$, $S_n = a + a + a + \ldots + a$ (n times)
$= na$.

Thus, sum of n terms of a G.P. is $\frac{a(1-r^n)}{1-r}$ provided $r \neq 1$.

In case $r = 1$, sum of G.P. is na.

Example 3: Find the sum of the first 14 terms of a G.P.
3, 9, 27, 81, 243, 729,...

Solution: In this case $a = 3$, $r = 3$, $n = 14$.

So, $S_n = \frac{a(1-r^n)}{1-r} = \frac{3(1-3^{14})}{1-3}$

$= \frac{3}{2}(3^{14} - 1)$.

Example 4: Find the sum of first 11 terms of a G.P. given by

$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8} \ldots, \ldots$

Solution: Here, $a = 1$, $r = -\frac{1}{2}$, $n = 11$.

So, $S_n = \frac{a(1-r^n)}{1-r} = \frac{1\left[1-\left(-\frac{1}{2}\right)^{11}\right]}{1+\frac{1}{2}}$

$= \frac{2^{11}+1}{3\times 2^{10}} = \frac{683}{1024}$.

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To Insert *n* Geometric Means between Two given Numbers *a* and *b*

Let $G_1, G_3, \ldots G_n$ be n geometric means between a and b. Thus, $a, G_1, G_2, \ldots G_n, b$ is a GP, b being $(n+2)$th term = ar^{n+1}, where r is the common ratio of GP

Thus, $b = ar^{n+1} \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

So, $G_1 = ar = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}} = (a^n b)^{\frac{1}{n+1}}$

$G_2 = ar^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}} = (a^{n-1}b^2)^{\frac{1}{n+1}}$

... .

$G_n = ar^{n-1} = a\left(\frac{b}{a}\right)^{\frac{n-1}{n+1}} = (a^2 b^{n-1})^{\frac{1}{n+1}}$

Example 5: Find 7 GM's between 1 and 256.

Solution: Let $G_1, G_2, \ldots G_7$ be 7 GM's between 1 and 256

Then, 256= 9th term of GP,

$= 1.\ r^8$, where r is the common ratio of the GP

This gives that, $r^8 = 256 \Rightarrow r = 2$

Thus, $G_1 = ar = 1.2 = 2$

$G_2 = ar^2 = 1.4 = 4$

$G_3 = ar^3 = 1.8 = 8$

$G_4 = ar^4 = 1.16 = 16$

$G_5 = ar^5 = 1.32 = 32$

$G_6 = ar^6 = 1.64 = 64$

$G_7 = ar^7 = 1.128 = 128$

Hence, required GM's are 2, 4, 8, 16, 32, 64, 128.

Example 6: Sum the series $1 + 3x + 5x^2 + 7x^3 + \ldots$ up to n terms, $x \neq 1$.

Solution: Note that nth term of this series = $(2n-1)\,x^{n-1}$

Let $S_n = 1 + 3x + 5x^2 + \ldots + (2n-1)\,x^{n-1}$

Then, $xS_n = x + 3x^2 + \ldots + (2n-3)\,x^{n-1} + (2n-1)\,x^n$

Subtracing, we get

$S_n(1-x) = 1 + 2x + 2x^2 + \ldots + 2x^{n-1} - (2n-1)\,x^n$

$= 1 + 2x\left(\frac{1-x^{n-1}}{1-x}\right) - (2n-1)\,x^n$

$= \frac{1 - x + 2x - 2x^n - (2n-1)\,x^n(1-x)}{1-x}$

$= \frac{1 + x - 2x^n - (2n-1)\,x^n + (2n-1)\,x^{n+1}}{1-x}$

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$$= \frac{1 + x - (2n+1)\,x^n + (2n-1)\,x^{n+1}}{1-x}$$

Hence, $$S = \frac{1 + x - (2n+1)\,x^n + (2n-1)\,x^{n+1}}{(1-x)^2}$$

Example 7: If in a GP $(p+q)$th term $= m$ and $(p-q)$th term $= n$, then find its pth and qth terms.

Solution: Suppose that the given GP be $a, ar, ar^2, ar^3, \ldots$

By hypothesis, $(p+q)$th term $= m = ar^{p+q-1}$

$(p-q)$th term $= n = ar^{p-q-1}$

Then, $$\frac{m}{n} = r^{2q} \Rightarrow r = \left(\frac{m}{n}\right)^{1/2q}$$

Hence, $$m = a\left(\frac{m}{n}\right)^{(p+q-1)/2q} \Rightarrow a = m^{(q-p+1)/2q}\, n^{(p+q-1)/2q}$$

Thus, $$p\text{th term} = ar^{p-1} = m^{1/2}\, n^{1/2} = \sqrt{mn}$$

$$q\text{th term} = ar^{q-1} = m^{\frac{2q-p}{2q}}\, n^{\frac{p}{2q}}$$

Example 8: Sum the series $5 + 55 + 555 + \ldots$ up to n terms.

Solution: Let $S_n = 5 + 55 + 555 + \ldots.$

$$S_n = 5\,(1 + 11 + 111 + \ldots.)$$

$$= \frac{5}{9}\,(9 + 99 + 999 + \ldots)$$

$$= \frac{5}{9}\,[(10-1) + (100-1) + (1000-1) + \ldots]$$

$$= \frac{5}{9}\,[(10 + 10^2 + 10^3 + \ldots + 10^n) - (1 + 1 + \ldots. n \text{ terms})]$$

$$= \frac{5}{9}\,[(10 + 10^2 + 10^3 + \ldots + 10^n) - n]$$

$$= \frac{5}{9}\left[\frac{10(1-10^n)}{1-10} - n\right]$$

$$= \frac{5}{9}\left[\frac{10(10^n-1)}{9} - n\right]$$

$$= \frac{50}{81}(10^n - 1) - \frac{5n}{9}$$

Example 9: If a, b, c, d are in GP, prove that $a^2 - b^2$, $b^2 - c^2$ and $c^2 - d^2$ are also in GP.

Solution: Since, $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = k$ (say)

we have, $b = ak,\ c = bk,\ d = ck$

i.e., $b = ak,\ c = ak^2,\ d = ak^3$

Now, $(b^2 - c^2)^2 = (a^2k^2 - a^2k^4)^2$

$= a^4k^4(1-k^2)^2$

Also, $(a^2 - b^2)\,(c^2 - d^2) = (a^2 - a^2k^2)\,(a^2k^4 - a^2k^6)$

$= a^4(1-k^2)\,(k^4 - k^6)$

$= a^4k^4\,(1-k^2)^2$

Hence, $(b^2 - c^2) = (a^2 - b^2)\,(c^2 - d^2)$

This gives that, $a^2 - b^2$, $b^2 - c^2$, $c^2 - d^2$ are in GP.

Example 10: Three numbers are in GP. Their product is 64 and sum is $\frac{124}{5}$. Find them.

Solution: Let the numbers be $\frac{a}{r}, a, ar$

Since, $\frac{a}{r} + a + ar = \frac{124}{5}$ and $\frac{a}{r} \times a \times ar = 64$,

we have, $a^3 = 64 \Rightarrow a = 4$

This gives, $\frac{4}{r} + 4 + 4r = \frac{124}{5}$

$\Rightarrow \frac{1}{r} + 1 + r = \frac{31}{5}$

$\Rightarrow \frac{r^2+1}{r} = \frac{26}{5}$

$\Rightarrow 5r^2 + 5 = 26r$

$\Rightarrow 5r^2 - 26r + 5 = 0$

$\Rightarrow 5r^2 - 25r - r + 5 = 0$

$\Rightarrow 5r\,(r-5) - 1\,(r-5) = 0$

$\Rightarrow (r-5)\,(5r-1) = 0$

$\Rightarrow r = \frac{1}{5}$ or 5

In either case, numbers are $\frac{4}{5}$, 4 and 20.

Example 11: If a, b, c are in GP and $a^x = b^y = c^z$, prove that

$$\frac{1}{x} + \frac{1}{z} = \frac{2}{y}$$

Solution: a, b, c are in GP, $b^2 = ac$

But, $b^y = a^x \Rightarrow a = b^{y/x}$

and, $b^y = c^z \Rightarrow c = b^{y/z}$

So, we get $b^2 = b^{y/x} \cdot b^{y/z}$

$= b^{y\left(\frac{1}{x} + \frac{1}{z}\right)}$

$\Rightarrow 2 = y\left(\frac{1}{x} + \frac{1}{z}\right)$

$\Rightarrow \frac{1}{x} + \frac{1}{z} = \frac{2}{y}$

Example 12: Sum to n terms the series

$0.7 + 0.77 + 0.777 + \ldots$

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Solution: Given series,

$$= 0.7 + 0.77 + 0.777 + \ldots \text{ up to } n \text{ terms}$$
$$= 7\,(0.1 + 0.11 + 0.111 + \ldots \text{ up to } n \text{ terms})$$
$$= \frac{7}{9}\,(0.9 + 0.99 + 0.999 + \ldots \text{ up to } n \text{ terms})$$
$$= \frac{7}{9}\left[\left(1-\frac{1}{10}\right)+\left(1-\frac{1}{10^2}\right)+\left(1-\frac{1}{10^3}\right)+\ldots\right]$$
$$= \frac{7}{9}\left[n-\frac{1}{10}-\frac{1}{10^2}-\ldots \text{ up to } n \text{ terms}\right]$$
$$= \frac{7}{9}\left[n-\frac{\frac{1}{10}(1-1/10^n)}{1-\frac{1}{10}}\right]$$
$$= \frac{7}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$$
$$= \frac{7}{9}\left[n-\frac{1}{9}\left(1-\frac{1}{10^n}\right)\right]$$

Example 13: The sum of three numbers in GP is 35 and their product is 1000. Find the numbers.

Solution: Let the numbers be $\frac{\alpha}{r}$, α, αr

The product of $\frac{\alpha}{r}\times\alpha\times\alpha r = 1000$

$$\alpha^3 = 1000$$
$$\Rightarrow \quad \alpha = 10$$

So, the numbers are $\frac{10}{r}$, 10, $10r$

The sum of these numbers = 35

$$\Rightarrow \quad \frac{10}{r} + 10 + 10r = 35$$
$$\Rightarrow \quad \frac{2}{r} + 2r = 5$$
$$\Rightarrow \quad 2r^2 - 5r + 2 = 0$$
$$\Rightarrow \quad (2r-1)\,(r-2) = 0$$
$$\Rightarrow \quad r = 2 \text{ or } \frac{1}{2}$$

$r = 2$ gives the numbers as 5, 10, 20

$r = \frac{1}{2}$, gives the numbers as 20, 10, 5, the same as the first set.

Hence, the required numbers are 5, 10 and 20.

Example 14: The sum of the first eight terms of a GP (of real terms) is five times the sum of the first four terms. Find the common ratio.

Solution: Let the GP be $a, ar, ar^2, \ldots$

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$$S_8 = \text{Sum of first eight terms} = \frac{a\,(1-r^8)}{1-r}$$

$$S_4 = \text{Sum of first four terms} = \frac{a\,(1-r^4)}{1-r}$$

By hypothesis, $S_8 = 5S_4 \Rightarrow \frac{a\,(1-r^8)}{1-r} = \frac{5a(1-r^4)}{1-r}$

$$\Rightarrow \quad 1 - r^8 = 5(1-r^4)$$
$$\Rightarrow \quad (1-r^4)\,(1+r^4) = 5(1-r^4)$$

In case, $r^4 - 1 = 0$ we get, $r^2 - 1 = 0 \Rightarrow r = \pm 1$

(Note that $r^2 + 1 = 0 \Rightarrow r$ is imaginary)

Now, $r = 1 \Rightarrow$ The given series is $a + a + a + \ldots$

but, $S_8 = 8a$ and $S_4 = 4a$

So, $S_8 \neq 5S_4$

In case $r = -1$, we get, $S_8 = 0$ and $S_4 = 0$, hence the hypothesis is satisfied.

Suppose now, $r^4 - 1 \neq 0$, then $1 + r^4 = 5$

$$\Rightarrow \quad r^4 = 4 \Rightarrow r^2 = 2 \quad (r^2 \neq -2)$$
$$\Rightarrow \quad r = \pm\sqrt{2}$$

Hence, $r = -1$ or $\pm\sqrt{2}$

Example 15: If S is the sum, P the product of n term of G.P. and R the sum of reciprocals of n terms in GP, then prove that

$$P^2R^n = S^n.$$

Solution: Let $a, ar, ar^2, \ldots$ be the given GP

Then, $S = a + ar + ar^2 + \ldots$ up to n terms

$$= \frac{a(1-r^n)}{1-r} \qquad \ldots(1)$$

$$P = a \cdot ar \cdot ar^2 \ldots\ ar^{n-1}$$
$$= a^n\, r^{1+2+3+\ldots+(n-1)}$$
$$= a^n\, r^{\frac{(n-1)}{2}(2+n-2)}$$
$$= a^n r^{\left(\frac{n-1}{2}\right)n} \qquad \ldots(2)$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots \quad \text{up to } n \text{ terms}$$

$$= \frac{\frac{1}{a}\left(1-\frac{1}{r^n}\right)}{1-\frac{1}{r}} = \frac{r}{a}\,\frac{(r^n-1)}{(r-1)\,r^n}$$

$$= \frac{(1-r^n)}{a\,(1-r)\,r^{n-1}} \qquad \ldots(3)$$

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By Equations (2) and (3),

$$P^2R^n = a^{2n}\, r^{n(n-1)}\, \frac{(1-r^n)^n}{a^n\,(1-r)^n\, r^{n(n-1)}}$$

$$= \frac{a^n(1-r^n)^n}{(1-r)^n} = S^n, \text{ by (1)}$$

Example 16: The ratio of the 4th to the 12th term of a GP with positive common ratio is $\frac{1}{256}$. If the sum of the two terms is 61.68, find the sum of series to 8 terms.

Solution: Let the series be $a, ar, ar^2, \ldots,$

$$T_4 = 4\text{th term} = ar^3$$

$$T_{12} = 12\text{th term} = ar^{11}$$

By hypothesis, $\frac{T_4}{T_{12}} = \frac{1}{256}$

i.e., $\frac{ar^3}{ar^{11}} = \frac{1}{256}$

$$\frac{1}{r^8} = \frac{1}{256}$$

$\Rightarrow$ $r^8 = 256$

$\Rightarrow$ $r = \pm 2$

Since r is given to be positive, we reject negative sign.

Again, it is given that

$$T_4 + T_{12} = 61.68$$

i.e., $a\,(r^3 + r^{11}) = 61.68$

$$a\,(8 + 2048) = 61.68$$

$$a = \frac{61.68}{2056} = 0.03$$

Hence, $S_8 = \text{Sum to eight terms}$

$$= \frac{a\,(1-r^8)}{1-r} = \frac{a\,(r^8-1)}{r-1}$$

$$= \frac{(0.03)\,(256-1)}{(2-1)} = 0.03 \times 255 = 7.65$$

Example 17: A manufacturer reckons that the value of a machine which costs him Rs 18750 will depreciate each year by 20%. Find the estimated value at the end of 5 years.

Solution: At the end of first year the value of machine is

$$= 18750 \times \frac{80}{100} = \frac{4}{5}\,(18750)$$

At the end of 2nd year it is equal to $\left(\frac{4}{5}\right)^2 (18750)$; proceeding in this manner, the estimated value of machine at the end of 5 years is $\left(\frac{4}{5}\right)^5 (18750)$

NOTES

$$= \frac{64 \times 16}{125 \times 25} \times 18750$$

$$= \frac{1024}{125} \times 750 = 1024 \times 6$$

$$= 6144 \text{ rupees}$$

Example 18: Show that a given sum of money accumulated at 20 % per annum, more than doubles itself in 4 years at compound interest.

Solution: Let the given sum be a rupees. After 1 year it becomes $\frac{6a}{5}$ (it is increased by $\frac{a}{5}$).

At the end of two years it becomes $\frac{6}{5}\left(\frac{6a}{5}\right) = \left(\frac{6}{5}\right)^2 a$

Proceeding in this manner, we get that at the end of 4th year, the amount will be $\left(\frac{6}{5}\right)^4 a = \frac{1296}{625}\, a$

Now, $\frac{1296}{625}\, a - 2a = \frac{46}{625}a$, since a is a + ve quantity, so the amount after 4 years is more than double of the original amount.

Example 19: If $x = a + \frac{a}{r} + \frac{a}{r^2} + \ldots \infty$

$$y = b - \frac{b}{r} + \frac{b}{r^2} + \ldots \infty$$

and $z = c + \frac{c}{r^2} + \frac{c}{r^4} + \ldots \infty$

Show that $\frac{xy}{z} = \frac{ab}{c}$

Solution: Clearly, $x = \frac{a}{1 - \frac{1}{r}} = \frac{ar}{r-1}$,

$$y = \frac{b}{1-(-1/r)} = \frac{br}{r+1}$$

and, $z = \frac{c}{1 - \frac{1}{r^2}} = \frac{cr^2}{r^2-1}$

Now, $\frac{xy}{z} = \frac{ab\,r^2}{(r^2-1)} \Big/ \left(\frac{cr^2}{r^2-1}\right) = \frac{ab}{c}$

Example 20: If $a^2 + b^2$, $ab + bc$ and $b^2 + c^2$ are in GP, prove that a, b, c are also in GP.

Solution: Since $a^2 + b^2$, $ab + bc$ and $b^2 + c^2$ are in GP, we get,

$$(ab + bc)^2 = (a^2 + b^2)\,(b^2 + c^2)$$

$$b^2(a^2 + 2ac + c^2) = a^2b^2 + a^2c^2 + b^4 + b^2c^2$$

NOTES

$$\Rightarrow \quad 2ab^2c = a^2c^2 + b^4$$

$$\Rightarrow \quad a^2c^2 - 2ab^2c + b^4 = 0$$

$$\Rightarrow \quad (ac - b^2)^2 = 0$$

$$\Rightarrow \quad ac = b^2$$

$\Rightarrow$ a, b, c are in GP.

5.2.4 Harmonic Mean

If a, b, c are in HP, then b is called a *Harmonic Mean* between a and c, written as HM.

Harmonical Progression

Non zero quantities whose reciprocals are in AP, or Arithmetic Progression are said to be in *Harmonical Progression*, written as HP.

Consider the following examples:

(*a*) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\ldots$

(*b*) $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots\ldots$

(*c*) $2, \frac{5}{2}, \frac{10}{3}, \ldots$

(*d*) $\frac{1}{a}, \frac{1}{a+b}, \frac{1}{a+2b}, \ldots\ldots \; a, b > 0$

(*e*) $5, \frac{55}{9}, \frac{55}{7}, 11, \ldots\ldots$

It can be easily checked that in each case, the series obtained by taking reciprocal of each of the term is an AP.

To Insert *n* Harmonic Means between *a* and *b*

Let $H_1, H_2, H_3, \ldots, H_n$ be the required Harmonic Means. Then, $a, H_1, H_2, \ldots, H_n, b$ are in HP

i.e., $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots, \frac{1}{H_n}, \frac{1}{b}$ are in AP

Then, $\frac{1}{b} = (n+2)$th term of an AP

$$= \frac{1}{a} + (n+1)d$$

Where d is the common difference of AP.

This gives, $d = \frac{a-b}{(n+1)ab}$

Now, $\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{(n+1)\,ab}$

$$= \frac{nb + b + a - b}{(n+1)\,ab} = \frac{a + nb}{(n+1)\,ab}$$

NOTES

So, $\frac{1}{H_1} = \frac{a + nb}{(n+1)\,ab}$

$\Rightarrow$ $H_1 = \frac{(n+1)\,ab}{a + nb}$

Again, $\frac{1}{H_2} = \frac{1}{a} + 2d = \frac{1}{a} + \frac{2\,(a-b)}{(n+1)\,ab}$

$$= \frac{nb + b + 2a - 2b}{(n+1)\,ab} = \frac{2a - b + nb}{(n+1)\,ab}$$

$\Rightarrow$ $H_2 = \frac{(n+1)\,ab}{2a - b + nb}$

Similarly, $\frac{1}{H_3} = \frac{1}{a} + 3d = \frac{3a - 2b + nb}{(n+1)\,ab}$

$\Rightarrow$ $H_3 = \frac{(n+1)\,ab}{3a - 2b + nb}$ and so on,

$$\frac{1}{H_n} = \frac{1}{a} + nd = \frac{1}{a} + \frac{n\,(a-b)}{(n+1)\,ab}$$

$$= \frac{nb + b + na - nb}{(n+1)\,ab}$$

$$= \frac{na + b}{(n+1)\,ab} \Rightarrow H_n = \frac{(n+1)\,ab}{na + b}$$

Example 21: Find the 5th term of $2, 2\frac{1}{2}, 3\frac{1}{3}, \ldots\ldots$

Solution: Let 5th term be x. Then, $\frac{1}{x}$ is 5th term of corresponding AP $\frac{1}{2}, \frac{2}{5}, \frac{3}{10}, \ldots\ldots$

Then, $\frac{1}{x} = \frac{1}{2} + 4\left(\frac{2}{5} - \frac{1}{2}\right) = \frac{1}{2} + 4\left(\frac{-1}{10}\right)$

$\Rightarrow$ $\frac{1}{x} = \frac{1}{2} - \frac{2}{5} = \frac{1}{10} \Rightarrow x = 10$

Example 22: Insert two harmonic means between $\frac{1}{2}$ and $\frac{4}{17}$.

Solution: Let H_1, H_2 be two harmonic means between $\frac{1}{2}$ and $\frac{4}{17}$.

Thus, $2, \frac{1}{H_1}, \frac{1}{H_2}, \frac{17}{4}$ are in AP Let d be their common difference.

Then, $\frac{17}{4} = 2 + 3d$

$\Rightarrow$ $3d = \frac{9}{4} \quad \Rightarrow \quad d = \frac{3}{4}$

Thus, $\frac{1}{H_1} = 2 + \frac{3}{4} = \frac{11}{4} \quad \Rightarrow \quad H_1 = \frac{4}{11}$

$$\frac{1}{H_2} = 2 + 2 \times \frac{3}{4} = \frac{7}{2} \quad \Rightarrow \quad H_2 = \frac{2}{7}$$

Required harmonic means are $\frac{4}{11}, \frac{2}{7}$.

NOTES

5.3 MEASURES OF DISPERSION

A measure of dispersion or simply dispersion may be defined as statistics signifying the extent of the scatteredness of items around a measure of central tendency.

A measure of dispersion may be expressed in an 'absolute form' or in a 'relative form'. It is said to be in an absolute form when it states the actual amount by which the value of an item on an average deviates from a measure of central tendency. Absolute measures are expressed in concrete units, i.e., units in terms of which the data have been expressed, e.g., rupees, centimetres, kilograms, etc., and are used to describe frequency distribution.

A relative measure of dispersion computed is a quotient obtained by dividing the absolute measures by a quantity in respect to which absolute deviation has been computed. It is as such a pure number and is usually expressed in a percentage form. Relative measures are used for making comparisons between two or more distributions.

A measure of dispersion should possess all those characteristics which are considered essential for a measure of central tendency, which are as follows:

- It should be based on all observations.
- It should be readily comprehensible.
- It should be fairly easily calculated.
- It should be affected as little as possible by fluctuations of sampling.
- It should be amenable to algebraic treatment.

Types of Measures of Dispersion

There are four measures of dispersion which are given below:

(a) Range (R)

(b) Quartile Deviation (Q)

(c) Average Deviation (AD)

(d) Standard Deviation (SD)

Each of the above measures of dispersion tells us how the individual scores are scattered or spread throughout the distribution or the given data.

5.3.1 Types of Measures

The following are the various types of measures:

Quartile Deviation (QD)

There are many types of measures of dispersion, one of this is the semi-interquartile range, usually termed as 'Quartile Deviation' or QD. Quartiles are the points which divide the array into four equal parts. More precisely, Q_1 gives the value of the item 1/4th the way up the distribution and Q_3 the value of the item 3/4th the way up the distribution. Between Q_1 and Q_3 are included half the total number of items. The difference between Q_1 and Q_3 includes only the central items but excludes the extremes. Since under most circumstances, the central half of the series tends to be fairly typical of all the items, the interquartile range $(Q_3 - Q_1)$ affords a convenient and often a good indicator of the absolute variability. The larger the interquartile range, the larger the variability.

NOTES

Usually, one-half of the difference between Q_3 and Q_1 is used and it is given the name of quartile deviation or semi-interquartile range. The interquartile range is divided by 2 for the reason that half of the interquartile range will, in a normal distribution, be equal to the difference between the median and any quartile. This means that 50 per cent items of a normal distribution will lie within the interval defined by the median plus and minus the semi-interquartile range.

Symbolically,

$$\text{Q.D.} = \frac{Q_3 - Q_1}{2}$$

Where, Q_1 and Q_3 represent the 1st and 3rd quartiles of dispersion under consideration. The value $Q_3 - Q_1$ is the difference or range between the 3rd and 1st quartiles and is the interquartile range.

For computing quartile deviation, this interquartile range is divided by 2 and, therefore, quartile deviation is also names as semi-interquartile range. In this way, for computing Q, the value of Q_1 and Q_3 are first determined by the method and then applying the above formula, we get the value of the quartile deviation.

Let us find quartile deviations for the weekly earnings of labour in the four workshops whose data is given in Table 5.1. The computations are as shown in Table 5.1.

Table 5.1 *Weekly Earnings of Labourers in Four Workshops of the Same Type*

Weekly Earnings ₹	*No. of Workers*			
	Workshop A	*Workshop B*	*Workshop C*	*Workshop D*
15–16	...	...	2	...
17–18	...	2	4	...
19–20	...	4	4	4
21–22	10	10	10	14
23–24	22	14	16	16
25–26	20	18	14	16
27–28	14	16	12	12
29–30	14	10	6	12
31–32	...	6	6	4
33–34	...	...	2	2
35–36	...	...	...	...
37–38	...	...	4	...
Total	80	80	80	80
Mean	25.5	25.5	25.5	25.5

The range is as follows:

Workshop	*Range*
A	9
B	15
C	23
D	15

NOTES

As shown in Table 5.2, Q.D. of workshop *A* is ₹ 2.12 and median value in 25.3. This means that if the distribution is symmetrical, the number of workers whose wages vary between (25.3–2.1) = ₹ 23.2 and (25.3 + 2.1) = ₹ 27.4, shall be just half of the total cases. The other half of the workers will be more than ₹ 2.1 removed from the median wage. As this distribution is not symmetrical, the distance between Q_1 and the median Q_2 is not the same as between Q_3 and the median. Hence, the interval defined by median plus and minus semi inter-quartile range will not be exactly the same as given by the value of the two quartiles. Under such conditions the range between ₹ 23.2 and ₹ 27.4 will not include precisely 50 per cent of the workers.

If quartile deviation is to be used for comparing the variability of any two series, it is necessary to convert the absolute measure to a coefficient of quartile deviation. To do this the absolute measure is divided by the average size of the two quartiles.

Symbolically,

$$\text{Coefficient of quartile deviation} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Applying this to our illustration of four workshops in Table 5.1 the coefficients of Q.D. are as given in Table 5.2.

Table 5.2 *Calculation of Quartile Deviation*

		Workshop A	*Workshop B*	*Workshop C*	*Workshop D*
Location of Q_2	$\frac{N}{2}$	$\frac{80}{2}=40$	$\frac{80}{2}=40$	$\frac{80}{2}=40$	$\frac{80}{2}=40$
	Q_2	$24.5+\frac{40-30}{22}\times 2$	$24.5+\frac{40-30}{18}\times 2$	$24.5+\frac{40-30}{16}\times 2$	$24.5+\frac{40-30}{16}\times 2$
		$= 24.5 + 0.9$	$= 24.5 + 1.1$	$= 24.5 + 0.75$	$= 24.5 + 0.75$
		$= 25.4$	$= 25.61$	$= 25.25$	$= 25.25$
Location of Q_1	$\frac{N}{4}$	$\frac{80}{4}=20$	$\frac{80}{4}=20$	$\frac{80}{4}=20$	$\frac{80}{4}=20$
	Q_1	$22.5+\frac{20-10}{22}\times 2$	$22.5+\frac{20-16}{14}\times 2$	$20.5+\frac{20-10}{10}\times 2$	$22.5+\frac{20-18}{16}\times 2$
		$= 22.5 + .91$	$= 22.5 + .57$	$= 20.5 + 2$	$= 22.5 + .25$
		$= 23.41$	$= 23.07$	$= 22.5$	$= 22.75$
Location of Q_3	$\frac{3N}{4}$	$3\times\frac{80}{4}=60$	60	60	60
	Q_3	$26.5+\frac{60-52}{14}\times 2$	$26.5+\frac{60-48}{16}\times 2$	$26.5+\frac{60-50}{12}\times 2$	$26.5+\frac{60-50}{12}\times 2$
		$= 26.5 + 1.14$	$= 26.5 + 1.5$	$= 26.5 + 1.67$	$= 26.5 + 1.67$
		$= 27.64$	$= 28.0$	$= 28.17$	$= 28.17$
Quartile Deviation	$\frac{Q_3-Q_1}{2}$	$\frac{27.64-23.41}{2}$	$\frac{28-23.07}{2}$	$\frac{28.17-22.5}{2}$	$\frac{28.17-22.75}{2}$
		$=\frac{4.23}{2}=₹\ 2.12$	$=\frac{4.93}{2}=₹\ 2.46$	$=\frac{5.67}{2}=₹\ 2.83$	$=\frac{5.42}{2}=₹.\ 2.71$
Coefficient of Quartile Deviation $=\frac{Q_3-Q_1}{Q_3+Q_1}$		$\frac{27.64-23.41}{27.64+23.41}$	$\frac{28-23.07}{28+23.07}$	$\frac{28.17-22.5}{28.17+22.5}$	$\frac{28.17-22.75}{28.17+22.75}$
		$= 0.083$	$= 0.097$	$= 0.112$	$= 0.106$

NOTES

Characteristics of Quartile Deviation

The following are the characteristics of quartile deviation:

(a) The size of the quartile deviation gives an indication about the uniformity or otherwise of the size of the items of a distribution. If the quartile deviation is small, it denotes large uniformity. Thus, a coefficient of quartile deviation may be used for comparing uniformity or variation in different distributions.

(b) Quartile deviation is not a measure of dispersion in the sense that it does not show the scatter around an average, but only a distance on scale. Consequently, quartile deviation is regarded as a measure of partition.

(c) It can be computed when the distribution has open-end classes.

Limitations of Quartile Deviation

Except for the fact that its computation is simple and it is easy to understand, a quartile deviation does not satisfy any other test of a good measure of variation.

Average Deviation (AD)

In the following section you will study that a weakness of the measures of dispersion, based upon the range or a portion thereof, is that the precise size of most of the variants has no effect on the result. As an illustration, the quartile deviation will be the same whether the variates between Q_1 and Q_3 are concentrated just above Q_1 or they are spread uniformly from Q_1 to Q_3. This is an important defect from the viewpoint of measuring the divergence of the distribution from its typical value. The Average Deviation (AD) is employed to answer the objection.

Average Deviation (AD), also called mean deviation, of a frequency distribution is the mean of the absolute values of the deviation from some measure of central tendency. In other words, mean deviation is the arithmetic average of the variations (deviations) of the individual items of the series from a measure of their central tendency.

Garrett in 1971 defines Average Deviation (AD) as the mean of deviations of all the separate scores in the series taken from their mean (occasionally from the median or mode). It is the simplest measure of variability that takes into account the fluctuation or variation of all the items in a series.

Computation of Average Deviation from Ungrouped Data

In the case of ungrouped data, the average deviation is calculated by the formula

$$AD = \sum |x| / N$$

Where, $x = X - M$ = Deviation of the raw score from the mean of the series and $|x|$ signifies that in the deviation values we ignore the algebraic signs +ve or –ve.

Computation of Average Deviation from Grouped Data

From the grouped data, AD can be computed by the formula

$$AD = \sum |fx| / N$$

We can measure the deviations from any measure of central tendency, but the most commonly employed ones are the median and the mean. The median is preferred

NOTES

because it has the important property that the average deviation from it is the least.

Calculation of mean deviation then involves the following steps:

(a) Calculate the median (or the mean) M_d (or $\bar{X}$).

(b) Record the deviations $|d| = |x - M_d|$ of each of the items, ignoring the sign.

(c) Find the average value of deviations.

$$\text{Mean Deviation} = \frac{\sum |d|}{N}$$

Example 23 explains it better.

Example 23: Calculate the mean deviation from the following data giving marks obtained by 11 students in a class test.

14, 15, 23, 20, 10, 30, 19, 18, 16, 25, 12.

Solution:

$$\text{Median} = \text{Size of } \frac{11+1}{2} \text{ th item}$$

$$= \text{Size of 6th item} = 18.$$

Serial No.	*Marks*	$\lvert x - \text{Median} \rvert$ $\lvert d \rvert$
1	10	8
2	12	6
3	14	4
4	15	3
5	16	2
6	18	0
7	19	1
8	20	2
9	23	5
10	25	7
11	30	12
		$\sum \lvert d \rvert = 50$

$$\text{Mean deviation from median} = \frac{\sum |d|}{N}$$

$$= \frac{50}{11} = 4.5 \text{ marks}$$

For grouped data, it is easy to see that the mean deviation is given by:

$$\text{Mean deviation} = \frac{\sum f|d|}{\sum f}$$

Where,

$|d| = |x - \text{Median}|$ for grouped discrete data.

$|d| = M - \text{Median}|$ for grouped continuous data with M as the mid-value of a particular group.

Examples 24 and 25 illustrate the use of this formula.

NOTES

Example 24: Calculate the mean deviation from the following data:

Size of Item	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

Solution:

Size	*Frequency (f)*	*Cumulative Frequency*	*Deviations from Median (9)* $\lvert d \rvert$	$f \lvert d \rvert$
6	3	3	3	9
7	6	9	2	12
8	9	18	1	9
9	13	31	0	0
10	8	39	1	8
11	5	44	2	10
12	4	48	3	12
	48			60

Median = The size of $\frac{48+1}{2} = 24.5$th item which is 9.

Therefore, deviations d are calculated from 9, i.e., $|d| = |x - 9|$.

$$\text{Mean deviation} = \frac{\sum f|d|}{\sum f} = \frac{60}{48} = 1.25$$

Example 25: Calculate the mean deviation from the following data:

x	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
f	18	16	15	12	10	5	2	2

Solution:

This is a frequency distribution with continuous variable. Thus, deviations are calculated from mid-values.

x	*Mid-Value*	(f)	*Less than (c.f.)*	*Deviation from Median* $\lvert d \rvert$	$f \lvert d \rvert$
0–10	5	18	18	19	342
10–20	15	16	34	9	144
20–30	25	15	49	1	15
30–40	35	12	61	11	132
40–50	45	10	71	21	210
50–60	55	5	76	31	155
60–70	65	2	78	41	82
70–80	75	2	80	51	102
		80			1182

NOTES

$$\text{Median} = \text{The size of } \frac{80}{2} \text{ th item}$$

$$= 20 + \frac{6}{15} \times 10 = 24$$

and then, mean deviation $= \dfrac{\Sigma f|d|}{\Sigma f}$

$$= \frac{1182}{80} = 14.775.$$

Merits and Demerits of the Average (Mean) Deviation

Merits

The merits are as follows:

1. It is easy to understand.
2. As compared to standard deviation, its computation is simple.
3. As compared to standard deviation, it is less affected by extreme values.
4. Since it is based on all values in the distribution, it is better than range or quartile deviation.

Demerits

The demerits are as follows:

1. It lacks those algebraic properties which would facilitate its computation and establish its relation to other measures.
2. Due to this, it is not suitable for further mathematical processing.

Coefficient of Mean or Average Deviation

The coefficient or relative dispersion is found by dividing the mean deviations recorded. Thus,

$$\text{Coefficient of MD} = \frac{\text{Mean Deviation}}{\text{Mean}}$$

(when deviations were recorded from the mean)

$$= \frac{\text{Mean Deviation}}{\text{Median}}$$

(when deviations were recorded from the median)

Applying the above formula to Example 1.7.

$$\text{Coefficient of MD} = \frac{14.775}{24}$$

$$= 0.616$$

Standard Deviation (SD)

By far the most universally used and the most useful measure of dispersion is the Standard Deviation (SD) or root mean square deviation about the mean. We have seen that all the methods of measuring dispersion so far discussed are not universally adopted for want of adequacy and accuracy. The range is not satisfactory as its magnitude is determined by most extreme cases in the entire group. Further, the range is notable because it is dependent on the item whose size is largely a matter of chance. Mean deviation method is also an unsatisfactory measure of scatter, as it ignores the algebraic signs of deviation. We desire a measure of scatter which is free from these shortcomings. To some extent standard deviation is one such measure.

NOTES

The calculation of standard deviation differs in the following respects from that of mean deviation. First, in calculating standard deviation, the deviations are squared. This is done so as to get rid of negative signs without committing algebraic violence. Further, the squaring of deviations provides added weight to the extreme items, a desirable feature for certain types of series.

Second, the deviations are always recorded from the arithmetic mean, because although the sum of deviations is the minimum from the median, the sum of squares of deviations is minimum when deviations are measured from the arithmetic average. The deviation from $\bar{x}$ is represented by σ.

Thus, standard deviation, σ (sigma) is defined as the square root of the mean of the squares of the deviations of individual items from their arithmetic mean.

Standard deviation of a set of scores is defined as the square root of the average of the squares of the deviations of each score from the mean. Symbolically, we can say that:

$$SD = \sqrt{\Sigma (X - M)^2/2}$$

$$= \sqrt{\Sigma x^2 / N}$$

Where,

X = Individual score

M = Mean of the given set of scores

N = Total number of the sores

x = Derivation of each score from the mean

Standard Deviation or SD is regarded as the most stable and reliable measure of variability as it employs the mean for its computation. It is often called *root mean square deviation* and is denoted by the Greek letter sigma (σ).

Computation of Standard Deviation (SD) from Ungrouped Data

Standard deviation can be computed from the ungrouped scores by the formula

$$\sigma = \sqrt{\Sigma x^2 / N}$$

Computation of Standard Deviation (SD) from Grouped Data

Standard deviation in case of grouped data can be computed by the formula

$$\sigma = \sqrt{\Sigma fx^2 / N}$$

Computation of Standard Deviation (SD) from Grouped Data by Short-Cut Method

Standard deviation from grouped data can be computed by the following formula:

$$\sigma = \sqrt{\Sigma fx'^2 / N - (\Sigma fx'/N)^2}$$

Example 26: Compute the standard deviation for the following data:

11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21.

NOTES

Solution:

We first calculate the mean as $\bar{x} = \sum x/N = 176/11 = 16$, and then calculate the deviation as follows:

x	$(x - \bar{x})$	$(x - \bar{x})^2$
11	−5	25
12	−4	16
13	−3	9
14	−2	4
15	−1	1
16	0	0
17	+1	1
18	+2	4
19	+3	9
20	+4	16
21	+5	25
176		110

Thus,

$$\sigma = \sqrt{\frac{110}{11}} = \sqrt{10} = 3.16$$

Example 27: Find the standard deviation of the data in the following distributions:

x	12	13	14	15	16	17	18	20
f	4	11	32	21	15	8	6	4

Solution:

Since for calculation of $\bar{x}$, we need $\sum fx$ and then for σ we need $\sum f(x - \bar{x})^2$, the calculations are conveniently made in the following format.

x	f	fx	$d = x - \bar{x}$	d^2	fd^2
12	4	48	−3	9	36
13	11	143	−2	4	44
14	32	448	−1	1	32
15	21	315	0	0	0
16	15	240	1	1	15
17	8	136	2	4	32
18	5	90	3	9	45
20	4	80	5	25	100
	$\sum f$ 100	$\sum fx$ 1500			$\sum fd^2$ 304

Here, $\bar{x} = \sum fx/\sum f = 1500/100 = 15$

and

$$\sigma = \sqrt{\frac{\sum fd^2}{\sum f}}$$

$$= \sqrt{\frac{304}{100}} = \sqrt{3.04} = 1.74$$

NOTES

Example 28: Compute the standard deviation by the short-cut method for the following data:

11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Solution: Let us assume that $A = 15$.

	$x' = (x - 15)$	x'^2
11	−4	16
12	−3	9
13	−2	4
14	−1	1
15	0	0
16	1	1
17	2	4
18	3	9
19	4	16
20	5	25
21	6	36
$N = 11$	$\sum x' = 11$	$\sum x'^2 = 121$

$$\sigma = \sqrt{\frac{\sum x'^2}{N} - \left(\frac{\sum x'}{N}\right)^2}$$

$$= \sqrt{\frac{121}{11} - \left(\frac{11}{11}\right)^2}$$

$$= \sqrt{11 - 1}$$

$$= \sqrt{10}$$

$$= 3.16$$

Another Method

If we assume A as zero, then the deviation of each item from the assumed mean is the same as the value of item itself. Thus, 11 deviates from the assumed mean of zero by 11, 12 deviates by 12, and so on. As such, we work with deviations without having to compute them, and the formula takes the following shape:

x	x^2
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400
21	441
176	2,926

NOTES

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

$$= \sqrt{\frac{2926}{11} - \left(\frac{176}{11}\right)^2} = \sqrt{266 - 256} = 3.16$$

Combining Standard Deviations of Two Distributions

If we were given two sets of data of N_1 and N_2 items with means $\bar{x}_1$ and $\bar{x}_2$ and standard deviations σ_1 and σ_2, respectively, we can obtain the mean and standard deviation $\bar{x}$ and σ of the combined distribution by the following formulae:

$$\bar{x} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2}$$

and $$\sigma = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1(\bar{x} - \bar{x}_1)^2 + N_2(\bar{x} - \bar{x}_2)^2}{N_1 + N_2}}$$

Example 29: The mean and standard deviations of two distributions of 100 and 150 items are 50, 5 and 40, 6, respectively. Find the standard deviation of all taken together.

Solution:

Combined mean,

$$\bar{x} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2}{N_1 + N_2} = \frac{100 \times 50 + 150 \times 40}{100 + 150}$$

$$= 44$$

Combined standard deviation,

$$\sigma = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1(\bar{x} - \bar{x}_1)^2 + N_2(\bar{x} - \bar{x}_2)^2}{N_1 + N_2}}$$

$$= \sqrt{\frac{100 \times (5)^2 + 150\,(6)^2 + 100\,(44 - 50)^2 + 150\,(44 - 40)^2}{100 + 150}}$$

$$= 7.46.$$

Example 30: A distribution consists of three components with 200, 250, 300 items having mean 25, 10 and 15 and standard deviation 3, 4 and 5, respectively. Find the standard deviation of the combined distribution.

Solution:

In the usual notations, we are given here

$$N_1 = 200,\ N_2 = 250,\ N_3 = 300$$

$$\bar{x}_1 = 25,\ \bar{x}_2 = 10,\ \bar{x}_3 = 15$$

For the combination of three series the formula will be:

$$\bar{x} = \frac{N_1\bar{x}_1 + N_2\bar{x}_2 + N_3\bar{x}_3}{N_1 + N_2 + N_3}$$

$$= \frac{200 \times 25 + 250 \times 10 + 300 \times 15}{200 + 250 + 300}$$

$$= \frac{12000}{750} = 16$$

and,

NOTES

$$\sigma = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1(\bar{x} - \bar{x}_1)^2 + N_2(\bar{x} - \bar{x}_2)^2 + N_3(\bar{x} - \bar{x}_3)^2}{N_1 + N_2 + N_3}}$$

$$= \sqrt{\frac{200 \times 9 + 250 \times 16 + 300 \times 25 + 200 \times 81 + 250 \times 36 + 300 \times 1}{200 + 250 + 300}}$$

$$= \sqrt{51.73} = 7.19$$

5.3.2 Range (R)

Range (R) of a set of data is the difference between the largest and smallest values. It is the simplest measure of variability or dispersion and is calculated by subtracting the lowest score in the series from the highest. But it is very rough measure of the variability of series. It takes only extreme scores into consideration and ignores the variation of individual items.

Thus the crudest measure of dispersion is the range of the distribution. The range of any series is the difference between the highest and the lowest values in the series. If the marks received in an examination taken by 248 students are arranged in ascending order, then the range will be equal to the difference between the highest and the lowest marks.

In a frequency distribution, the range is taken to be the difference between the lower limit of the class at the lower extreme of the distribution and the upper limit of the class at the upper extreme.

Consider the data on weekly earnings of worker on four workshops given in Table 5.1.

From these figure in Table 5.1, it is clear that the greater the range, the greater is the variation of the values in the group.

The range is a measure of absolute dispersion and as such cannot be usefully employed for comparing the variability of two distributions expressed in different units. The amount of dispersion measured, say, in pounds, is not comparable with dispersion measured in inches. Thus, the need of measuring relative dispersion arises.

An absolute measure can be converted into a relative measure if we divide it by some other value regarded as standard for the purpose. We may use the mean of the distribution or any other positional average as the standard.

For Table 5.1, the relative dispersion would be,

$$\text{Workshop } A = \frac{9}{25.5} \qquad \text{Workshop } C = \frac{23}{25.5}$$

$$\text{Workshop } B = \frac{15}{25.5} \qquad \text{Workshop } D = \frac{15}{25.5}$$

An alternate method of converting an absolute variation into a relative one would be to use the total of the extremes as the standard. This will be equal to dividing the difference of the extreme items by the total of the extreme items. Thus,

NOTES

$$\text{Relative Dispersion} = \frac{\text{Difference of extreme items, i.e, Range}}{\text{Sum of extreme items}}$$

The relative dispersion of the series is called the coefficient or ratio of dispersion. In our example of weekly earnings of workers considered earlier, the coefficients would be,

$$\text{Workshop } A = \frac{9}{21+30} = \frac{9}{51} \qquad \text{Workshop } B = \frac{15}{17+32} = \frac{15}{49}$$

$$\text{Workshop } C = \frac{23}{15+38} = \frac{23}{53} \qquad \text{Workshop } D = \frac{15}{19+34} = \frac{15}{53}$$

Merits and Limitations of Range

Merits

Of the various characteristics that a good measure of dispersion should possess, the range has only two, which are as follows:

1. It is easy to understand.
2. Its computation is simple.

Limitations

Besides the aforesaid two qualities, the range does not satisfy the other test of a good measure and hence it is often termed as a crude measure of dispersion.

The following are the limitations that are inherent in the range as a concept of variability:

1. Since it is based upon two extreme cases in the entire distribution, the range may be considerably changed if either of the extreme cases happens to drop out, while the removal of any other case would not affect it at all.
2. It does not tell anything about the distribution of values in the series relative to a measure of central tendency.
3. It cannot be computed when distribution has open-end classes.
4. It does not take into account the entire data. These can be illustrated by the following illustration. Consider the data given in Table 5.3.

Table 5.3 *Distribution with the Same Number of Cases, but Different Variability*

Class	No. of Students		
	Section A	Section B	Section C
0–10	...	...	...
10–20	1	...	...
20–30	12	...	...
30–40	17	12	19
40–50	29	20	18
50–60	18	35	16
60–70	16	25	18
70–80	6	10	18
80–90	11	8	21
90–100	...	...	...
Total	110	...	...
Range	80	110	110
		60	60

NOTES

The table is designed to illustrate three distributions with the same number of cases but different variability. The removal of two extreme students from Section *A* would make its range equal to that of *B* or *C*.

The greater range of *A* is not a description of the entire group of 110 students, but of the two most extreme students only. Further, though sections *B* and *C* have the same range, the students in Section B cluster more closely around the central tendency of the group than they do in Section *C*. Thus, the range fails to reveal the greater homogeneity of *B* or the greater dispersion of *C*. Due to this defect, it is seldom used as a measure of dispersion.

Specific Uses of Range

In spite of the numerous limitations of the range as a measure of dispersion, it is the most appropriate under the following circumstances:

(a) In situations where the extremes involve some hazard for which preparation should be made, it may be more important to know the most extreme cases to be encountered than to know anything else about the distribution. For example, an explorer, would like to know the lowest and the highest temperatures on record in the region he is about to enter; or an engineer would like to know the maximum rainfall during 24 hours for the construction of a storm water drain.

(b) In the study of prices of securities, range has a special field of activity. Thus to highlight fluctuations in the prices of shares or bullion, it is a common practice to indicate the range over which the prices have moved during a certain period of time. This information, besides being of use to the operators, gives an indication of the stability of the bullion market, or that of the investment climate.

(c) In statistical quality control, range is used as a measure of variation. We, for example, determine the range over which variations in quality are due to random causes, which is made the basis for the fixation of control limits.

5.3.3 Skewness

Skewness refers to lack of symmetry in a distribution. In a symmetrical distribution, the mean, median and mode coincide. Positive and negative skewness is shown in Figure 5.1.

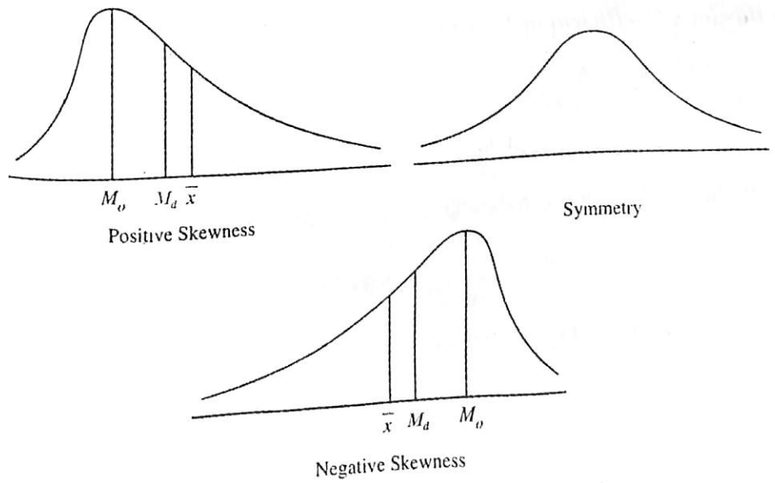

Fig. 5.1 *Skewness*

NOTES

In a positively skewed distribution, the longer tail is on the right side and the mean is on the right of the median.

In a negatively skewed distribution, the longer tail is on the left and the mean is on the left of the median.

In a skewed distribution, the distance between the mean and the median is nearly one-third of that between the mean and the mode.

How to Check the Presence of Skewness in a Distribution

In the following cases skewness is present in the data:

(a) The graph is not symmetrical.

(b) The mean, median and mode do not coincide.

(c) The quartiles are not equidistant from the mean.

(d) The sum of positive and negative deviations from the median is not zero.

(e) Frequencies are not similarly distributed on either side of the mode.

Measure of Skewness

A measure of skewness gives a numerical expression and the direction of asymmetry in a distribution. It gives information about the shape of the distribution and the degree of variation on either side of the central value.

We consider some relative measures of skewness that are as follows:

(a) *Pearson's Coefficient of Skewness*

$$PSk = \frac{\bar{x} - M_o}{s} = \frac{3(\bar{x} - M_d)}{s}$$

It may have any value, but usually it lies between –1 and +1.

Illustration 1: If for a given data it is found that

$\bar{x} = 10$, Mode = 8, s = 4, we have

$$PSk = \frac{\bar{x} - M_o}{s} = \frac{10 - 8}{4} = 0.5$$

(b) *Bowley's Coefficient of Skewness*

$$BSk = \frac{Q_3 - Q_1 - 2M_d}{Q_3 - Q_1}$$

Its value lies between –1 and +1.

Illustration 2: If for a given data $Q_1 = 2$, $Q_3 = 8$, $M_d = 5$

$$BSk = \frac{Q_3 + Q_1 - 2M_d}{Q_3 - Q_1} = \frac{8 + 2 - 5}{8 - 2} = 0.83$$

(c) *Kelley's Coefficient of Skewness*

$$KSk = P_{50} - \frac{1}{2}(P_{10} + P_{90})$$

where P_{10}, P_{50} and P_{90} are the 10th, 50th and 90th percentiles of the data.

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(d) *Method of Moments*

If μ_2, μ_3 are moments about the mean we have the coefficient of skewness

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \mu_3^2 / \sigma^6$$

Sometimes, we define the coefficient of skewness as follows:

$$\gamma_1 = \sqrt{\beta_1} = \sqrt{\frac{\mu_3^2}{\mu_2^3}} = \frac{\mu_3}{\sigma^3}$$

5.3.4 Kurtosis

Kurtosis is a measure of peakedness of a distribution. It shows the degree of convexity of a frequency curve.

If the normal curve is taken as the standard, symmetrical and bell-shaped curve then kurtosis gives a measure of departure from the normal convexity of a distribution. The normal curve is mesokurtic. It is of intermediate peakedness. The flat-topped curve, broader than the normal, is termed platykurtic. The slender, highly peaked curve is termed leptokurtic.

Measures of Kurtosis

(a) Moment Coefficient of Kurtosis : $\beta_2 = \frac{\mu_4}{\mu_2^2}$

Instead of β_2 we often use $\gamma_2 = \beta_2 - 3$ which is positive for a leptokurtic distribution, negative for a platykurtic distribution and zero for the normal distribution.

(b) Percentile Coefficient of Kurtosis $k = \frac{Q}{P_{90} - P_{10}}$, where $Q = \frac{1}{2}(Q_3 - Q_1)$ is the semi-interquartile range.

5.3.5 Comparison of Various Measures of Dispersion

The range is the easiest to calculate the measure of dispersion, but since it depends on extreme values, it is extremely sensitive to the size of the sample and to the sample variability. In fact, as the sample size increases the range increases dramatically, because the more the items one considers, the more likely it is that one item will turn up which is larger than the previous maximum or smaller than the previous minimum. So, it is, in general, impossible to interpret properly the significance of a given range unless the sample size is constant. It is for this reason that there appears to be only one valid application of the range, namely in statistical quality control where the same sample size is repeatedly used so that comparison of ranges is not distorted by differences in sample size.

The quartile deviations and other such positional measures of dispersions are also easy to calculate, but suffer from the disadvantage that they are not amenable to algebraic treatment. Similarly, the mean deviation is not suitable because we cannot obtain the mean deviation of a combined series from the deviations of component series. However, it is easy to interpret and easier to calculate than the standard deviation.

NOTES

The standard deviation of a set of data, on the other hand, is one of the most important statistics describing it. It lends itself to rigorous algebraic treatment, is rigidly defined and is based on all observations. It is, therefore, quite insensitive to sample size (provided the size is 'large enough') and is least affected by sampling variations.

It is used extensively in testing of hypothesis about population parameters based on sampling statistics.

In fact, the standard deviations has such stable mathematical properties that it is used as a standard scale for measuring deviations from the mean. If we are told that the performance of an individual is 10 points better than the mean, it really does not tell us enough, for 10 points may or may not be a large enough difference to be of significance. However, if we know that the s for the score is only 4 points, so that on this scale, the performance is $1.5s$ better than the mean, the statement becomes meaningful. This indicates an extremely good performance. This sigma scale is a very commonly used scale for measuring and specifying deviations which immediately suggest the significance of the deviation.

The only disadvantages of the standard deviation lies in the amount of work involved in its calculation, and the large weight it attaches to extreme values because of the process of squaring involved in its calculations.

5.4 COEFFICIENT OF VARIATION

The square of standard deviation, namely σ^2, is termed as variance and is more often specified than the standard deviation. Clearly, it has the same properties as standard deviation.

As is clear, the standard deviation σ or its square, the variance, cannot be very useful in comparing two series where either the units are different or the mean values are different. Thus, a σ of 5 on an examination where the mean score is 30 has an altogether different meaning than on an examination where the mean score is 90. Clearly, the variability in the second examination is much less. To take care of this problem, we define and use a coefficient of variation, V. Where,

$$V = \frac{\sigma}{\bar{x}} \times 100$$

This is expressed as percentage.

Example 31: The following are the scores of two batsmen A and B in a series of innings:

A	12	115	6	73	7	19	119	36	84	29
B	47	12	76	42	4	51	37	48	13	0

Who is the better run-getter? Who is more consistent?

Solution: In order to decide as to which of the two batsmen, A and B, is the better run-getter, we should find their batting averages. The one whose average is higher will be considered as a better batsman.

To determine the consistency in batting we should determine the coefficient of variation. The less this coefficient the more consistent will be the player.

NOTES

	A			B	
Score x	x	x^2	Scores x	x	x^2
12	–38	1,444	47	14	196
115	+65	4,225	12	–21	441
6	–44	1,936	76	43	1,849
73	+23	529	42	9	81
7	–43	1,849	–4	– 29	841
19	–31	961	51	18	324
119	+69	4,761	37	4	16
36	–14	196	48	15	225
84	+34	1,156	13	–20	400
29	–21	441	0	–33	1,089
$\Sigma x = 500$		17,498	$\Sigma x = 330$		5,462

Batsman A:

$$\bar{x} = \frac{500}{10} = 50$$

$$\sigma = \sqrt{\frac{17,498}{10}} = 41.83$$

$$V = \frac{41.83 \times 100}{50}$$

= 83.66 per cent

Batsman B:

$$\bar{x} = \frac{330}{10} = 33$$

$$\sigma = \sqrt{\frac{5,462}{10}} = 23.37$$

$$V = \frac{23.37}{33} \times 100$$

= 70.8 per cent

A is a better batsman since his average is 50 as compared to 33 of B, but B is more consistent since the variation in his case is 70.8 as compared to 83.66 of A.

Example 32: The following table gives the age distribution of students admitted to a college in the years 1914 and 1918. Find which of the two groups is more variable in age.

Age	Number of Students in	
	1914	1918
15	–	1
16	1	6
17	3	34
18	8	22
19	12	35
20	14	20
21	13	7
22	5	19
23	2	3
24	3	–
25	1	–
26	–	–
27	1	–

NOTES

Solution:

Age	Assumed Mean–21 1914				Assumed Mean–19 1918			
	f	x'	fx'	fx'^2	f	x'	fx	fx'^2
15	0	–6	0	0				
16	1	–5	–5	25	1	–4	–4	16
17	3	–4	–12	48	6	–3	–18	54
18	8	–3	–24	72	34	–2	–68	136
19	12	–2	–24	48	22	–1	–22	22
20	14	–1	–14	14			–112	
			–79					
21	13	0	0	0	35	0	0	0
22	5	1	5	5	20	1	20	20
23	2	2	4	8	7	2	14	28
24	3	3	9	27	19	3	57	171
25	1	4	4	16	3	4	12	48
26	0	5	0	0	147		+103	495
27	1	6	6	36			–9	
	63		+28	299				
			–51					

1914 Group:

$$\sigma = \sqrt{\frac{\Sigma fx'^2}{N} - \left[\frac{\Sigma(fx')}{N}\right]^2}$$

$$= \sqrt{\frac{299}{63} - \left(\frac{-51}{63}\right)^2}$$

$$= \sqrt{4.476 - 0.655} = \sqrt{4.091}$$

$$= 2.02.$$

$$\bar{x} = 21 + \left(\frac{-51}{63}\right) = 21 - 8 = 20.2$$

$$V = \frac{2.02}{20.2} \times 100$$

$$= \frac{202}{20.2} = 10$$

1918 Group:

$$\sigma = \sqrt{\frac{495}{147} - \left(\frac{-9}{147}\right)^2} = \sqrt{3.3673 - 0.0037}$$

$$= \sqrt{3.3636} = 1.834$$

$$\bar{x} = 19 + \left(\frac{-9}{147}\right)$$

$$= 19 - .06 = 18.94$$

$$V = \frac{1.834}{18.94} \times 100$$

$$= 9.68$$

Check Your Progress

1. Name three measures of central tendency.
2. What is a mode?
3. What do you mean by median?
4. What is a geometric mean?
5. What do you understand by harmonic mean?
6. What is range?
7. What is a quartile?
8. What is mean deviation?
9. Define the term skewness?
10. What is kurtosis?

The coefficient of variation of the 1914 group is 10 and that of the 1918 group 9.68. This means that the 1914 group is more variable, but only barely so.

NOTES

ACTIVITY

1. Determine the median and the value corresponding to the first and third quartiles in the following data set:

   46, 47, 49, 49, 51, 53, 54, 54, 55, 55, 59

2. Calculate mean deviation and its coefficient about median, arithmetic mean and mode for the following figures, and show that the mean deviation about the median is least.

   103, 50, 68, 110, 108, 105, 174, 103, 150, 200, 225, 350, 103

DID YOU KNOW

The choice of a particular measure of central tendency of location depends on the purpose of investigation. It should be noted that the Arithmetic Mean (AM) is quite precisely defined and is therefore more amenable to further mathematical manipulation. On the other hand, the mode can be located by inspection but cannot be manipulated easily. There are cases when we have the following condition:

1. Mean < Median < Mode
2. Mean > Median > Mode
3. Mean = Median = Mode

5.5 SUMMARY

- There are several commonly used measures of central tendency, such as arithmetic mean, mode and median. These values are very useful not only in presenting the overall picture of the entire data but also for the purpose of making comparisons among two or more sets of data.
- Tate in 1955 defines the measures of central tendency as, "A sort of average or typical value of the items in the series and its function is to summarize the series in terms of this average value".
- Arithmetic mean is also commonly known as simply the mean. Even though average, in general, means any measure of central location, when we use the word average in our daily routine, we always mean the arithmetic average.
- The simplest but most useful measure of central tendency is the arithmetic mean. It can be defined as the sum of all the values of the items in a series divided by the number of items. It is represented by the symbol M.
- The mode is another form of average and can be defined as the most frequently occurring value in the data. The mode is not affected by extreme values in the data and can easily be obtained from an ordered set of data.
- Mode is defined as the size of a variable which occurs most frequently. It is the point on the score sale that corresponds to the maximum frequency of the

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distribution. In any series, it is the value of the item which is most characteristics or common and is usually repeated the maximum number of times.

- The median is a measure of central tendency and it appears in the centre of an ordered data. It divides the list of ordered values in the data into two equal parts so that half of the data will have values less than the median and half will have values greater than the median.
- If the items of a series are arranged in ascending or descending order of magnitude, the measure or value of the central item in the series is termed as median. The median of a distribution can thus be said as the point on the score scale below which half (or 50 per cent) of the scores fall. Thus median is the score or the value of that central item which divides the series into two equal parts.
- In the computation of arithmetic mean we had given equal importance to each observation in the series. The weighted arithmetic mean is particularly useful where we have to compute the mean of means.
- Non zero quantities whose reciprocals are in AP or Arithmetic Progression are said to be in Harmonical Progression, written as HP.
- A measure of dispersion or simply dispersion may be defined as statistics signifying the extent of the scatteredness of items around a measure of central tendency.
- A measure of dispersion may be expressed in an 'absolute form' or in a 'relative form'. It is said to be in an absolute form when it states the actual amount by which the value of an item on an average deviates from a measure of central tendency.
- A relative measure of dispersion computed is a quotient obtained by dividing the absolute measures by a quantity in respect to which absolute deviation has been computed. It is as such a pure number and is usually expressed in a percentage form. Relative measures are used for making comparisons between two or more distributions.
- There are four measures of dispersion which are Range (R), Quartile Deviation (Q), Average Deviation (AD) and Standard Deviation (SD). Each of the above measures of dispersion tells us how the individual scores are scattered or spread throughout the distribution or the given data.
- There are many types of measures of dispersion. One of this is the semi-interquartile range usually termed as 'Quartile Deviation' or QD.
- Quartiles are the points which divide the array into four equal parts. More precisely, Q_1 gives the value of the item 1/4th the way up the distribution and Q_3 the value of the item 3/4th the way up the distribution. Between Q_1 and Q_3 are included half the total number of items.
- The difference between Q_1 and Q_3 includes only the central items but excludes the extremes. Where, Q_1 and Q_3 represent the 1st and 3rd quartiles of dispersion under consideration. The value $Q_3 - Q_1$ is the difference or range between the 3rd and 1st quartiles and is the interquartile range.
- The size of the quartile deviation gives an indication about the uniformity of otherwise of the size of the items of a distribution. If the quartile deviation is small, it denotes large uniformity.

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- Average Deviation (AD), also called mean deviation, of a frequency distribution is the mean of the absolute values of the deviation from some measure of central tendency. In other words, mean deviation is the arithmetic average of the variations (deviations) of the individual items of the series from a measure of their central tendency.
- Garrett in 1971 defines Average Deviation (AD) as the mean of deviations of all the separate scores in the series taken from their mean (occasionally from the median or mode). It is the simplest measure of variability that takes into account the fluctuation or variation of all the items in a series.
- By far the most universally used and the most useful measure of dispersion is the Standard Deviation (SD) or root mean square deviation about the mean.
- Standard Deviation or is regarded as the most stable and reliable measure of variability as it employs the mean for its computation. It is often called root mean square deviation and is denoted by the Greek letter sigma (σ).
- Thus, standard deviation, σ (sigma) is defined as the square root of the mean of the squares of the deviations of individual items from their arithmetic mean.
- Range (R) of a set of data is the difference between the largest and smallest values. It is the simplest measure of variability or dispersion and is calculated by subtracting the lowest score in the series from the highest. But it is very rough measure of the variability of series. It takes only extreme scores into consideration and ignores the variation of individual items.
- The crudest measure of dispersion is the range of the distribution. The range of any series is the difference between the highest and the lowest values in the series.
- In a frequency distribution, the range is taken to be the difference between the lower limit of the class at the lower extreme of the distribution and the upper limit of the class at the upper extreme.
- An absolute measure can be converted into a relative measure if we divide it by some other value regarded as standard for the purpose. We may use the mean of the distribution or any other positional average as the standard.
- Skewness refers to lack of symmetry in a distribution. In a symmetrical distribution, the mean, median and mode coincide. In a positively skewed distribution, the longer tail is on the right side and the mean is on the right of the median. In a negatively skewed distribution, the longer tail is on the left and the mean is on the left of the median.
- In a skewed distribution, the distance between the mean and the median is nearly one-third of that between the mean and the mode.
- A measure of skewness gives a numerical expression and the direction of asymmetry in a distribution. It gives information about the shape of the distribution and the degree of variation on either side of the central value.
- Kurtosis is a measure of peakedness of a distribution. It shows the degree of convexity of a frequency curve.
- If the normal curve is taken as the standard, symmetrical and bell-shaped curve then kurtosis gives a measure of departure from the normal convexity of a distribution. The normal curve is mesokurtic. It is of intermediate peakedness. The flat-topped curve, broader than the normal, is termed platykurtic. The slender, highly peaked curve is termed leptokurtic.

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- The square of standard deviation, namely σ^2, is termed as variance and is more often specified than the standard deviation. Clearly, it has the same properties as standard deviation.

5.6 KEY TERMS

- **Arithmetic mean:** The most commonly used measure of central location, otherwise referred to with the term 'average'
- **Mode:** The most frequently occurring value in a data
- **Median:** A measure that divides values in a data into two equal parts
- **Weighted arithmetic mean:** A measurement of a mean of means
- **Harmonic mean:** An average of different rates
- **Range:** This is the crudest measure of dispersion
- **Dispersion:** The extent of scatteredness of items around a measure of central tendency.
- **Skewness:** Lack of symmetry in a distribution
- **Kurtosis:** A measure of peakedness of a distribution

5.7 ANSWERS TO 'CHECK YOUR PROGRESS'

1. The three measures of dispersion are arithmetic mean, median and mode.
2. The mode is another form of average and can be defined as the most frequently occurring value in the data.
3. The median is a measure of central tendency and it appears in the centre of an ordered data.
4. If α, β, γ are in GP, then β is called a geometric mean between α and γ written as GM.
5. If a, b, c are in HP, then b is called a Harmonic Mean between a and c, written as HM.
6. Range is the difference between the highest and lowest values in a series.
7. Quartiles are points that divide an array into four equal parts.
8. Mean deviation is the arithmetic average of the variations of the individual items in a series from a measure of their central tendency.
9. Skewness refers to lack of symmetry in a distribution.
10. Kurtosis is a measure of peakedness of a distribution

5.8 QUESTIONS AND EXERCISES

Short-Answer Questions

1. How is central tendency measured?
2. Define the term arithmetic mean.
3. Write three characteristics of mean.
4. What is the importance of arithmetic mean in statistics?
5. Define the term median with example.
6. How is location of median calculated using graphic analysis?
7. Define the terms quartiles, deciles and percentiles with suitable examples.
8. What is mode? How is it calculated?
9. Differentiate between a mean and a mode.
10. What is a harmonic mean?
11. When a measure of dispersion is expressed in an absolute form and in relative form?
12. What is coefficient of mean or average deviation?
13. How is the calculation of standard deviation different from that of average deviation?
14. What are the merits and demerits of range?
15. How will you measure skewness?
16. How will you measure the degree of convexity of a frequency curve using kurtosis?
17. The price of a commodity was four times higher in 1970 than what it was a decade back. Find the average rate of growth of price of the commodity.
18. Arithmetic mean of a group of 100 items is 50 and of another group of 150 items is 100. What will be the mean of all the items?
19. Arithmetic mean of 98 items is 50. Two items 60 and 70 were left out at the time of calculation. What is the correct mean of all the items?

Long-Answer Questions

1. Discuss the various measures of central tendency. What purposes do their measurement serve?
2. Explain harmonic mean with the help of examples.
3. Eight coins were tossed together and the number of heads resulting was observed. The operation was performed 256 times and the frequencies that were obtained for the different values of x, the number of heads, are shown in the following table. Calculate mean, median and quartiles of the distribution of x.

x	0	1	2	3	4	5	6	7	8
Frequency	1	9	26	59	72	52	29	7	1

4. Find the mean of the following distribution:

Breadth in mm	19–21	22–24	25–27	28–30	31–33	34–36	37–39
	6	13	19	23	18	12	9

5. The following table shows the number of persons employed in certain units of an industry. Find the average number of persons employed.

No. of Persons: below	20	20–30	30–50	50–100	100–200	200 and above
	2	5	6	3	2	2

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6. Calculate arithmetic mean for the following data:

Class Interval	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45
Frequency	6	5	15	10	5	4	3	2

7. From the following table, calculate mean and median.

Crop Cutting Experimental Data on Plot Yields of Wheat

Yield (in lbs)	*No. of Plots*	*Yield (in lbs.)*	*No. of Plots*
Over 0	216	Over 300	31
Over 60	210	Over 360	13
Over 120	156	Over 420	7
Over 180	98	Over 480	2
Over 240	57	Up to 540	216

8. Find arithmetic mean, median and mode from the following:

Marks below	10	20	30	40	50	60	70	80
No. of Students	15	35	60	84	96	127	198	250

9. The wages of 1060 employees range from ₹ 300 to ₹ 450. They are grouped in 15 classes with a common class interval of ₹ 10. Class frequencies from lowest to the highest are 6, 17, 35, 48, 65, 90, 131, 173, 155, 177, 75, 52, 9, 6. Tabulate the data and calculate the mean wage.

10. From the table given below, find the mean.

Salary Per Day	*No. of Persons*	*Salary Per Day*	*No. of Persons*
1–5	7	26–30	18
6–10	10	31–35	10
11–15	16	36–40	5
16–20	32	41–45	1
21–25	24		

11. (a) From the data given below, find the mode.

Ages	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60
No. of Persons	50	70	80	180	150	120	70	50

(b) If the mode and mean of a moderately asymmetrical series are respectively 16 inches and 20.2 inches, compute the most probable median.

12. Following is the distribution of the size of certain farms selected at random from a district. Calculate the mode of distribution.

Central Size of the Farm in Acres	10	20	30	40	50	60	70
No. of Farms	7	12	17	29	31	5	3

13. Draw a histogram from the following data and measure the modal value:

Class Size	*Frequency*	*Class Size*	*Frequency*
0–10	5	50–60	10
10–20	11	60–70	8
20–30	19	70–80	6
30–40	21	80–90	3
40–50	16	90–100	1

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14. Monthly incomes of the families are given below in rupees:
2000, 35, 400, 15, 40, 1500, 300, 6, 90, 250, 20, 12, 450, 10, 150, 8, 25, 30, 1200, 60.
Calculate the geometric mean and harmonic mean of the above series.

15. The following table gives weights of 31 persons in a sample inquiry. Calculate mean weight using (a) Geometric mean and (b) Harmonic mean.

Weight in lbs.	130	135	140	145	146	148	149	150	157
No. of Persons	3	4	6	6	3	5	2	1	1

16. Peter travelled by car for 4 days. He drove 10 hours each day. He drove: first day at the rate of 45 km per hour and fourth day at the rate of 37 km per hour. What was his average speed?

17. The price of certain articles becomes $1\frac{1}{2}$ times in first year, $1\frac{5}{8}$ times in the second year and $\frac{7}{9}$ times in the third year. What is the average change per year?

18. You take a trip which entails travelling 900 miles by train at an average speed of 60 mph 3000 miles by boat at an average of 25 mph, 400 miles by plane at 350 mph and finally 15 miles by taxi at 25 mph. What is your average speed for the entire distance?

19. Calculate the simple average and weighted average of the following items:

Items	68	85	101	102	108	110	112	113	124	128	143	146	151	153	172
Weights	1	46	31	1	11	7	23	17	9	14	2	4	6	5	2

Account for the difference in the two averages.

20. Find the quartile deviation from the mean of the series 5, 7, 10, 12, 6.

21. Calculate the mean deviation from the mean and median and their coefficients of the following data:

Size of Shoes:	3	6	11	2	4	10	5	7	8	9
Pairs Sold:	10	15	25	6	4	3	2	8	9	4

22. Explain standard deviation in detail. Also, discuss the short-cut method of calculating standard deviation.

23. The following are the frequencies, means and standard deviations of two series. Find the standard deviation of the combined series.

	Series I	Series II
Frequency	35	45
Mean	30	20
Standard Deviation	10	5

24. Discuss the instances when range can specifically be used.

25. Explain the different types of skewness.

27. Find out the range and its coefficient in the following series:

Size	3	4	5	6	7	8	9	10
Frequency	35	30	20	10	6	3	2	1

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28. Find out the range and its coefficient in the following series:

Size	10–60	60–120	120–180	180–240	240–300
Frequency	3	5	6	3	2

29. Calculate the quartile deviation and its coefficient from the following data:

Size	6	7	8	9	10	11	12
Frequency	3	6	9	13	8	5	4

30. The following table gives monthly wages (in hundreds of ₹) of 72 workers in a factory. Compute the quartile deviation.

Wages (Rs.)	*No. of Workers*	*Wages (Rs.)*	*No. of Workers*
12.5 – 17.5	2	37.5 – 42.5	4
17.5 – 22.5	22	42.5 – 47.5	6
22.5 – 27.5	19	47.5 – 52.5	1
27.5 – 32.5	14	52.5 – 57.5	1
32.5 – 37.5	3		

31. Find the mean deviation of the set of numbers 3, 10, 9, 9, 4, 7, 14, (a) From mean (b) From median.

32. Calculate the mean deviation from the median of the following discrete series and find its coefficient:

Size	2	3	4	5	6	7
Frequency	5	4	10	8	3	2

33. Find the mean deviation from the mean of the following series :

x	56	63	70	77	84	91	98
f	3	6	14	16	13	6	2

34. Calculate the mean deviation from arithmetic mean of the following grouped data:

Class	0–10	10–20	20–30	30–40	40–50
Frequency	2	8	10	3	4

35. Calculate the mean deviation from the median of the following data:

Class	140–150	150–160	160 –170	170–180	180–190	190–200
Frequency	4	6	10	18	9	3

36. Find the standard deviation of the set of numbers: 3, 10, 9, 9, 4, 7, 14

37. Calculate the standard deviation the following distribution :

x	25	35	45	55	65	75	85
f	3	61	132	153	140	51	2

38. Calculate the standard deviation and its coefficient from the following table by short-cut method:

Class	5–15	15–25	25–35	35–45	45–55	55–65
Frequency	15	32	51	78	97	109

39. The no. of workers employed, the mean wages (in Rs.) per month and standard deviation (in Rs.) in each section of a factory are given below. Calculate the mean wages and standard deviation of all the workers taken together.

Section	*No. of Workers Employed*	*Mean Wages (in ₹)*	*Standard Deviation (in ₹)*
A	50	1113	60
B	60	1120	70
C	90	1115	80

40. The mean and standard deviation of 200 items are found to be 60 and 20 respectively. If at the time of calculations, two items were wrongly taken as 3 and 67 instead of 13 and 17, find the correct mean and standard deviation. What is the correct coefficient of variation?

5.9 FURTHER READING

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UNIT 6 CORRELATION

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Structure

6.0 INTRODUCTION

In this unit, you will learn about correlation analysis. This technique looks at indirect relationships and establishes the variables that are most closely associated with a given data or mindset. It is the process of finding how accurately the line fits using the observations. Correlation analysis can be referred to as the statistical tool used to describe the degree to which one variable is related to another. The relationship, if any, is assumed to be a linear one. In fact, the word 'correlation' refers to the relationship or the interdependence between two variables. There are various phenomena that are related to each other. The theory by means of which quantitative connections between two sets of phenomena are determined is called the 'Theory of Correlation'. On the basis of this theory, you can study the comparative changes occurring in two related phenomena and their cause–effect relation can also be examined. Thus, correlation is concerned with the relationship between two related and quantifiable variables and can be positive or negative.

In this unit, you will also learn about regression analysis, which is the mathematical process of using observations to find the line of best fit through the data in order to make estimates and predictions about the behaviour of variables. This technique is used to determine the statistical relationship between two or more variables and to make prediction of one variable on the basis of one or more other variables.

6.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Explain the types and applications of correlation
- Describe the various methods of computing

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- Discuss the significance of Karl Pearson's coefficient
- Describe what rank correlation is
- Discuss the significance of regression analysis

6.2 CORRELATION: CONCEPTS AND ITS APPLICATIONS

Correlation analysis is the statistical tool generally used to describe the degree to which one variable is related to another. The relationship, if any, is usually assumed to be a linear one. This analysis is used quite frequently in conjunction with regression analysis to measure how well the regression line explains the variations of the dependent variable. In fact, the word correlation refers to the relationship or interdependence between two variables. There are various phenomena which are related to each other. For instance, when demand of a certain commodity increases, its price goes up and when its demand decreases, its price comes down. Similarly, with age the height of children, with height the weight of children, with money the supply and the general level of prices go up. Such sort of relationships can as well be noticed for several other phenomena. The theory by means of which quantitative connections between two sets of phenomena are determined is called the 'Theory of Correlation'.

On the basis of the theory of correlation, one can study the comparative changes occurring in two related phenomena and their cause–effect relation can be examined. It should, however, be borne in mind that relationships like 'black cat causes bad luck', 'filled up pitchers result in good fortune' and similar other beliefs of the people cannot be explained by the theory of correlation, since they are all imaginary and are incapable of being justified mathematically. Thus, correlation is concerned with relationship between two related and quantifiable variables. If two quantities vary in sympathy, so that a movement (an increase or decrease) in one tends to be accompanied by a movement in the same or opposite direction in the other and the greater the change in one, the greater is the change in the other, the quantities are said to be correlated. This type of relationship is known as correlation or what is sometimes called, in statistics, as covariation.

For correlation, it is essential that the two phenomena should have cause–effect relationship. If such relationship does not exist then one should not talk of correlation. For example, if the height of the students as well as the height of the trees increases, then one should not call it a case of correlation because the two phenomena, viz., the height of students and the height of trees are not even casually related. However, the relationship between the price of a commodity and its demand, the price of a commodity and its supply, the rate of interest and savings, etc. are examples of correlation, since in all such cases the change in one phenomenon is explained by a change in another phenomenon.

It is appropriate here to mention that correlation in case of phenomena pertaining to natural sciences can be reduced to absolute mathematical term, e.g., heat always increases with light. However, in phenomena pertaining to social sciences, it is often difficult to establish any absolute relationship between two phenomena. Hence, in social sciences, we must take the fact of correlation being established if in a large number of cases, two variables always tend to move in the same or opposite direction.

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Correlation can either be positive or it can be negative. Whether correlation is positive or negative would depend upon the direction in which the variables are moving. If both variables are changing in the same direction, then correlation is said to be positive, but when the variations in the two variables take place in opposite direction, the correlation is termed as negative. This can be explained as follows:

Changes in Independent Variable	*Changes in Dependent Variable*	*Nature of Correlation*
Increase (+)↑	Increase (+)↑	Positive (+)
Decrease (–)↓	Decrease (–)↓	Positive (+)
Increase (+)↑	Decrease (–)↓	Negative (–)
Decrease (–)↓	Increase (+)↑	Negative (–)

Statisticians have developed two measures for describing the correlation between two variables, viz., the coefficient of determination and the coefficient of correlation. These two methods are explained in detail in the following sections.

6.3 METHODS OF COMPUTING

The following are the various methods of computing:

Coefficient of Determination

The **coefficient of determination** (symbolically indicated as r^2, though some people would prefer to put it as R^2) is a measure of the degree of linear association or correlation between two variables, say X and Y, one of which happens to be an independent variable and the other being a dependent variable. This coefficient is based on the following two types of variations:

(a) The variation of the Y values around the fitted regression line, viz., $\Sigma\left(Y-\hat{Y}\right)^2$, technically known as the unexplained variation.

(b) The variation of the Y values around their own mean, viz., $\Sigma\left(Y-\overline{Y}\right)^2$, technically known as the total variation.

If we subtract the unexplained variation from the total variation, we obtain what is known as the explained variation, i.e., the variation explained by the line of regression.
Thus, Explained Variation = (Total variation) – (Unexplained variation)

$$= \Sigma\left(Y-\overline{Y}\right)^2 - \Sigma\left(Y-\hat{Y}\right)^2$$

$$= \Sigma\left(\hat{Y}-\overline{Y}\right)^2$$

The Total and Explained as well as Unexplained variations can be shown as given in Figure. 6.1.

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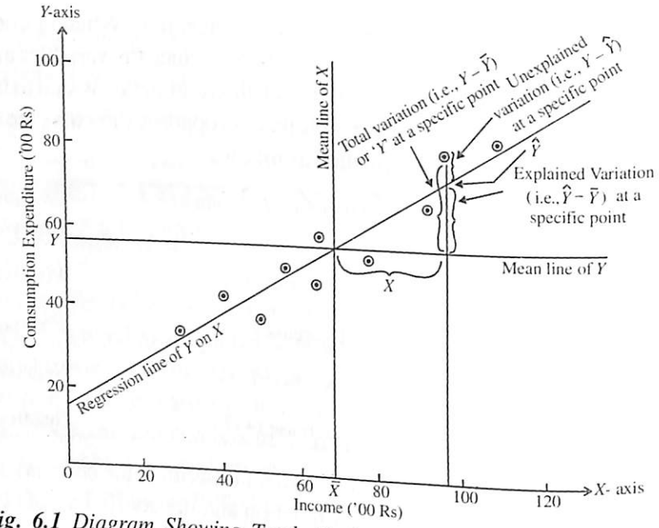

Fig. 6.1 Diagram Showing Total, Explained and Unexplained Variations

Coefficients of determination is that fraction of the total variation of Y which is explained by the regression line. In other words, coefficient of determination is the ratio of explained variation to total variation in the Y variable related to the X variable. Coefficient of determination can be algebraically stated as,

$$r^2 = \frac{\text{Explained variation}}{\text{Total variation}}$$

$$= \frac{\Sigma\left(\hat{Y}-\bar{Y}\right)^2}{\Sigma\left(Y-\bar{Y}\right)^2}$$

Alternatively, r^2 can also be stated as,

$$r^2 = 1 - \frac{\text{Explained variation}}{\text{Total variation}}$$

$$= 1 - \frac{\Sigma\left(\hat{Y}-\bar{Y}\right)^2}{\Sigma\left(Y-\bar{Y}\right)^2}$$

Interpreting r^2

The coefficient of determination can have a value ranging from 0 – 1. The value of 1 can occur only if the unexplained variation is 0, which simply means that all the data points in the Scatter diagram fall exactly on the regression line. For a 0 value to occur, $\Sigma(Y-\bar{Y})^2 = \Sigma(Y-\hat{Y})^2$, which simply means that X tells us nothing about Y and hence there is no regression relationship between X and Y variables. Values between 0 and 1 indicate the 'Goodness of fit' of the regression line to the sample data. The higher the value of r^2, the better the fit. In other words, the value of r^2 will lie somewhere between 0 and 1. If r^2 has a 0 value then it indicates no correlation, but if it has a value equal to 1 then it indicates that there is perfect correlation and as such the regression line is a perfect estimator. However, in most cases, the value of r^2 will lie somewhere between these two extremes of 1 and 0. One should remember that r^2 close to 1 indicates a strong correlation between X and Y, while an r^2 near 0 means there is little correlation between these two variables. r^2 value can as well be interpreted by looking at the amount of the variation in Y, the dependant variable, that is explained by the regression line. Supposing, we get a value of $r^2 = 0.925$ then this would mean that the variations in independent variable (say X) would explain 92.5 per cent of the variation in the dependent variable (say Y). If r^2 is close to 1 then it indicates that the regression equation explains most of the variations in the dependent variable (see Example 1).

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Example 1: Calculate the coefficient of determination (r^2) using the provided data. Calculate and analyse the result.

Observations	1	2	3	4	5	6	7	8	9	10
Income (X) ('00 ₹)	41	65	50	57	96	94	110	30	79	65
Consumption Expenditure (Y) ('00 ₹)	44	60	39	51	80	68	84	34	55	48

Solution:

r^2 can be worked out as follows:

Since, $$r^2 = 1 - \frac{\text{Unexplained variation}}{\text{Total variation}} = 1 - \frac{\Sigma\left(Y-\hat{Y}\right)^2}{\Sigma\left(Y-\bar{Y}\right)^2}$$

As, $\Sigma\left(Y-\bar{Y}\right)^2 = \Sigma Y^2 = \left(\Sigma Y^2 - n\bar{Y}^2\right)$, we can write,

$$r^2 = 1 - \frac{\Sigma\left(Y-\hat{Y}\right)^2}{\Sigma Y^2 - n\bar{Y}^2}$$

Calculating and putting the various values, we have the following equation:

$$= 1 - \frac{260.54}{34223 - 10(56.3)^2} = 1 - \frac{260.54}{2526.10} = 0.897$$

Analysis of Result: The regression equation used to calculate the value of the coefficient of determination (r^2) from the sample data shows that, about 90 per cent of the variations in consumption expenditure can be explained. In other words, it means that the variations in income explain about 90 per cent of variations in consumption expenditure.

Observation	1	2	3	4	5	6	7	8	9	10
Income (X) ('00 ₹)	41	65	50	57	96	94	110	30	79	65
Consumption Expenditure (Y) ('00 ₹)	44	60	39	51	80	68	84	34	55	48

Properties of Correlation Coefficient

The coefficient of correlation, symbolically denoted by 'r', is another important measure to describe how well one variable is explained by another. It measures the degree of relationship between the two casually related variables. The value of this coefficient can never be more than +1 or less than –1. Thus, +1 and –1 are the limits of this coefficient. For a unit change in independent variable, if there happens to be a constant change in the dependent variable in the same direction, then the value of the coefficient will be +1 indicative of the perfect positive correlation; but if such a change occurs in the opposite direction, the value of the coefficient will be –1, indicating the perfect negative correlation. In practical life, the possibility of obtaining either a perfect positive or perfect negative

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correlation is very remote particularly in respect of phenomena concerning social sciences. If the coefficient of correlation has a zero value then it means that there exists no correlation between the variables under study.

There are several methods of finding the coefficient of correlation, but the following ones are considered important:

(a) Coefficient of correlation by the method of least squares.

(b) Coefficient of correlation using simple regression coefficients.

(c) Coefficient of correlation through product moment method or Karl Pearson's coefficient of correlation.

Whichever of these three methods we adopt, we get the same value of *r.*

6.3.1 Method of Least Squares

Under this method, first, the estimating equation is obtained using the least square method of simple regression analysis. The equation is worked out as,

$$\hat{Y} = a + bX_i$$

$$\text{Total variation} = \Sigma\left(Y - \overline{Y}\right)^2$$

$$\text{Unexplained variation} = \Sigma\left(Y - \hat{Y}\right)^2$$

$$\text{Explained variation} = \Sigma\left(\hat{Y} - \overline{Y}\right)^2$$

Then, by applying the following formulae, we can find the value of the coefficient of correlation as,

$$r = \sqrt{r^2} = \sqrt{\frac{\text{Explained variation}}{\text{Total variation}}}$$

$$= \sqrt{1 - \frac{\text{Unexplained variation}}{\text{Total variation}}}$$

$$= \sqrt{1 - \frac{\Sigma\left(Y - \hat{Y}\right)^2}{\Sigma\left(Y - \overline{Y}\right)^2}}$$

This clearly shows that the coefficient of correlation happens to be the square root of the coefficient of determination.

Short-cut formula for finding the value of '*r*' by the method of least squares can be repeated and readily written as,

$$r = \sqrt{\frac{a\Sigma Y + b\Sigma XY - n\overline{Y}^2}{\Sigma Y^2 - n\overline{Y}^2}}$$

Where,

a = *Y*-intercept

b = Slope of the estimating equation

X = Values of the independent variable

Y = Values of dependent variable

$\overline{Y}$ = Mean of the observed values of Y

n = Number of items in the sample (i.e., pairs of observed data)

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The plus (+) or the minus (–) sign of the coefficient of correlation worked out by the method of least squares, is related to the sign of '*b*' in the estimating equation, viz., $\hat{Y} = a + bX_i$. If '*b*' has a minus sign, the sign of '*r*' will also be minus, but if '*b*' has a plus sign, then the sign of '*r*' will also be plus. The value of '*r*' indicates the degree along with the direction of the relationship between the two variables *X* and *Y*.

6.3.2 Simple Regression Coefficients

Under this method, the estimating equation of *Y* and the estimating equation of *X* is worked out using the method of least squares. From these estimating equations we find the regression coefficient of *X* on *Y*, i.e., the slope of the estimating equation of *X* (symbolically written as b_{XY}) and this happens to be equal to $r\frac{\sigma_X}{\sigma_Y}$ and similarly, we find the regression coefficient of *Y* on *X*, i.e., the slope of the estimating equation of *Y* (symbolically written as b_{YX}) and this happens to be equal to $r\frac{\sigma_Y}{\sigma_X}$. For finding '*r*', the square root of the product of these two regression coefficients are worked out as[1]

$$r = \sqrt{b_{XY}.b_{YX}}$$

$$= \sqrt{r\frac{\sigma_X}{\sigma_Y}.r\frac{\sigma_Y}{\sigma_X}}$$

$$= \sqrt{r^2} = r$$

As stated earlier, the sign of '*r*' will depend upon the sign of the regression coefficients. If they have minus sign, then '*r*' will take minus sign but the sign of '*r*' will be plus if regression coefficients have plus sign.

Other Measures

Two other measures are often talked about along with the coefficients of determinations and that of correlation. These are as follows:

(a) Coefficient of Non-Determination: Instead of using coefficient of determination, sometimes coefficient of nondetermination is used. Coefficient of non-determination (denoted by k^2) is the ratio of unexplained variation to total variation in the *Y* variable related to the *X* variable. Algebrically,

$$k^2 = \frac{\text{Unexplained variation}}{\text{Total variation}} = \frac{\Sigma\left(Y - \hat{Y}\right)^2}{\Sigma\left(Y - \overline{Y}\right)^2}$$

1. The short-cut formulae to workout b_{XY} and b_{YX}:

$$b_{XY} = \frac{\Sigma XY - n\overline{X}\overline{Y}}{\Sigma Y^2 - n\overline{Y}^2}$$

and

$$b_{YX} = \frac{\Sigma XY - n\overline{X}\overline{Y}}{\Sigma X^2 - n\overline{X}^2}$$

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Concerning the data of Example 1, coefficient of nondetermination will be calculated as follows:

$$k^2 = \frac{260.54}{2526.10} = 0.103$$

The value of k^2 shows that about 10 per cent of the variation in consumption expenditure remains unexplained by the regression equation we had worked out, viz., $\hat{Y} = 14.000 + 0.616X_i$. In simple terms, this means that variable other than X is responsible for 10 per cent of the variations in the dependent variable Y in the given case.

Coefficient of non-determination can as well be worked out as,

$$k^2 = 1 - r^2$$

Accordingly for Example 1, it will be equal to 1–0.897 = 0.103.

Note: Always remember that $r^2 + k^2 = 1$.

(b) Coefficient of Alienation: Based on k^2, we can work out one more measure, namely the coefficient of alienation, symbolically written as 'k'. Thus, coefficient of alienation, i.e., 'k' $= \sqrt{k^2}$.

Unlike $r + k^2 = 1$, the sum of 'r' and 'k' will not be equal to 1 unless one of the two coefficients is 1 and in this case the remaining coefficients must be zero. In all other cases, 'r' + 'k' > 1. Coefficient of alienation is not a popular measure from a practical point of view and is used very rarely.

6.4 KARL PEARSON'S COEFFICIENT

Karl Pearson's method is the most widely used method of measuring the relationship between two variables. This coefficient is based on the following assumptions:

(a) There is a linear relationship between the two variables, which means that a straight line would be obtained if the observed data is plotted on a graph.

(b) The two variables are casually related, which means that one of the variables is independent and the other one is dependent.

(c) A large number of independent causes operate on both the variables so as to produce a normal distribution.

According to Karl Pearson, 'r' can be worked out as,

$$r = \frac{\sum XY}{n\sigma_X \sigma_Y}$$

Where,

$X = (X - \bar{X})$

$Y = (Y - \bar{Y})$

σ_X = Standard deviation of X series and is equal to $\sqrt{\frac{\sum X^2}{n}}$

σ_Y = Standard deviation of Y series and is equal to $\sqrt{\frac{\sum Y^2}{n}}$

n = Number of pairs of X and Y observed

NOTES

A short-cut formula, known as the Product Moment Formula, can be derived from the above stated formula as,

$$r = \frac{\sum XY}{n\sigma_X \sigma_Y}$$

$$= \frac{\sum XY}{\sqrt{\frac{\sum X^2}{n} \cdot \frac{\sum Y^2}{n}}}$$

$$n = \frac{\sum XY}{\sqrt{\sum X^2 \sum Y^2}}$$

The above formulae are based on obtaining true means (viz., $\bar{X}$ and $\bar{Y}$) first and then doing all other calculations. This happens to be a tedious task, particularly if the true means are in fractions. To avoid difficult calculations, we make use of the assumed means in taking out deviations and doing the related calculations. In such a situation, we can use the following formula for finding the value of 'r':[2]

(a) In Case of Ungrouped Data:

$$r = \frac{\frac{\sum dX.dY}{n} - \left(\frac{\sum dX}{n} \cdot \frac{\sum dY}{n}\right)}{\sqrt{\frac{\sum dX^2}{n} - \left(\frac{\sum dX}{n}\right)^2} \cdot \sqrt{\frac{\sum dY^2}{n} - \left(\frac{\sum dY}{n}\right)^2}}$$

$$= \frac{\sum dX.dY - \left(\frac{\sum dX \times \sum dY}{n}\right)}{\sqrt{\sum dX^2 - \frac{(\sum dX)^2}{n}} \sqrt{\sum dY^2 - \frac{(\sum dY)^2}{n}}}$$

Where, $\sum dX = \sum(X - X_A)$ X_A = Assumed average of X

$\sum dY = \sum(Y - Y_A)$ Y_A = Assumed average of Y

2. In case we take assumed mean to be zero for X variable as for Y variable, then our formula will be as,

$$r = \frac{\frac{\sum XY}{n} - \left(\frac{\sum X}{n}\right)\left(\frac{\sum Y}{n}\right)}{\sqrt{\frac{\sum X^2}{n} - \left(\frac{\sum X}{n}\right)^2} \sqrt{\frac{\sum Y^2}{n} - \left(\frac{\sum Y}{n}\right)^2}}$$

or

$$r = \frac{\frac{\sum XY}{n} - \bar{X}\bar{Y}}{\sqrt{\frac{\sum X^2}{n} - \bar{X}^2} \sqrt{\frac{\sum Y^2}{n} - \bar{Y}^2}}$$

$$r = \frac{\sum XY - n\bar{X}\bar{Y}}{\sqrt{\sum X^2 - n\bar{X}^2} \sqrt{\sum Y^2 - n\bar{Y}^2}}$$

NOTES

$$\Sigma dX^2 = \Sigma(X - X_A)^2$$

$$\Sigma dY^2 = \Sigma(Y - Y_A)^2$$

$$\Sigma dX \,.\, dY = \Sigma(X - X_A)\,(Y - Y_A)$$

n = Number of pairs of observations of X and Y

(b) In Case of Grouped Data:

$$r = \frac{\frac{\Sigma fdX.dY}{n} - \left(\frac{\Sigma fdX}{n} \cdot \frac{\Sigma fdY}{n}\right)}{\sqrt{\frac{\Sigma fdX^2}{n} - \left(\frac{\Sigma fdX}{n}\right)^2}\sqrt{\frac{\Sigma fdY^2}{n} - \left(\frac{\Sigma fdY}{n}\right)^2}}$$

or

$$r = \frac{\Sigma fdX.dY - \left(\frac{\Sigma fdX.\Sigma fdY}{n}\right)}{\sqrt{\Sigma fdX^2 - \left(\frac{\Sigma fdX}{n}\right)^2}\sqrt{\Sigma fdY^2 - \left(\frac{\Sigma fdY}{n}\right)^2}}$$

Where, $\Sigma fdX.dY = 0\Sigma f\,(X - X_A)\,(Y - Y_A)$

$\Sigma fdX = \Sigma f\,(X - X_A)$

$\Sigma fdY = \Sigma f\,(Y - Y_A)$

$\Sigma fdY^2 = \Sigma f\,(Y - Y_A)^2$

$\Sigma fdX^2 = \Sigma f\,(X - X_A)^2$

n = Number of pairs of observations of X and Y

Probable Error (P.E.) of the Coefficient of Correlation

Probable Error (P.E.) of r is very useful in interpreting the value of r and is worked out for Karl Pearson's coefficient of correlation as,

$$\text{P.E.} = 0.6745\frac{1-r^2}{\sqrt{n}}$$

If r is less than its P.E., it is not at all significant. If r is more than P.E., there is correlation. If r is more than six times its P.E. and greater than ± 0.5, then it is considered significant. Let us consider Example 2.

Example 2: From the following data calculate 'r' between X and Y applying the following three methods:

(a) The method of least squares.

(b) The method based on regression coefficients.

(c) The product moment method of Karl Pearson.

Verify the obtained result of any one method with that of another.

X	1	2	3	4	5	6	7	8	9
Y	9	8	10	12	11	13	14	16	15

NOTES

Solution:

Let us develop the following table for calculating the value of 'r':

X	Y	X^2	Y^2	XY
1	9	1	81	9
2	8	4	64	16
3	10	9	100	30
4	12	16	144	48
5	11	25	121	55
6	13	36	169	78
7	14	49	196	98
8	16	64	256	128
9	15	81	225	135
$n = 9$				
$\Sigma X = 45$	$\Sigma Y = 108$	$\Sigma X^2 = 285$	$\Sigma Y^2 = 1356$	$\Sigma XY = 597$

$\therefore \quad \bar{X} = 5; \qquad \bar{Y} = 12$

(a) Coefficient of correlation by the method of least squares is worked out as follows:
First find out the estimating equation,

$$\hat{Y} = a + bX_i$$

Where, $$b = \frac{\Sigma XY - n\bar{X}\bar{Y}}{\Sigma X^2 - n\bar{X}^2}$$

$$= \frac{597 - 9(5)(12)}{285 - 9(25)} = \frac{597 - 540}{285 - 225} = \frac{57}{60} = 0.95$$

and $$a = \bar{Y} - b\bar{X}$$

$$= 12 - 0.95(5) = 12 - 4.75 = 7.25$$

Hence, $$\hat{Y} = 7.25 + 0.95X_i$$

Now 'r' can be worked out as follows by the method of least squares,

$$r = \sqrt{1 - \frac{\text{Unexplained variation}}{\text{Total variation}}}$$

$$= \sqrt{1 - \frac{\Sigma\left(Y - \hat{Y}\right)^2}{\Sigma\left(Y - \bar{Y}\right)^2}} = \sqrt{\frac{\Sigma\left(\hat{Y} - \bar{Y}\right)^2}{\Sigma\left(Y - \bar{Y}\right)^2}}$$

$$= \sqrt{\frac{a\Sigma Y + b\Sigma XY - n\bar{Y}^2}{\Sigma Y^2 - n\bar{Y}^2}}$$

NOTES

This is as per short-cut formula,

$$r = \sqrt{\frac{7.25(108)+0.95(597)-9(12)^2}{1356-9(12)^2}}$$

$$= \sqrt{\frac{783+567.15-1296}{1356-1296}}$$

$$= \sqrt{\frac{54.15}{60}} = \sqrt{0.9025} = 0.95$$

(b) Coefficient of correlation by the method based on regression coefficients is worked out as,

$\because$ Regression coefficients of Y on X,

i.e., $$b_{YX} = \frac{\sum XY - n\overline{XY}}{\sum X^2 - n\overline{X}^2}$$

$$= \frac{597-9\times5\times12}{285-9(5)^2} = \frac{597-540}{285-225} = \frac{57}{60}$$

Regression coefficient of X on Y,

i.e., $$b_{XY} = \frac{\sum XY - n\overline{XY}}{\sum Y^2 - n\overline{Y}^2}$$

$$= \frac{597-9\times5\times12}{1356-9(12)^2} = \frac{597-540}{1356-1296} = \frac{57}{60}$$

Hence, $$r = \sqrt{b_{YX}.b_{XY}}$$

$$= \sqrt{\frac{57}{60}\times\frac{57}{60}} = \frac{57}{60} = 0.95$$

(c) Coefficient of correlation by the product moment method of Karl Pearson is worked out as,

$$r = \frac{\sum XY - n\overline{XY}}{\sqrt{\sum X^2 - n\overline{X}^2}\sqrt{\sum Y^2 - n\overline{Y}^2}}$$

$$= \frac{597-9(5)(12)}{\sqrt{285-9(5)^2}\sqrt{1356-9(12)^2}}$$

$$= \frac{597-540}{\sqrt{285-225}\sqrt{1356-1296}} = \frac{57}{\sqrt{60}\sqrt{60}} = \frac{57}{60} = 0.95$$

Hence, we get the value of $r = 0.95$. We get the same value by applying the other two methods also. Therefore, whichever method we apply, the results will be the same.

6.4.1 Product Moment Correlation

In statistics, the Pearson Product Moment Correlation Coefficient is a measure of the correlation (linear dependence) between two variables X and Y, giving a value between +1 and −1 inclusive. It is sometimes referred the PPMCC or PCC or Pearson's r. It is widely used in the sciences as a measure of the strength of linear dependence between two variables. It was developed by Karl Pearson from a related idea introduced by Francis Galton in the 1880s.

NOTES

Pearson's correlation coefficient between two variables is defined as the covariance of the two variables divided by the product of their standard deviations. The form of the definition involves a 'product moment', i.e., the mean (the first moment about the origin) of the product of the mean adjusted random variables; hence the modifier *product moment* in the name.

For a Population

Pearson's correlation coefficient when applied to a population is commonly represented by the Greek letter ρ (rho) and may be referred to as the *population correlation coefficient* or the *population Pearson correlation coefficient*. The formula for ρ is:

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X\sigma_Y} = \frac{E\left[(X-\mu_X)(Y-\mu_Y)\right]}{\sigma_X\sigma_Y}$$

For a Sample

Pearson's correlation coefficient when applied to a sample is commonly represented by the letter r and may be referred to as the *sample correlation coefficient* or the *sample Pearson correlation coefficient*. We can obtain a formula for r by substituting estimates of the covariances and variances based on a sample into the above formula. That formula for r is:

$$r = \frac{\sum_{i=1}^{n}\left(X_i-\overline{X}\right)\left(Y_i-\overline{Y}\right)}{\sqrt{\sum_{i=1}^{n}\left(X_i-\overline{X}\right)^2}\sqrt{\sum_{i=1}^{n}\left(Y_i-\overline{Y}\right)^2}}$$

An equivalent expression gives the correlation coefficient as the mean of the products of the standard scores. Based on a sample of paired data (X_i, Y_i), the sample Pearson correlation coefficient is as follows:

$$r = \frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{X_i-\overline{X}}{s_X}\right)\left(\frac{Y_i-\overline{Y}}{s_Y}\right)$$

Where,

$\frac{X_i-\overline{X}}{s_X}$, $\overline{X}$, and s_X are the standard score, sample mean, and sample standard deviation, respectively.

How to Calculate Product Moment Correlation Coefficient?

The product moment correlation coefficient allows you to work out the linear dependence of two variables (referred to as X and Y). Let us consider an example, suppose you are the owner of a restaurant. You record the time of every 10th customer stayed in your restaurant (X in minutes) and the amount spend (Y, in rupees). If it is considered that the longer time the customer stayed the bigger is the amount spend, then this would be a positive correlation. Or it can also be considered in the other way, i.e., the richer the client the lesser time he takes for lunch in restaurant, then this would be a negative correlation. Pearson Product-Moment Correlation Coefficient or PMCC can be calculated to find the correlation in a situation.

NOTES

Step 1: Remove Incomplete Pairs: After removing incomplete pairs, use only those observations where *both* X and Y are known. However, do not exclude observations just because one of the values equals zero.

Step 2: Summarize the Data into the Values needed for the Calculation: These are:

- n: The number of data.
- ΣX: The sum of all the X values.
- ΣX^2: The sum of the squares of the X values.
- ΣY: The sum of all the Y values.
- ΣY^2: The sum of the squares of the Y values.
- ΣXY: The sum of each X value multiplied by its corresponding Y value.

Step 3: Calculate S_{XY}, S_{XX} and S_{YY} using these values:

- $S_{XY} = \Sigma XY - (\Sigma XY \div n)$
- $S_{XX} = \Sigma X^2 - (\Sigma X \; \Sigma X \div n)$
- $S_{XY} = \Sigma Y^2 - (\Sigma Y \; \Sigma Y \div n)$

Step 4: Insert these Values into the Equation below to Calculate the Product Moment Correlation Coefficient (r): The value should be between 1 and –1.

$$r = \frac{S_{xy}}{\sqrt{S_{xy} S_{yy}}}$$

- If a value is close to 1 implies strong positive correlation. The higher the X, the higher the Y.
- If a value close to 0 implies little or no correlation.
- If a value close to –1 implies strong negative correlation. The higher the X, the lower the Y.

6.5 RANK CORRELATION

If observations on two variables are given in the form of ranks and not as numerical values, it is possible to compute what is known as rank correlation between the two series.

The **rank correlation**, written as ρ, is a descriptive index of agreement between ranks over individuals. It is the same as the ordinary coefficient of correlation computed on ranks, but its formula is simpler.

$$\rho = 1 - \frac{6\Sigma D_i^2}{n(n^2 - 1)}$$

Here, n is the number of observations and D_i, the positive difference between ranks associated with the individuals i.

Like r, the rank correlation lies between –1 and +1. Consider Examples 3 and 4 for better understanding

NOTES

Example 3: The ranks given by two judges to 10 individuals are as follows:

Individual	Rank given by Judge I x	Judge II y	$D = x - y$	D^2
1	1	7	6	36
2	2	5	3	9
3	7	8	1	1
4	9	10	1	1
5	8	9	1	1
6	6	4	2	4
7	4	1	3	9
8	3	6	3	9
9	10	3	7	49
10	5	2	3	9
				$\Sigma D^2 = 128$

Solution:

The rank correlation is given by,

$$\rho = 1 - \frac{6\Sigma D^2}{n^3 - n} = 1 - \frac{6 \times 128}{10^3 - 10} = 1 - 0.776 = 0.224$$

The value of $\rho = 0.224$ shows that the agreement between the judges is not high.

Example 4: Consider Example 3 and compute r and compare.

Solution:

The simple coefficient of correlation r for the previous data is calculated as follows:

x	y	x^2	y^2	xy
1	7	1	49	7
2	5	4	25	10
7	8	49	64	56
9	10	81	100	90
8	9	64	81	72
6	4	36	16	24
4	1	16	1	4
3	6	9	36	18
10	3	100	9	30
5	2	25	4	10
$\Sigma x = 55$	$\Sigma y = 55$	$\Sigma x^2 = 385$	$\Sigma y^2 = 385$	$\Sigma xy = 321$

$$r = \frac{321 - 10 \times \frac{55}{10} \times \frac{55}{10}}{\sqrt{385 - 10 \times \left(\frac{55}{10}\right)^2} \sqrt{385 - 10 \times \left(\frac{55}{10}\right)^2}} = \frac{18.5}{\sqrt{82.5 \times 82.5}} = \frac{18.5}{82.5} = 0.224$$

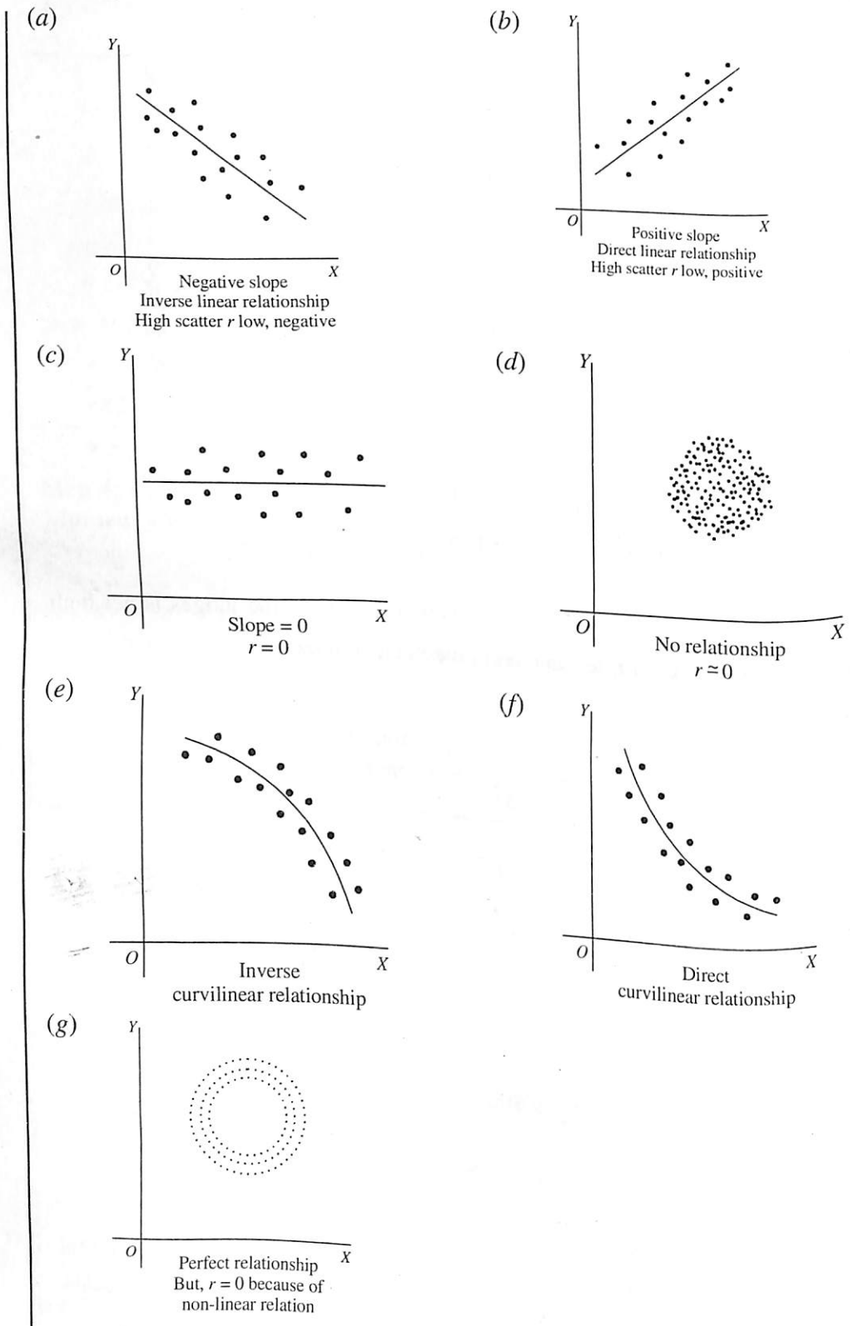
This shows that the Spearman ρ for any two sets of ranks is the same as the Pearson r for the set of ranks. However, it is much easier to compute ρ.

Often, the ranks are not given. Instead, the numerical values of observations are given. In such a case, we must attach the ranks to these values to calculate ρ.

NOTES

Example 5: Show by means of diagrams various cases of scatter expressing correlation between x, y.

Solution:



Correlation analysis helps us in determining the degree to which two or more variables are related to each other.

When there are only two variables, we can determine the degree to which one variable is linearly related to the other. Regression analysis helps in determining the pattern of relationship between one or more independent variables and a dependent variable. This is done by an equation estimated with the help of data.

NOTES

6.6 REGRESSION ANALYSIS

The term 'regression' was first used in 1877 by Sir Francis Galton who made a study that showed that the height of children born to tall parents will tend to move back or 'regress' toward the mean height of the population. He designated the word regression as the name of the process of predicting one variable from another variable. He coined the term multiple regression to describe the process by which several variables are used to predict another. Thus, when there is a well-established relationship between variables, it is possible to make use of this relationship in making estimates and to forecast the value of one variable (the unknown or the dependent variable) on the basis of the other variable/s (the known or the independent variable/s). A banker, for example, could predict deposits on the basis of per capita income in the trading area of bank. A marketing manager, may plan his advertising expenditures on the basis of the expected effect on total sales revenue of a change in the level of advertising expenditure. Similarly, a hospital superintendent could project his need for beds on the basis of total population. Such predictions may be made by using regression analysis. An investigator may employ regression analysis to test his theory having the cause and effect relationship. All these explain that regression analysis is an extremely useful tool especially in problems of business and industry involving predictions.

Assumptions in Regression Analysis

While making use of the regression techniques for making predictions, the following are always assumed:

(a) There is an actual relationship between the dependent and independent variables.

(b) The values of the dependent variable are random but the values of the independent variable are fixed quantities without error and are chosen by the experimentor.

(c) There is a clear indication of direction of the relationship. This means that dependent variable is a function of independent variable. (For example, when we say that advertising has an effect on sales, then we are saying that sales has an effect on advertising).

(d) The conditions (that existed when the relationship between the dependent and independent variable was estimated by the regression) are the same when the regression model is being used. In other words, it simply means that the relationship has not changed since the regression equation was computed.

(e) The analysis is to be used to predict values within the range (and not for values outside the range) for which it is valid.

Simple Linear Regression Model

In case of simple linear regression analysis, a single variable is used to predict another variable on the assumption of linear relationship (i.e., relationship of the type defined by $Y = a + bX$) between the given variables. The variable to be predicted is called the dependent variable and the variable on which the prediction is based is called the independent variable.

NOTES

Simple linear regression model[3] (or the Regression Line) is stated as,

$$Y_i = a + bX_i + e_i$$

Where, Y_i = The dependent variable

X_i = The independent variable

e_i = Unpredictable random element (usually called residual or error term)

(a) *a* represents the *Y*-intercept, i.e., the intercept specifies the value of the dependent variable when the independent variable has a value of zero. (However, this term has practical meaning only if a zero value for the independent variable is possible).

(b) *b* is a constant, indicating the slope of the regression line. Slope of the line indicates the amount of change in the value of the dependent variable for a unit change in the independent variable.

If the two constants (viz., *a* and *b*) are known, the accuracy of our prediction of *Y* (denoted by $\hat{Y}$ and read as *Y*-hat) depends on the magnitude of the values of e_i. If in the model, all the e_i tend to have very large values then the estimates will not be very good, but if these values are relatively small, then the predicted values ($\hat{Y}$) will tend to be close to the true values (Y_i).

Estimating the Intercept and Slope of the Regression Model (or Estimating the Regression Equation)

The two constants or the parameters viz., '*a*' and '*b*' in the regression model for the entire population or universe are generally unknown and as such are estimated from sample information. The following are the two methods used for estimation:

(a) Scatter diagram method

(b) Least squares method

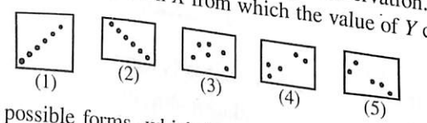
1. Scatter Diagram Method

This method makes use of the Scatter diagram also known as Dot diagram. *Scatter diagram*[4] is a diagram representing two series with the known variable, i.e., independent variable plotted on the *X*-axis and the variable to be estimated, i.e., dependent variable to be plotted on the *Y*-axis on a graph paper (see Figure 6.2) to get the follow ing information illustrated in Table 6.1:

3. Usually, the estimate of *Y* denoted by $\hat{Y}$ is written as,

$$\hat{Y} = a + bX_i$$

on the assumption that the random disturbance to the system averages out or has an expected value of zero (i.e., $e = 0$) for any single observation. This regression model is known as the Regression line of *Y* on *X* from which the value of *Y* can be estimated for the given value of *X*.

4. 

Five possible forms, which Scatter diagram may assume has been depicted in the above five diagrams. Diagram (1) is indicative of perfect positive relationship. Diagram (2) shows perfect negative relationship. Diagram (3) shows no relationship, Diagram (4) shows positive relationship and *Diagram* (5) shows negative relationship between the two variables under consideration.

NOTES

Table 6.1 *Table Derived from Scatter Diagram*

Income *X* *(Hundreds of Rupees)*	*Consumption Expenditure* *Y* *(Hundreds of Rupees)*
41	44
65	60
50	39
57	51
96	80
94	68
110	84
30	34
79	55
65	48

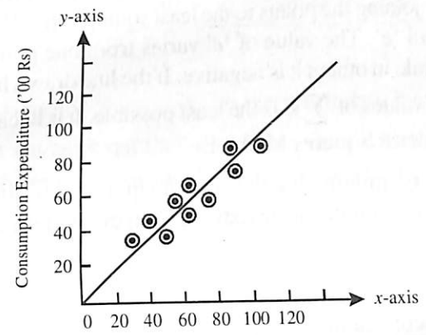


Fig. 6.2 *Scatter Diagram*

The scatter diagram by itself is not sufficient for predicting values of the dependent variable. Some formal expression of the relationship between the two variables is necessary for predictive purposes. For the purpose, one may simply take a ruler and draw a straight line through the points in the scatter diagram and this way can determine the intercept and the slope of the said line and then the line can be defined as $\hat{Y} = a + bX_i$, with the help of which we can predict *Y* for a given value of *X*. However, there are shortcomings in this approach. For example, if five different persons draw such a straight line in the same scatter diagram, it is possible that there may be five different estimates of *a* and *b*, especially when the dots are more dispersed in the diagram. Hence, the estimates cannot be worked out only through this approach. A more systematic and statistical method is required to estimate the constants of the predictive equation. The least squares method is used to draw the best fit line.

2. Least Square Method

The least squares method of fitting a line (the line of best fit or the regression line) through the scatter diagram is a method which minimizes the sum of the squared vertical deviations from the fitted line. In other words, the line to be fitted will pass through the points of the scatter diagram in such a way that the sum of the squares of the vertical deviations of these points from the line will be a minimum.

The meaning of the least squares criterion can be easily understood through Figure 6.3, where the earlier Figure 6.2 in scatter diagram has been reproduced along with a line which represents the least squares line to fit the data.

NOTES

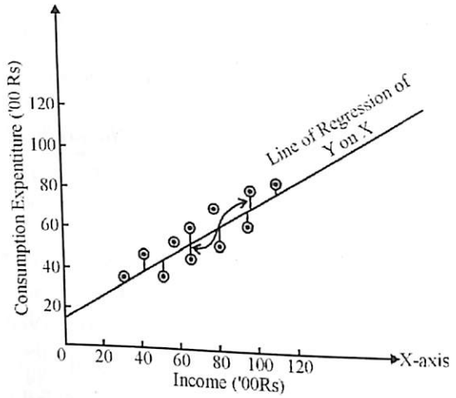


Fig. 6.3 Scatter Diagram, Regression Line and Short Vertical Lines Representing 'e'

In Figure 6.3, the vertical deviations of the individual points from the line are shown as the short vertical lines joining the points to the least squares line. These deviations will be denoted by the symbol '*e*'. The value of '*e*' varies from one point to another. In some cases it is positive, while in others it is negative. If the line drawn happens to be the least squares line, then the values of $\sum e_i$ is the least possible. It is because of this feature the method is known as Least Squares Method.

Why we insist on minimizing the sum of squared deviations is a question that needs explanation. If we denote the deviations from the actual value Y to the estimated value $\hat{Y}$ as $(Y - \hat{Y})$ or e_i, it is logical that we want the $\Sigma(Y - \hat{Y})$ or $\sum_{i=1}^{n} e_i$, to be as small as possible. However, mere examining $\Sigma(Y - \hat{Y})$ or $\sum_{i=1}^{n} e_i$, is inappropriate, since any e_i can be positive or negative. Large positive values and large negative values could cancel one another. However, large values of e_i regardless of their sign, indicate a poor prediction. Even if we ignore the signs while working out $\sum_{i=1}^{n} |e_i|$, the difficulties may continue. Hence, the standard procedure is to eliminate the effect of signs by squaring each observation. Squaring each term accomplishes two purposes, viz., (i) it magnifies (or penalizes) the larger errors, and (ii) it cancels the effect of the positive and negative values (since a negative error when squared becomes positive). The choice of minimizing the squared sum of errors rather than the sum of the absolute values implies that there are many small errors rather than a few large errors. Hence, in obtaining the regression line, we follow the approach that the sum of the squared deviations be minimum and on this basis work out the values of its constants viz., '*a*' and '*b*' also known as the intercept and the slope of the line. This is done with the help of the following two normal equations:[5]

5. If we proceed centering each variable, i.e., setting its origin at its mean, then the two equations will be as under:

$$\Sigma Y = na + b\Sigma X$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

But since ΣY and ΣX will be zero, the first equation and the first term of the second equation will disappear and we shall simply have the following equations:

$$\Sigma XY = b\Sigma X^2$$
$$b = \Sigma XY/\Sigma X^2$$

The value of '*a*' can then be worked out as:

$$a = \bar{Y} - b\bar{X}$$

$$\Sigma Y = na + b\Sigma X$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

In these two equations, '*a*' and '*b*' are unknowns and all other values viz., ΣX, ΣY, ΣX^2, ΣXY, are the sum of the products and cross products to be calculated from the sample data, and '*n*' means the number of observations in the sample.

Example 6 explains the Least squares method.

Example 6: Fit a regression line $\hat{Y} = a + bX_i$ by the method of Least squares to the following sample information.

Observations	1	2	3	4	5	6	7	8	9	10
Income (*X*) ('00 ₹)	41	65	50	57	96	94	110	30	79	65
Consumption Expenditure (*Y*) ('00 ₹)	44	60	39	51	80	68	84	34	55	48

Solution:

We are to fit a regression line $\hat{Y} = a + bX_i$ to the given data by the method of Least Squares. Accordingly, we work out the '*a*' and '*b*' values with the help of the normal equations as stated above and also for the purpose, work out ΣX, ΣY, ΣXY, ΣX^2 values from the given sample information table on summations for regression equation.

Summations for Regression Equation

Observations	*Income X* ('00 ₹)	*Consumption Expenditure Y* ('00 ₹)	*XY*	X^2	Y^2
1	41	44	1804	1681	1936
2	65	60	3900	4225	3600
3	50	39	1950	2500	1521
4	57	51	2907	3249	2601
5	96	80	7680	9216	6400
6	94	68	6392	8836	4624
7	110	84	9240	12100	7056
8	30	34	1020	900	1156
9	79	55	4345	6241	3025
10	65	48	3120	4225	2304
$n = 10$	$\Sigma X = 687$	$\Sigma Y = 563$	$\Sigma XY = 42358$	$\Sigma X^2 = 53173$	$\Sigma Y^2 = 34223$

Putting the values in the required normal equations we have,

$$563 = 10a + 687b$$
$$42358 = 687a + 53173b$$

Solving these two equations for a and b we obtain,

$$a = 14.000 \quad \text{and} \quad b = 0.616$$

Hence, the equation for the required regression line is,

$$\hat{Y} = a + bX_i$$

or,

$$\hat{Y} = 14.000 + 0.616X_i$$

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This equation is known as the regression equation of Y on X from which Y values can be estimated for given values of X variable.[6]

Checking the Accuracy of Equation

After finding the regression line, one can check its accuracy also. The method to be used for the purpose follows from the mathematical property of a line fitted by the method of least squares, viz., the individual positive and negative errors must sum to zero. In other words, using the estimating equation one must find out whether the term $\Sigma(Y-\hat{Y})$ is zero and if this is so, then one can reasonably be sure that he has not committed any mistake in determining the estimating equation.

The Problem of Prediction

When we talk about prediction or estimation, we usually imply that if the relationship $Y_i = a + bX_i + e_i$ exists, then the regression equation, $\hat{Y} = a + bX_i$ provides a base for making estimates of the value of Y which will be associated with particular values of X. In Example 6, we worked out the regression equation for the income and consumption data as,

$$\hat{Y} = 14.000 + 0.616X_i$$

On the basis of this equation, we can make a point estimate of Y for any given value of X. Suppose we wish to estimate the consumption expenditure of individuals with income of ₹ 10,000. We substitute $X = 100$ for the same in our equation and get an estimate of consumption expenditure as,

$$\hat{Y} = 14.000 + 0.616(100) = 75.60$$

Thus, the regression relationship indicates that individuals with ₹ 10,000 of income may be expected to spend approximately ₹ 7,560 on consumption. However, this is only an expected or an estimated value and it is possible that actual consumption expenditure of the individual with that income may deviate from this amount and if so, then our estimate will be an error, the likelihood of which will be high if the estimate is applied to any one individual. The interval estimate method is considered better and it states an interval in which the expected consumption expenditure may fall. Remember that the wider the interval, the greater the level of confidence we can have, but the width of the interval (or what is technically known as the precision of the estimate) is associated with a specified level of confidence and is dependent on the variability (consumption expenditure in our case) found in the sample. This variability is measured by the standard deviation of the error term, 'e', and is popularly known as the standard error of the estimate.

Standard Error of the Estimate

Standard error of estimate is a measure developed by statisticians for measuring the reliability of the estimating equation. Like the standard deviation, the Standard Error (S.E.) of $\hat{Y}$ measures the variability or scatter of the observed values of Y around the regression line. Standard Error of Estimate (S.E. of $\hat{Y}$) is worked out as,

6. It should be pointed out that the equation used to estimate the Y variable values from values of X should not be used to estimate the values of X variable from given values of Y variable. Another regression equation (known as the regression equation of X on Y of the type $X = a + bY$) that reverses the two value should be used if it is desired to estimate X from value of Y.

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$$\text{S.E. of } \hat{Y} \text{ (or } S_e) = \sqrt{\frac{\sum(Y-\hat{Y})^2}{n-2}} = \sqrt{\frac{\sum e^2}{n-2}}$$

where,

S.E. of $\hat{Y}$ (or S_e) = Standard error of the estimate
Y = Observed value of Y
$\hat{Y}$ = Estimated value of Y
e = The error term = $(Y-\hat{Y})$
n = Number of observations in the sample

Note: In the above formula, $n-2$ is used instead of n because of the fact that two degrees of freedom are lost in basing the estimate on the variability of the sample observations about the line with two constants viz., 'a' and 'b' whose position is determined by those same sample observations.

The square of the S_e, also known as the variance of the error term, is the basic measure of reliability. The larger the variance, the more significant are the magnitudes of the e's and the less reliable is the regression analysis in predicting the data.

Interpreting the Standard Error of Estimate and Finding the Confidence Limits for the Estimate in Large and Small Samples

The larger the S.E. of estimate (SE_e), the greater happens to be the dispersion, or scattering, of given observations around the regression line. However, if the S.E. of estimate happens to be zero, then the estimating equation is a 'perfect' estimator (i.e., cent per cent correct estimator) of the dependent variable.

(a) In case of large samples, i.e., where $n > 30$ in a sample, it is assumed that the observed points are normally distributed around the regression line and we may find that,

- 68 per cent of all points lie within $\hat{Y} \pm 1$ SE_e limits.
- 95.5 per cent of all points lie within $\hat{Y} \pm 2$ SE_e limits.
- 99.7 per cent of all points lie within $\hat{Y} \pm 3$ SE_e limits.

This can be stated as,

- The observed values of Y are normally distributed around each estimated value of $\hat{Y}$ and;
- The variance of the distributions around each possible value of $\hat{Y}$ is the same.

(b) In case of small samples, i.e., where $n \leq 30$ in a sample the 't' distribution is used for finding the two limits more appropriately.

This is done as follows:

Upper limit = $\hat{Y}$ + 't' (SE_e)
Lower limit = $\hat{Y}$ – 't' (SE_e)

Where,
$\hat{Y}$ = The estimated value of Y for a given value of X.
SE_e = The standard error of estimate.
't' = Table value of 't' for given degrees of freedom for a specified confidence level.

NOTES

Some other Details Concerning Simple Regression

Sometimes, the estimating equation of Y also known as the regression equation of Y on X, is written as,

$$\left(\hat{Y}-\overline{Y}\right) = r\frac{\sigma_Y}{\sigma_X}\left(X_i-\overline{X}\right)$$

or,
$$\hat{Y} = r\frac{\sigma_Y}{\sigma_X}\left(X_i-\overline{X}\right)+\overline{Y}$$

Where, r = Coefficient of simple correlation between X and Y

σ_Y = Standard deviation of Y

σ_X = Standard deviation of X

$\overline{X}$ = Mean of X

$\overline{Y}$ = Mean of Y

$\hat{Y}$ = Value of Y to be estimated

X_i = Any given value of X for which Y is to be estimated

This is based on the formula we have used, i.e., $\hat{Y}=a+bX_i$. The coefficient of X_i is defined as,

$$\text{Coefficient of } X_i = b = r\frac{\sigma_Y}{\sigma_X}$$

(Also known as regression coefficient of Y on X or slope of the regression line of Y on X) or b_{YX}.

$$= \frac{\sum XY - n\overline{X}\,\overline{Y}\times\sqrt{\sum Y^2 - n\overline{Y}^2}}{\sqrt{\sum Y^2 - n\overline{Y}^2}\sqrt{\sum X^2 - n\overline{X}^2}\sqrt{\sum X^2 - n\overline{X}^2}}$$

$$= \frac{\sum XY - n\overline{X}\,\overline{Y}}{\sum X^2 - n\overline{X}^2}$$

and
$$a = -r\frac{\sigma_Y}{\sigma_X}\overline{X}+\overline{Y}$$

$$= \overline{Y}-b\overline{X} \qquad \left(\text{since } b = r\frac{\sigma_Y}{\sigma_X}\right)$$

Similarly, the estimating equation of X, also known as the regression equation of X on Y, can be stated as,

$$\left(\hat{X}-\overline{X}\right) = r\frac{\sigma_X}{\sigma_Y}\left(Y-\overline{Y}\right)$$

or
$$\hat{X} = r\frac{\sigma_X}{\sigma_Y}\left(Y-\overline{Y}\right)+\overline{X}$$

and the

Regression coefficient of X on Y (or b_{XY}) $= r\frac{\sigma_X}{\sigma_Y} = \frac{\sum XY - n\overline{X}\,\overline{Y}}{\sum Y^2 - n\overline{Y}^2}$

NOTES

If we are given the two regression equations as stated above, along with the values of 'a' and 'b' constants to solve the same for finding the value of X and Y, then the values of X and Y so obtained, are the mean values of X (i.e., $\overline{X}$) and the mean value of Y (i.e., $\overline{Y}$).

If we are given the two regression coefficients (viz., b_{XY} and b_{YX}), then we can work out the value of coefficient of correlation by just taking the square root of the product of the regression coefficients as shown,

$$r = \sqrt{b_{YX}.b_{XY}}$$

$$= \sqrt{r\frac{\sigma_Y}{\sigma_X}.r\frac{\sigma_X}{\sigma_Y}}$$

$$= \sqrt{r.r} = r$$

The (±) sign of r will be determined on the basis of the sign of the given regression coefficients. If regression coefficients have minus sign then r will be taken with minus (–) sign and if regression coefficients have plus sign then r will be taken with plus (+) sign, (Remember that both regression coefficients will necessarily have the same sign, whether it is minus or plus, for their sign is governed by the sign of coefficient of correlation.) To understand it better, see Examples 7 and 8.

Example 7: Given is the following information:

	$\overline{X}$	$\overline{Y}$
Mean	39.5	47.5
Standard Deviation	10.8	17.8

Simple correlation coefficient between X and Y is = + 0.42.

Find the estimating equation of Y and X.

Solution:

Estimating equation of Y can be worked out as,

$$\because \quad \left(\hat{Y}-\overline{Y}\right) = r\frac{\sigma_Y}{\sigma_X}\left(X_i-\overline{X}\right)$$

or
$$\hat{Y} = r\frac{\sigma_Y}{\sigma_X}\left(X_i-\overline{X}\right)+\overline{Y}$$

$$= 0.42\frac{17.8}{10.8}(X_i-39.5)+47.5$$

$$= 0.69X_i - 27.25 + 47.5$$

$$= 0.69X_i + 20.25$$

Similarly, the estimating equation of X can be worked out as

$$\because \quad \left(\hat{X}-\overline{X}\right) = r\frac{\sigma_X}{\sigma_Y}\left(Y_i-\overline{Y}\right)$$

or
$$\hat{X} = r\frac{\sigma_X}{\sigma_Y}\left(Y_i-\overline{Y}\right)+\overline{X}$$

or
$$= 0.42\frac{10.8}{17.8}(Y_i-47.5)+39.5$$

or
$$= 0.26Y_i - 12.35 + 39.5$$

$$= 0.26Y_i + 27.15$$

NOTES

Example 8: The following is the given data:

Variance of $X = 9$

Regression equations:

$$4X - 5Y + 33 = 0$$

$$20X - 9Y - 107 = 0$$

Find: (a) Mean values of X and Y.

(b) Coefficient of Correlation between X and Y.

(c) Standard deviation of Y.

Solution:

(a) For finding the mean values of X and Y, we solve the two given regression equations for the values of X and Y as follows:

$$4X - 5Y + 33 = 0 \quad ...(1)$$

$$20X - 9Y - 107 = 0 \quad ...(2)$$

If we multiply Equation (1) by 5, we have the following equations:

$$20X - 25Y = -165 \quad ...(3)$$

$$20X - 9Y = 107 \quad ...(2)$$

$$- \quad + \quad -$$

$$-16Y = -272$$

Subtracting Equation (2) from (3)

or $$Y = 17$$

Putting this value of Y in Equation (1) we have,

$$4X = -33 + 5(17)$$

or

$$X = \frac{-33+85}{4} = \frac{52}{4} = 13$$

Hence, $$\bar{X} = 13 \quad \text{and} \quad \bar{Y} = 17$$

(b) For finding the coefficient of correlation, we first presume one of the two given regression equations as the estimating equation of X. Let equation $4X - 5Y + 33 = 0$ be the estimating equation of X, then we have,

and $$\hat{X} = \frac{5Y_i}{4} - \frac{33}{4}$$

From this we can write $b_{XY} = \frac{5}{4}$.

The other given equation is then taken as the estimating equation of Y and can be written as,

$$\hat{Y} = \frac{20X_i}{9} - \frac{107}{9}$$

and from this we can write $b_{YX} = \frac{20}{9}$.

NOTES

If the above equations are correct then r must be equal to,

$$r = \sqrt{5/4 \times 20/9} = \sqrt{25/9} = 5/3 = 1.6$$

which is an impossible equation, since r can in no case be greater than 1. Hence, we change our supposition about the estimating equations and by reversing it, we re-write the estimating equations as,

$$\hat{X} = \frac{9Y_i}{20} + \frac{107}{20}$$

and $$\hat{Y} = \frac{4X_i}{5} + \frac{33}{5}$$

Hence, $$r = \sqrt{9/20 \times 4/5}$$

$$= \sqrt{9/25}$$

$$= 3/5$$

$$= 0.6$$

Since, regression coefficients have plus signs, we take $r = +0.6$.

(c) Standard deviation of Y can be calculated,

$\because$ Variance of $X = 9$ $\quad \therefore$ Standard deviation of $X = 3$

$\because$ $$b_{YX} = r\frac{\sigma_Y}{\sigma_X} = \frac{4}{5} = 0.6\frac{\sigma_Y}{3} = 0.2\sigma_Y$$

Hence, $\sigma_Y = 4$

Alternatively, we can work it out as,

$\because$ $$b_{XY} = r\frac{\sigma_X}{\sigma_Y} = \frac{9}{20} = 0.6\frac{\sigma_Y}{3} = \frac{1.8}{\sigma_Y}$$

Hence, $\sigma_Y = 4$

ACTIVITY

1. Using the various correlation methods discussed in the unit, compute the correlation for the following data:

Person	Height (x)	Self Esteem (y)
1	68	4.1
2	71	4.6
3	62	3.8
4	75	4.4
5	58	3.2
6	60	3.1

2. Two random variables have the regression with equations,

$3X + 2Y - 26 = 0$

$6X + Y - 31 = 0$

Find the mean value of X as well as of Y and the correlation coefficient between X and Y. If the variance of X is 25, find $\sum Y$ from the data given above.

Check Your Progress

1. What is the theory of correlation?
2. What is coefficient of determination?
3. Define the term coefficient of correlation.
4. What are dependent and independent variables?
5. List the two methods used for estimation.

NOTES

DID YOU KNOW

The technique of **correlation** is used to test the statistical significance of the association. The r value will **always** lie between 1 and +1. If you have an r value outside of this range you have made an error in the calculations.

6.7 SUMMARY

- Correlation analysis is the statistical tool generally used to describe the degree to which one variable is related to another.
- The coefficient of determination (symbolically indicated as r^2, though some people would prefer to put it as R^2) is a measure of the degree of linear association or correlation between two variables, say X and Y, one of which happens to be the independent variable and the other being the dependent variable.
- The coefficient of correlation, symbolically denoted by 'r', is an important measure to describe how well one variable is explained by another. It measures the degree of relationship between two casually-related variables.
- In case of simple linear regression analysis, a single variable is used to predict another variable on the assumption of linear relationship (i.e., relationship of the type defined by $Y = a + bX$) between the given variables.
- In statistics, the Pearson Product Moment Correlation Coefficient is a measure of the correlation (linear dependence) between two variables X and Y, giving a value between +1 and −1 inclusive. It is sometimes referred the PPMCC or PCC or Pearson's r.
- Pearson's correlation coefficient between two variables is defined as the covariance of the two variables divided by the product of their standard deviations.
- Scatter diagram is a diagram representing two series with the known variable, i.e., independent variable plotted on the X-axis and the variable to be estimated, i.e., dependent variable to be plotted on the Y-axis on a graph paper
- The least squares method of fitting a line (the line of best fit or the regression line) through the scatter diagram is a method which minimizes the sum of the squared vertical deviations from the fitted line.
- The square of the S_e, also known as the variance of the error term, is the basic measure of reliability. The larger the variance, the more significant are the magnitudes of the e's and the less reliable is the regression analysis in predicting the data.

6.8 KEY TERMS

- **Correlation analysis:** The statistical tool used to describe the degree to which one variable is related to another

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- **Coefficient of determination:** A measure of the degree of linear association or correlation between two variables, say X and Y, one of which happens to be an independent variable and the other being a dependent variable
- **Coefficient of non-determination:** The ratio of unexplained variation to total variation in the Y variable related to the X variable
- **Rank correlation:** A descriptive index of agreement between ranks over individuals
- **Standard error of the estimate:** A measure developed by statisticians for measuring the reliability of the estimating equation

6.9 ANSWERS TO 'CHECK YOUR PROGRESS'

1. The theory by means of which quantitative connections between two sets of phenomena are determined is called the 'Theory of Correlation'.
2. The coefficient of determination (symbolically indicated as r^2, though some people would prefer to put it as R^2) is a measure of the degree of linear association or correlation between two variables, say X and Y, one of which happens to be independent variable and the other being dependent variable.
3. The coefficient of correlation, symbolically denoted by 'r', is another important measure to describe how well one variable is explained by another. It measures the degree of relationship between the casually-related variables.
4. The variable to be predicted is called the dependent variable and the variable on which the prediction is based is called the independent variable.
5. The following are the two methods used for estimation:
   (a) Scatter diagram method.
   (b) Least squares method.

6.10 QUESTIONS AND EXERCISES

Short-Answer Questions

1. What is the importance of correlation analysis?
2. How will you determine the coefficient of determination?
3. When is correlation positive and when is it negative?
4. What are the assumptions in Karl Pearson's coefficient?
5. What is the relationship between coefficient of nondetermination and coefficient of alienation?
6. List the basic precautions and limitations of regression and correlation analyses.
7. Differentiate between scatter diagram and least square method.

Long-Answer Questions

1. Explain the method to calculate the coefficient of correlation using simple regression coefficient.

NOTES

2. Describe Karl Pearson's method of measuring coefficient of correlation.
3. Explain Spearman's rank correlation.
4. What is regression analysis? What are the assumptions in it? Discuss.
5. Explain scatter diagram and the least square method in detail. Also, mention how scatter diagram helps in studying correlation between two variables.

6.11 FURTHER READING

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NOTES

UNIT 7 NORMAL PROBABILITY CURVE AND STATISTICAL SIGNIFICANCE

Structure

7.0 INTRODUCTION

In this unit, you will learn about the significant characteristics and applications of normal distribution or normal probability curve. The normal distribution is illustrated using the normal probability curve and is the most common type of distribution. Typically, a normal distribution is a very important statistical data distribution pattern occurring in many natural phenomena, such as height, blood pressure, lengths of objects produced by machines, etc. Certain data, when graphed as a histogram (data on the horizontal axis, amount of data on the vertical axis), creates a bell-shaped curve known as a normal probability curve or normal distribution.

The graph of the normal distribution depends on two factors - the mean (average) and standard deviation (σ, sigma). The mean of the distribution determines the location of the center of the graph and the standard deviation determines the height and width of the graph. If the standard deviation is large, the curve is short and wide while if the standard deviation is small, the curve is tall and narrow. All normal distributions look like a symmetric, bell-shaped curve. Thus a normal probability curve shows the theoretical shape of a normally distributed histogram. The shape of the normal probability curve is also based on two parameters: mean (average) and standard deviation (σ, sigma). Thus, the normal distribution is a very important class of statistical distribution. All normal distributions are symmetric and have bell-shaped density curves with a single peak. Specifically, in any normal distribution, two quantities have to be specified, the mean μ where the peak of the density occurs and the standard deviation σ which indicates the spread or girth of the bell curve.

The Normal Probability Curve (NPC), simply known as normal curve, is a symmetrical bell-shaped curve. This curve is based upon the law of probability and discovered by French mathematician Abraham Demoivre (1667–1754) in the 18th century. In this curve, the mean, median and mode lie at the middle point of the distribution. The total area of the curve represents the total number of cases and the middle point represents

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the mean, median and mode. The base line is divided into six sigma units (σ units). The scores more than the mean come on the + σ side and the scores less than the mean come on the – σ side. The mean point (middle point) is marked as zero (0). All the scores are expected to lie between – 3σ to + 3σ.

7.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Explain the significance and characteristics of normal probability curve or normal distribution
- Describe the family of normal probability distributions
- Explain the significance of normal probability curve
- Discuss how to measure the area under the normal curve
- Explain the properties and uses of the normal probability curve
- Understand the consequences of statistical significance

7.2 NORMAL PROBABILITY CURVE OR DISTRIBUTION: BASIC CONCEPTS

In probability theory, the normal probability curve or normal (or Gaussian) distribution is considered as the most frequently occurring continuous probability distribution. Normal distributions are exceptionally significant in statistics and are typically used in the context of natural and social sciences generally for real-valued random variables whose distributions are not known. The curve of normal distribution is illustrated using the normal probability curve and is the most common type of distribution. Typically, a normal distribution is a very important statistical data distribution pattern occurring in many natural phenomena, such as height, blood pressure, lengths of objects produced by machines, etc. Certain data, when graphed as a histogram (data on the horizontal axis, amount of data on the vertical axis), creates a bell-shaped curve known as a normal probability curve or normal distribution.

The graph of the normal distribution depends on two factors - the mean (average) and standard deviation (σ, sigma). The mean of the distribution determines the location of the center of the graph and the standard deviation determines the height and width of the graph. If the standard deviation is large, the curve is short and wide while if the standard deviation is small, the curve is tall and narrow. All normal distributions look like a symmetric, bell-shaped curve. Thus a normal probability curve shows the theoretical shape of a normally distributed histogram. The shape of the normal probability curve is also based on two parameters: mean (average) and standard deviation (σ, sigma). Thus, the normal distribution is a very important class of statistical distribution. All normal distributions are symmetric and have bell-shaped density curves with a single peak. Specifically, in any normal distribution, two quantities have to be specified, the mean μ where the peak of the density occurs and the standard deviation σ which indicates the spread or girth of the bell curve.

Among all the probability distributions, the normal probability distribution is by far the most important and frequently used continuous probability distribution. This is so because this distribution fits well in many types of problems. This distribution is of special significance in inferential statistics since it describes probabilistically the link between a statistic and a parameter (i.e., between the sample results and the population from which the sample is drawn). The name Karl Gauss, 18th century mathematician–astronomer, is associated with this distribution and in honour of his contribution, this distribution is often known as the Gaussian distribution.

Normal distribution can be theoretically derived as the limiting form of many discrete distributions. For instance, if in the binomial expansion of $(p + q)^n$, the value of 'n' is infinity and $p = q = \frac{1}{2}$, then a perfectly smooth symmetrical curve would be obtained. Even if the values of p and q are not equal but if the value of the exponent 'n' happens to be very very large, we get a curve of normal probability smooth and symmetrical. Such curves are called normal probability curves (or at times known as normal curves of error) and such curves represent the normal distributions.[1]

The probability function in case of normal probability distribution[2] is given as:

$$f(x) = \frac{1}{\sigma.\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where, μ = The mean of the distribution.

σ^2 = Variance of the distribution.

The normal distribution is thus defined by two parameters viz., μ and σ^2. This distribution can be represented graphically as in Figure 7.1.

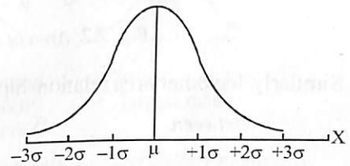

Fig. 7.1 Curve Representing Normal Distribution

7.2.1 Characteristics of Normal Distribution

The characteristics of the normal distribution or that of a normal curve are, as given as follow:

1. It is a symmetric distribution.[3]
2. The mean μ defines where the peak of the curve occurs. In other words, the ordinate at the mean is the highest ordinate. The height of the ordinate at a distance of one standard deviation from the mean is 60.653% of the height of the mean ordinate and similarly the height of other ordinates at various standard deviations (σ_s) from mean happens to be a fixed relationship with the height of the mean ordinate (see Figure 7.2).
3. The curve is asymptotic to the base line which means that it continues to approach but never touches the horizontal axis.
4. The variance (σ^2) defines the spread of the curve.

1. Quite often, mathematicians use the normal approximation of the binomial distribution whenever 'n' is equal to or greater than 30 and np and nq each are greater than 5.
2. Equation of the normal curve in its simplest form is,

$$y = y_0.e^{\left(x^2/2\sigma^2\right)}$$

Where, y = The computed height of an ordinate at a distance of X from the mean.

y_0 = The height of the maximum ordinate at the mean. It is a constant in the equation and is worked out as under:

$$y_0 = \frac{N_i}{\sigma\sqrt{2\pi}}$$

3. A symmetric distribution is one which has no skewness. As such it has the following statistical properties:
   (a) Mean=Mode=Median (i.e., $X=Z=M$)
   (b) (Upper Quantile – Median)=(Median – Lower Quantile) (i.e., $Q_3 - M = M - Q_1$)
   (c) Mean Deviation=0.7979(Standard Deviation)
   (d) $\frac{Q_3 - Q_1}{2} = 0.6745$ (Standard Deviation)

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5. Area enclosed between mean ordinate and an ordinate at a distance of one standard deviation from the mean is always 34.134% of the total area of the curve. It means that the area enclosed between two ordinates at one sigma (SD) distance from the mean on either side would always be 68.268% of the total area. This can be shown as follows:

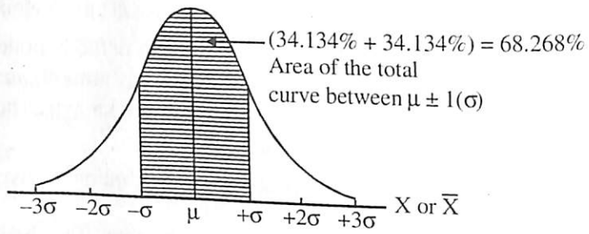


Fig. 7.2 Area of the Total Curve between μ ± 1 (σ)

Similarly, the other area relationships are as follows:

Between		*Area Covered to Total Area of the Normal Curve*[4]
μ ± 1	S.D.	68.27%
μ ± 2	S.D.	95.45%
μ ± 3	S.D.	99.73%
μ ± 1.96	S.D.	95%
μ ± 2.578	S.D.	99%
μ ± 0.6745	S.D.	50%

6. The normal distribution has only one mode since the curve has a single peak. In other words, it is always a unimodal distribution.
7. The maximum ordinate divides the graph of normal curve into two equal parts.
8. In addition to all the above stated characteristics the curve has the following properties:
   (a) $\mu = \bar{x}$
   (b) $\mu_2 = \sigma^2 =$ Variance
   (c) $\mu_4 = 3\sigma^4$
   (d) Moment Coefficient of Kurtosis = 3

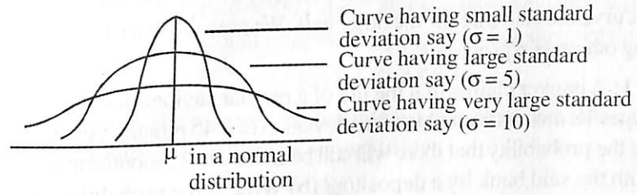
7.2.2 Family of Normal Distributions

We can have several normal probability distributions but each particular normal distribution is being defined by its two parameters viz., the mean (μ) and the standard deviation (σ). There is, thus, not a single normal curve but rather a family of normal curves (see Figure 7.3). We can exhibit some of these as under:

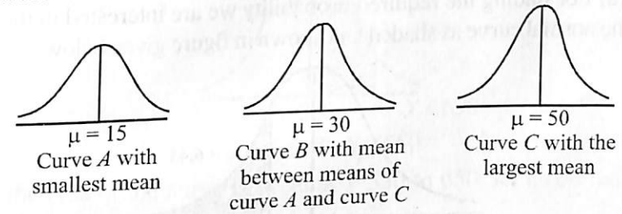
4. This also means that in a normal distribution, the probability of area lying between various limits are as follows:

Limits	Probability of area lying within the stated limits
μ ± 1 S.D.	0.6827
μ ± 2 S.D.	0.9545
μ ± 3 S.D.	0.9973 (This means that almost all cases lie within μ ± 3 S.D. limits)

Normal curves with identical means but different standard deviations:



Normal curves with identical standard deviation but each with different means:



Normal curves each with different standard deviations and different means:

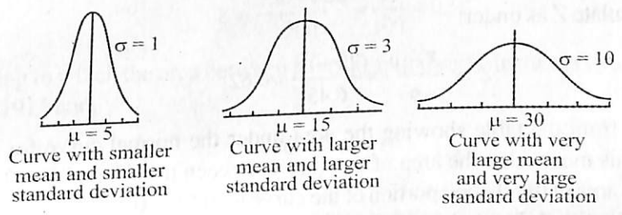


Fig. 7.3 Family of Curves

7.2.3 How to Measure the Area under the Normal Curve

We have stated above some of the area relationships involving certain intervals of standard deviations (plus and minus) from the means that are true in case of a normal curve. But what should be done in all other cases? We can make use of the statistical tables constructed by mathematicians for the purpose. Using these tables we can find the area (or probability, taking the entire area of the curve as equal to 1) that the normally distributed random variable will lie within certain distances from the mean. These distances are defined in terms of standard deviations. While using the tables showing the area under the normal curve we talk in terms of standard variate (symbolically Z) which really means standard deviations without units of measurement and this 'Z' is worked out as under:

$$Z = \frac{X - \mu}{\sigma}$$

Where, Z = The standard variate (or number of standard deviations from X to the mean of the distribution).

X = Value of the random variable under consideration.

μ = Mean of the distribution of the random variable.

σ = Standard deviation of the distribution.

The table showing the area under the normal curve (often termed as the standard normal probability distribution table) is organized in terms of standard variate (or Z) values. It

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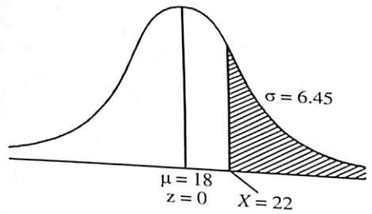
gives the values for only half the area under the normal curve, beginning with $Z = 0$ at the mean. Since the normal distribution is perfectly symmetrical the values true for one half of the curve are also true for the other half. We now illustrate the use of such a table for working out certain problems.

Example 1: A banker claims that the life of a regular savings account opened with his bank averages 18 months with a standard deviation of 6.45 months. Answer the following: (a) What is the probability that there will still be money in 22 months in a savings account opened with the said bank by a depositor? (b) What is the probability that the account will have been closed before two years?

Solution: (a) For finding the required probability we are interested in the area of the portion of the normal curve as shaded and shown in figure given below:

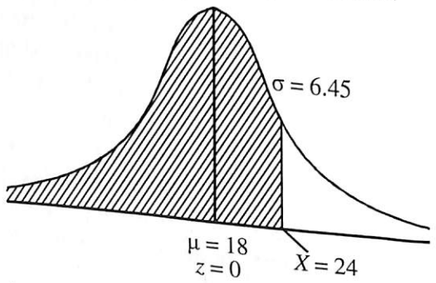


Let us calculate Z as under:

$$Z = \frac{X - \mu}{\sigma} = \frac{22 - 18}{6.45} = 0.62$$

The value from the table showing the area under the normal curve for $Z = 0.62$ is 0.2324. This means that the area of the curve between $\mu = 18$ and $X = 22$ is 0.2324. Hence, the area of the shaded portion of the curve is $(0.5) - (0.2324) = 0.2676$ since the area of the entire right hand portion of the curve always happens to be 0.5. Thus, the probability that there will still be money in 22 months in a savings account is 0.2676.

(b) For finding the required probability we are interested in the area of the portion of the normal curve as shaded and shown in figure given below:



For the purpose we calculate,

$$Z = \frac{24 - 18}{6.45} = 0.93$$

The value from the concerning table, when $Z = 0.93$, is 0.3238 which refers to the area of the curve between $\mu = 18$ and $X = 24$. The area of the entire left hand portion of the curve is 0.5 as usual.

Hence, the area of the shaded portion is $(0.5) + (0.3238) = 0.8238$ which is the required probability that the account will have been closed before two years, i.e., before 24 months.

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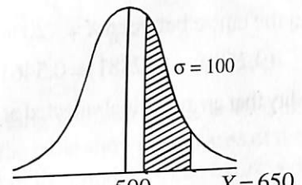
Example 2: Regarding a certain normal distribution concerning the income of the individuals we are given that mean=500 rupees and standard deviation =100 rupees. Find the probability that an individual selected at random will belong to income group,

(a) ₹ 550 to ₹ 650 (b) ₹ 420 to ₹570

Solution: (a) For finding the required probability we are interested in the area of the portion of the normal curve as shaded and shown below:



For finding the area of the curve between $X = 550$ to 650, let us do the following calculations:

$$Z = \frac{550 - 500}{100} = \frac{50}{100} = 0.50$$

Corresponding to which the area between $\mu = 500$ and $X = 550$ in the curve as per table is equal to 0.1915 and,

$$Z = \frac{650 - 500}{100} = \frac{150}{100} = 1.5$$

Corresponding to which, the area between $\mu = 500$ and $X = 650$ in the curve, as per table, is equal to 0.4332.

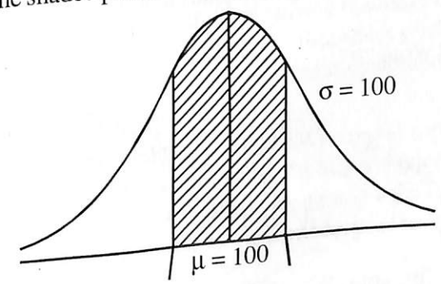
Hence, the area of the curve that lies between $X = 550$ and $X = 650$ is,

$$(0.4332) - (0.1915) = 0.2417$$

This is the required probability that an individual selected at random will belong to income group of ₹ 550 to ₹ 650.

(b) For finding the required probability we are interested in the area of the portion of the normal curve as shaded and shown below:

To find the area of the shaded portion we make the following calculations:



$$Z = \frac{570 - 500}{100} = 0.70$$

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Corresponding to which the area between $\mu = 500$ and $X = 570$ in the curve as per table is equal to 0.2580.

and $$Z = \frac{420 - 500}{100} = -0.80$$

Corresponding to which the area between $\mu = 500$ and $X = 420$ in the curve as per table is equal to 0.2881.

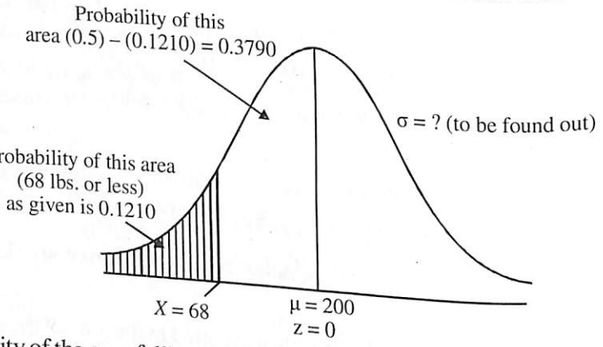
Hence, the required area in the curve between $X = 420$ and $X = 570$ is,

$$(0.2580) + (0.2881) = 0.5461$$

This is the required probability that an individual selected at random will belong to income group of ₹ 420 to ₹ 570.

Example 3: A certain company manufactures $1\frac{1''}{2}$ all-purpose rope using imported hemp. The manager of the company knows that the average load-bearing capacity of the rope is 200 lbs. Assuming that normal distribution applies, find the standard deviation of load-bearing capacity for the $1\frac{1''}{2}$ rope if it is given that the rope has a 0.1210 probability of breaking with 68 lbs. or less pull.

Solution: Given information can be depicted in a normal curve as shown below:



If the probability of the area falling within $\mu = 200$ and $X = 68$ is 0.3790 as stated above, the corresponding value of Z as per the standard statistical tables showing the area of the normal curve is – 1.17 (minus sign indicates that we are in the left portion of the curve).

Now to find σ, we can write,

$$Z = \frac{X - \mu}{\sigma}$$

or $$-1.17 = \frac{68 - 200}{\sigma}$$

or $$-1.17\sigma = -132$$

or $$\sigma = 112.8 \text{ lbs. approx.}$$

Thus, the required standard deviation is 112.8 lbs. approximately.

Example 4: In a normal distribution, 31 per cent items are below 45 and 8 per cent are above 64. Find the $\bar{X}$ and σ of this distribution.

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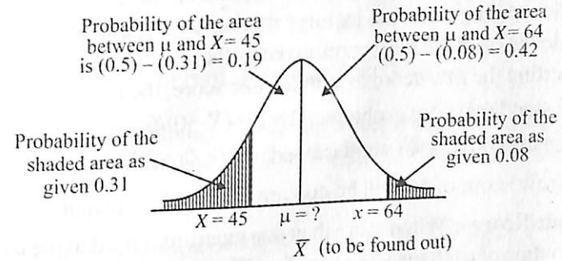
Solution: We can depict the given information in a normal curve as shown below:



If the probability of the area falling within μ and $X = 45$ is 0.19 as stated above, the corresponding value of Z from the table showing the area of the normal curve is – 0.50. Since, we are in the left portion of the curve, we can express this as under,

$$-0.50 = \frac{45 - \mu}{\sigma} \quad (1)$$

Similarly, if the probability of the area falling within μ and $X = 64$ is 0.42, as stated above, the corresponding value of Z from the area table is, +1.41. Since, we are in the right portion of the curve we can express this as under,

$$1.41 = \frac{64 - \mu}{\sigma} \quad (2)$$

If we solve Equations (1) and (2) above to obtain the value of μ or $\bar{X}$, we have,

$$-0.5\,\sigma = 45 - \mu \quad (3)$$

$$1.41\,\sigma = 64 - \mu \quad (4)$$

By subtracting Equation (4) from Equation (3) we have,

$$-1.91\,\sigma = -19$$

$$\therefore \quad \sigma = 10$$

Putting $\sigma = 10$ in Equation (3) we have,

$$-5 = 45 - \mu$$

$$\therefore \quad \mu = 50$$

Hence, $\bar{X}$ (or μ)=50 and σ =10 for the concerning normal distribution or probability curve.

7.2.4 Applications of Normal Distribution or Probability Curve

The following are the applications of normal distribution or probability curve:

1. **Random Processes:** Many naturally occurring random processes tend to have a distribution that is approximately normal. Examples can be found in any field, these include: SAT test scores of college bound students and body temperature of a healthy adult.
2. **Approximation of Binomial Distribution:** When $np>5$ and $n(1-p)>5$, the normal distribution provides an good approximation of the binomial distribution. Distributions that are based on multiple observations, for example the Binomial distribution, approach the normal distribution when n gets large. The value $n>30$ is usually considered large.
3. **Standardization:** It is used where it is usually hypothesize that the theoretical distribution of a certain variable is normal, whereas the measurement of such variable may not give a normal distribution.

   For example, in the introductory classes of Statistics there are 200 students and it has been assumed that the performance of all the students in the examination

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should be normally distributed. In addition, for giving reasonable distribution of marks, the mean should be 55 and the standard deviation should be 10. After the examinations being over, the lecturer marked all the papers, and the mean and standard deviation of the raw scores given by the lecturer are 50 and 6, respectively. For converting the raw score to standardize score, the follows steps were taken:

(a) The standard score is obtained by $Z = (X-50)/6$.

(b) Then the converted (standardized) = $10(Z) + 55$.

Hence, a raw score of 56 will be converted into 65.

4. **Composite Scores:** When more than one measure is used to measure a variable, the distribution of each measure usually differs from each other. In order to obtain an unbiased measure using several different measurements, each sub-measure is standardized before added together.

For example, if the marks are awarded according to the average of the marks given by the Marker I and Marker II, then clearly the final grades are greatly affected by the Marker I than by Marker II as Marker I is awarded the marks with higher standard deviation as shown in table below:

Students	Marker I	Marker II	Average
A	80	50	65.0
B	70	55	62.5
C	60	60	60.0
D	50	65	57.5
E	40	70	55.0
Mean	60	60	
σ	14	7	

For computing the composite score, the standardized scores of Marker I and Marker II should be averaged. If the ideal average score (μ) and standard deviation (σ) is taken to be 60 and 10, respectively, then the Z scores is converted into the standard score for each marker. The following table shows the resulted average standardized score 60 for every student.

	Marker I			Marker II		
Student	Raw score	$z=(x-\mu)/\sigma$	Standard score	Raw score	$z=(x-\mu)/\sigma$	Standard score
A	80	1.4	74	50	-1.4	46
B	70	0.7	67	55	-0.7	53
C	60	0	60	60	0	60
D	50	-0.7	53	65	0.7	67
E	40	-1.4	46	70	1.4	74

5. **Probability Distribution:** The probability distribution of $\bar{X}$ for large n is the normal distribution. The Central Limit Theorem states that if the observations are independent for one population which has a mean (μ) and standard deviation (σ) then for large n (n>30) $\bar{X}$ has a normal distribution with the same mean and a standard deviation of $\sigma / \sqrt{n}$.

Check Your Progress

1. What is a normal distribution?
2. List any four characteristics of normal distribution.
3. What is the significance of probability distribution table?
4. Write the properties of normal distribution curve.
5. How the normal distribution can be theoretically derived?

7.3 NORMAL PROBABILITY CURVE AND ITS PROPERTIES AND USES

The Normal Probability Curve (NPC), simply known as normal curve, is a symmetrical bell-shaped curve. This curve is based upon the law of probability and discovered by French mathematician Abraham Demoivre (1667–1754) in the 18th century. In this curve, the mean, median and mode lie at the middle point of the distribution. The total area of the curve represents the total number of cases and the middle point represents the mean, median and mode. The base line is divided into six sigma units (σ units). The scores more than the mean come on the + σ side and the scores less than the mean come on the – σ side. The mean point (middle point) is marked as zero (0). All the scores are expected to lie between – 3σ to + 3σ.

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7.3.1 Properties of Normal Probability Curve

The NPC or Normal Probability Curve has several features which are essential to understand for its use. The major characteristics are limited. They are as follows:

- It is a bell-shaped curve.
- The measures of central tendency are equal, i.e, mean, mode and median concentrate on one point.
- The height of the curve is 0.3989.
- It is an asymptotic curve. The ends of the curve approach but never touch the *X*-axis at the extremes because of the possibility of locating in the population, in cases where scores are still higher than our highest score or lower than our lowest score. Therefore theoretically, it extends from minus infinity to plus infinity as illustrated in Figure 7.4. Here, *M* is the mean or expectation (location of the peak) and σ is the standard deviation.

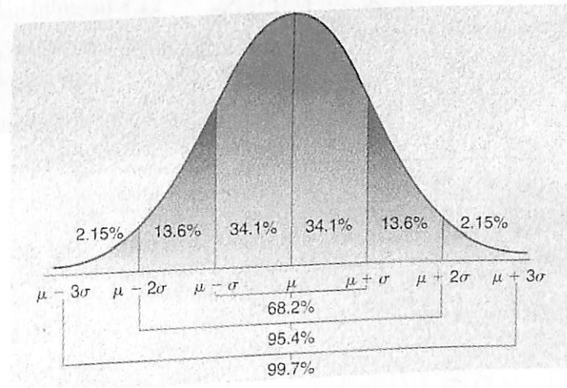

Fig. 7.4 Normal Curve Showing Areas at Different Distances from the Mean

- It has 50 per cent frequency above and 50 per cent below the mean. The mean is zero and it is always reference point.
- Standard deviation of a normal curve is always 1.
- The points of inflection of the curve occur at points –1 unit above and below mean.
- The distribution of frequency per cent has the definite limits.
- There is a definite relation between quartile deviation and standard deviation in a normal distribution curve.
- It is a mathematical curve and is an open-ended curve.

Some limits are as follows:

- The middle 68 per cent frequency is between –1 and +1.
- The middle 95 per cent frequency is between –1.96 and + 1.96.
- The middle 99 per cent frequency is between –2.58 and + 2.58.

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The total area under the normal curve is arbitrarily taken as 10,000. Every score should be converted into standard score (Z score) by using the following formula:

$$Z = \frac{X - M}{\sigma}$$

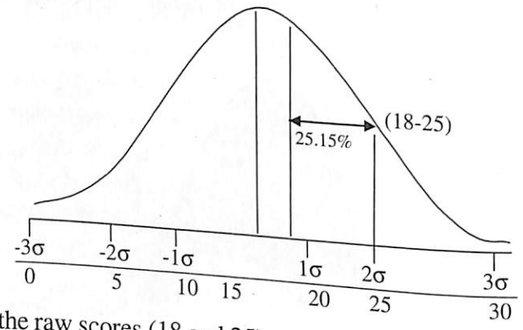
The area in proportion should be converted into a percentage at the time of reading the table. From the table, we can see the areas from mean to σ and also we can read the value of σ scores from the mean for the corresponding fractional area.

7.3.2 Uses of Normal Probability Curve: Determining Mean and Median

The uses of normal probability curve are discussed in this section.

NPC is used to Determine the Percentage of Cases within Given Limits

Example 5: Given a distribution of scores with a mean of 15 and a standard deviation of 5, what percentage of cases lie between 18 and 25. Refer to Figure given below to calculate the answer.



Solution: Both the raw scores (18 and 25) are to be converted into Z scores.

$$\text{Z score of } 18 = \frac{X - M}{\sigma} = \frac{18 - 15}{5}$$

$$= \frac{3}{5}$$

$$= 0.6\sigma$$

$$\text{Z score of } 25 = \frac{X - M}{\sigma} = \frac{25 - 15}{5}$$

$$= \frac{10}{5}$$

According to the table of area of a normal probability curve, the total percentage of cases lie between the mean and 0.6σ is 22.57. The percentage of cases lie between the mean and 2σ is 47.72. So, the total percentage of cases that fall between the scores 18 and 25 is 47.72 – 22.57 = 25.15.

NPC is used to determine the limit which includes a given percentage of cases

Example 6: Given a distribution of scores with a mean of 12 and an σ of 6, what limits will include the middle 70 per cent of the cases? Refer to Figure given below to calculate the answer.

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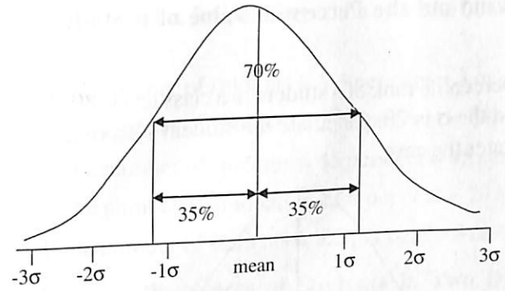


Mean = 12, σ = 6

Solution: The middle 70 per cent of the cases in a normal distribution signifies that 35 per cent cases above the mean and also 35 per cent cases below the mean. According to the table of area under NPC, 35 per cent of cases fall between the mean and 1.04 σ. So the middle 70 per cent of the cases will lie between –1.04σ to + 1.04σ.

The value of 1 σ = 6

So 1.04 σ = 6 × 1.04 = 6.24

The value of mean = 12

So the lowest limit for the middle 70 per cent cases of the distribution is:

12 – 6.24 = 5.76.

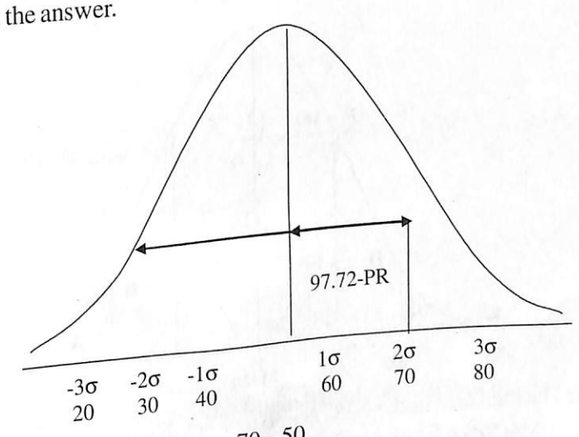
The highest limit for the middle 70 per cent cases of the distribution is:

12 + 6.24 = 18.24.

Thus, the middle 70 per cent cases lie in between 5.76 and 18.24.

NPC is used to Determine the Percentile Rank of a Student in his Class

Example 7: The score of a student in a class test is 70. The mean for the whole class is 50 and the σ is 10. Find the percentile rank of the student in the class. Refer the Figure given below to find the answer.



Solution: The Z score for the score 70 is $\frac{70 - 50}{10} = 2\sigma$

As per the table of area under the NPC, the area of the curve that lies between mean and 2σ is 47.72 per cent. The total percentage of cases below 70 is:

50 + 47.72 = 97.72 per cent or 98 per cent.

Thus, the percentile rank of the student is 98.

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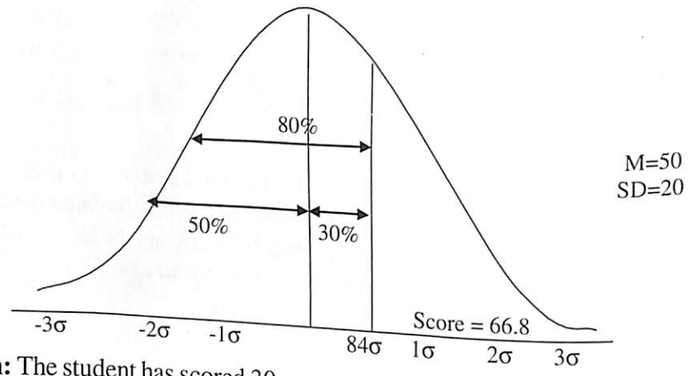
NPC is used to Find out the Percentile Value of a Student whose Percentile Rank is Known

Example 8: The percentile rank of a student in a class test is 80. The mean of the class in the test is 50 and the σ is 20. Calculate the student's score in the class test. Figure given below illustrates the case.



Solution: The student has scored 30 per cent scores above the mean. According to the table of area under NPC, 30 per cent cases from the mean is 0.84σ.

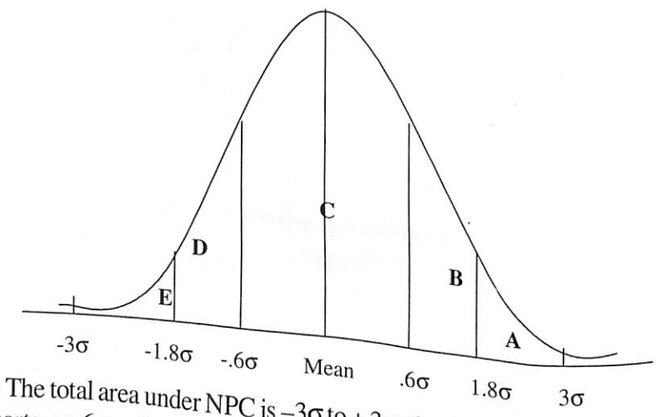
$1\sigma = 20.$

$0.84\sigma = 20 \times .84 = 16.8$

Thus, the percentile value of the student is $50 + 16.8 = 66.8$.

NPC is used to Divide a Group into Sub-Groups According to their Capacity

Example 9: Suppose there is a group of 100 students in a Commerce class. We want to divide them into five small groups A, B, C, D and E according to their ability, the range of ability being equal in each sub-group. Find out how many students should be placed in each category.



Solution: The total area under NPC is -3σ to $+3\sigma$, that is 6σ. This 6σ should be divided into five parts, so $6\sigma \div 5 = 1.2\sigma$.

According to the table of area under NPC:

3.5 per cent of the cases lie between 1.8 σ to 3σ (Group A, the high scorers), 23.8 per cent of the cases lie between .6σ to 1.8σ (23.8 per cent of the cases for B and also 23.8 per cent of the cases for D), the middle 45 per cent of the cases lie – 0.6σ to + 0.6σ (Group C), and the lowest 3.5 per cent of the cases lie between –3σ to – 1.8σ (Group E)

In category 'A' the number of students = 3.5 per cent = 3 or 4 students.

In category 'B' the number of students = 23.8 per cent = 24 students.

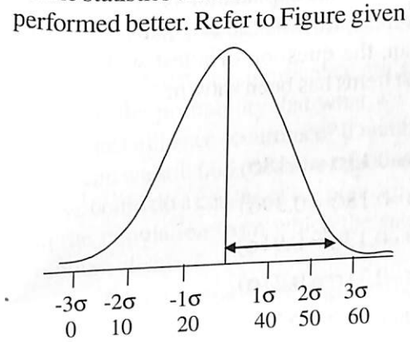
In category 'C' the number of students = 45 per cent = 45 students.

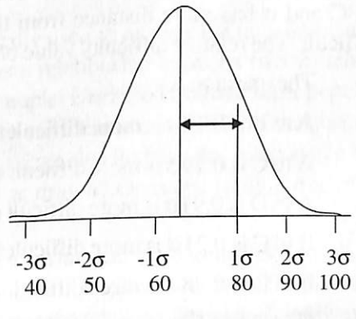
In category 'D' the number of students = 23.8 per cent = 24 students.

In category 'E' the number of students = 3.5 per cent = 3 or 4 students.

NPC is used to Compare the Scores of Students in Two Different Tests

Example 10: Suppose, a student scored 60 marks in English test and 80 marks in statistics test. The mean and SD for the English test is 30 and 10 respectively, whereas for the statistics test the mean is 70 and SD is 10. Find out, in which subject the student performed better. Refer to Figure given below.





Solution: In case of the English test:

Raw score = 60

Mean = 30

SD = 10

So Z score for the English test = $\frac{X - M}{\sigma} = \frac{60 - 30}{10} = \frac{30}{10} = 3\sigma$

In case of statistics test raw score = 80

Mean = 70

SD = 10

So Z Score for the statistics test = $\frac{X - M}{\sigma} = \frac{80 - 70}{10} = \frac{10}{10} = 1\sigma$

So, the student has done better in the English than the statistics on.

NPC is Used to Determine the Relative Difficulty Level of Test Items

Example 11: In a standardized test of psychology, question numbers A, B, C and D were solved by the students, 45 per cent, 38 per cent, 30 per cent and 15 per cent respectively. Assuming the normality, find out the relative difficulty level of the questions. Also explain the difficulty levels of questions. Table given below displays the information in tabular form.

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Solution:

Table 7.1 Determining the Difficulty Level of Test Items

Question Number	Percentage of Successful Students	Percentage of Unsuccessful Students	Percentage distance of Mean of Unsuccessful Students	Difficulty Level
A	45	55	55-50=5	0.13σ
B	38	62	62-50=12	0.31σ
C	30	70	70-50=20	0.52σ
D	15	85	85-50=35	1.04σ

As we know that in an NPC, 50 – 50 cases lie both the sides of mean. The mean of NPC is that point which is shown as 0. In an NPC, the explanation of difficulty level is done on the basis of σ — distance. Therefore, if a question is at the positive side of the NPC and σ has more distance from the mean, the question of a test will be much difficult. The relative difficulty value of the test items has been shown:

The question

A to B is 0.18σ is more difficult (0.31σ –0.13σ = 0.18σ)

A to C is 0.39σ is more difficult (0.52σ –0.13σ = 0.39σ)

A to D is 0.91σ is more difficult (1.04σ –0.13σ = 0.91σ)

B to C is 0.21σ is more difficult (0.52σ –0.31σ = 0.21σ)

B to D is 0.73σ is more difficult (1.04σ –0.31σ = 0.73σ)

C to D is 0.52σ is more difficult (1.04σ –0.152σ = 0.52σ)

7.4 STATISTICAL SIGNIFICANCE

Statistical significance is the result that is not likely to occur randomly, but rather it is likely to be attributable to a specific cause. Statistical significance can be strong or weak and is important feature of research in many mathematics and science related fields. Statistical significance does not always indicate practical significance. In addition, it can be misinterpreted when researchers do not use language carefully in reporting their results.

The calculation of statistical significance (significance testing) is subject to a certain degree of error. The researcher must define in advance the probability of a sampling error which exists in any test that does not include the entire population. Sample size is considered as an important component of statistical significance because larger samples are less prone to accidents. Only random, representative samples should be used in significance testing.

The level at which one can accept whether an event is statistically significant is known as the significance level or P value. Hence, statistical significance is the number called a P value and defines the probability of the result being observed given that the null hypothesis is true. If this P value is sufficiently small, the experimenter can safely assume that the null hypothesis is false.

In experiments of statistics one must define a level of significance at which a correlation will be estimated to have been verified, though the option is often actually

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made after the event. It is significant to know that however small the value of P is there is always a finite chance that the result is a pure accident. A typical level at which the threshold of P is set would be 0.01, which means there is a one per cent chance that the result was accidental. The significance of such a result would then be indicated by the statement P < 0.01. Further, in some cases the researcher can use the much lower levels of significance. A level frequently referenced is P < 0.05. This means that there is a one in twenty chance that the whole object was accidental.

It is difficult to generalize, but on the whole P < 0.01 would normally be considered significant and P < 0.001 highly significant. The origin of the P < 0.05 criterion goes back to the great pioneer of significance testing, R A Fisher, who did not in fact proved this. Many leading scientists and mathematicians today believe that the emphasis on significance testing is grossly overdone. P < 0.05 had become an end in itself and the determinant of a successful outcome to an experiment.

Tests for Statistical Significance

Tests for statistical significance are specifically used to answer the question, such as what is the probability that what we think is a relationship between two variables is really just a chance occurrence? If number of samples is selected from the same population then can we still find the same relationship between these two variables in every sample? If we could do a census of the population, would we also find that this relationship exists in the population from which the sample was drawn? Or is our finding due only to random chance?

Tests for statistical significance tell us what the probability is and also the relationship that occurs only due to random chance. This illustrates what the probability is and what would be the error if it is assumed that the relationship exists. It is always not 100% certain that a relationship exists between two variables. There are too many sources of error to be controlled, for example, sampling error, researcher bias, problems with reliability and validity, simple mistakes, etc. But using probability theory and the normal probability curve, the probability of being wrong can be estimated if it is assumed that the finding a relationship is true. If the probability of being wrong is small, then it is assumed that the observation of the relationship is a statistically significant finding.

Statistical significance means that there is a good chance that one may accurately find that a relationship exists between two variables. But statistical significance is not the same as practical significance. We can have a statistically significant finding, but the implications of that finding may have no practical application. The researcher must always examine both the statistical and the practical significance of any research finding. For example, consider that there is a statistically significant relationship between a citizen's age and the satisfaction level with city recreation services. It may be that older citizens are 5% less satisfied than younger citizens with city recreation services. But is 5% large enough difference to be concerned. At times, when differences are small but statistically significant which is due to a very large sample size then in a sample of a smaller size, the differences would not be enough to be statistically significant. The following are some significant tests for testing statistical significance.

Steps in Testing for Statistical Significance

1. State the Research Hypothesis
2. State the Null Hypothesis
3. Select a Probability of Error Level (α or Alpha Level)

NOTES

4. Select and Compute the Test for Statistical Significance
5. Interpret the Results

There is always a possibility that the researcher will make a mistake regarding the relationship between the two variables. There are two possible mistakes or errors. The first is called a Type I error. This occurs when the researcher assumes that a relationship exists when in fact the evidence is that it does not. In a Type I error, the researcher should accept the null hypothesis and reject the research hypothesis, but the opposite occurs. The probability of committing a Type I error is called alpha (α). The second is called a Type II error. This occurs when the researcher assumes that a relationship does not exist when in fact the evidence is that it does. In a Type II error, the researcher should reject the null hypothesis and accept the research hypothesis, but the opposite occurs. The probability of committing a Type II error is called beta (β).

Researchers generally specify the probability of committing a Type I error, i.e., the value of alpha. Most researchers select an alpha as 0.05. This means that there is a probability of 5% of making a Type I error assuming that a relationship between two variables exists when it is really does not. However, an alpha of 0.01 is also used when researchers do not want to have a probability of being wrong more than 0.1% of the time or one time in a thousand.

The level of alpha can vary, but the smaller the value, the more stringent the requirement for reaching statistical significance becomes. Alpha levels are often written as the 'P value' or 'P = 0.05'. Usual levels are P = 0.05 or the chance of one in 20 of making an error; P = 0.01 or the chance of one in 100 of making an error; P = 0.001 or the chance of one in 1,000 of making an error. When accounting the level of alpha, it is usually accounted as being 'less than' some level, using the 'less than' sign or '<'. Thus, it is accounted as $P < 0.05$ or $P < 0.01$, etc.

For nominal and ordinal data, Chi-square test is used as a test for statistical significance. To calculate Chi-square, we compare the original, observed frequencies with the new, expected frequencies. *t*-test is considered as the important test for statistical significance and is used with interval and ratio level data. *t*-tests can be used in several different types of statistical tests.

Tests for statistical significance are used to estimate the probability that a relationship observed in the data occurred only by chance and that the probability variables are really unrelated in the population.

Check Your Progress

6. What is the shape of the normal probability curve?
7. From Normal table find the area between mean ordinate for which Z or sigma score has the value – 1.65.
8. Enumerate two important characteristics of NPC.
9. Define the term statistical significance.

ACTIVITY

1. Using the NPC determine the percentile of a student whose score is 80. The mean for the class is 60 and the σ is 10.

DID YOU KNOW

The terms statistical inference, statistical induction and inferential statistics are specifically used to describe systems of procedures that can be used to draw conclusions from datasets arising from systems affected by random variation, such as observational errors, random sampling or random experimentation.

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7.5 SUMMARY

- In probability theory, the normal probability curve or normal (or Gaussian) distribution is considered as the most frequently occurring continuous probability distribution.
- Normal distributions are exceptionally significant in statistics and are typically used in the context of natural and social sciences generally for real-valued random variables whose distributions are not known.
- The curve of normal distribution is illustrated using the normal probability curve and is the most common type of distribution. Certain data, when graphed as a histogram (data on the horizontal axis, amount of data on the vertical axis), creates a bell-shaped curve known as a normal probability curve or normal distribution.
- The graph of the normal distribution depends on two factors - the mean (average) and standard deviation (σ, sigma). The mean of the distribution determines the location of the center of the graph and the standard deviation determines the height and width of the graph.
- All normal distributions look like a symmetric, bell-shaped curve. Specifically, in any normal distribution, two quantities have to be specified, the mean μ where the peak of the density occurs and the standard deviation s which indicates the spread or girth of the bell curve.
- Among all the probability distributions, the normal probability distribution is by far the most important and frequently used continuous probability distribution. This is so because this distribution fits well in many types of problems.
- Normal distribution is of special significance in inferential statistics since it describes probabilistically the link between a statistic and a parameter (i.e., between the sample results and the population from which the sample is drawn).
- The name Karl Gauss, 18th century mathematician–astronomer, is associated with this distribution and in honour of his contribution, this distribution is often known as the Gaussian distribution.
- Normal distribution can be theoretically derived as the limiting form of many discrete distributions.
- The mean μ defines where the peak of the curve occurs. In other words, the ordinate at the mean is the highest ordinate.
- The height of the ordinate at a distance of one standard deviation from the mean is 60.653% of the height of the mean ordinate and similarly the height of other ordinates at various standard deviations (σ_s) from mean happens to be a fixed relationship with the height of the mean ordinate.
- The table showing the area under the normal curve (often termed as the standard normal probability distribution table) is organized in terms of standard variate (or *Z*) values. It gives the values for only half the area under the normal curve, beginning with $Z = 0$ at the mean.
- The Normal Probability Curve (NPC), simply known as normal curve, is a symmetrical bell shaped curve.
- This curve is based upon the law of probability and discovered by French mathematician Abraham Demoivre (1667–1754) in the 18th century. In this curve, the mean, median and mode lie at the middle point of the distribution.

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- The uses of a NPC are, to determine the percentage of cases within given limits, to determine the limit which includes a given percentage of cases, to determine the percentile rank of a student in his class, to divide a group into sub-groups according to their capacity and to determine the relative difficulty level of test items.
- Statistical significance is the result that is not likely to occur randomly, but rather it is likely to be attributable to a specific cause. Statistical significance can be strong or weak and is important feature of research in many mathematics and science related fields.
- The calculation of statistical significance (significance testing) is subject to a certain degree of error. The researcher must define in advance the probability of a sampling error which exists in any test that does not include the entire population.
- Sample size is considered as an important component of statistical significance because larger samples are less prone to accidents. Only random, representative samples should be used in significance testing.
- The level at which one can accept whether an event is statistically significant is known as the significance level or P value. Hence, statistical significance is the number called a P value and defines the probability of the result being observed given that the null hypothesis is true. If this P value is sufficiently small, the experimenter can safely assume that the null hypothesis is false.
- Tests for statistical significance tell us what the probability is and also the relationship that occurs only due to random chance. It is always not 100% certain that a relationship exists between two variables.
- There is always a possibility that the researcher will make a mistake regarding the relationship between the two variables. There are two possible mistakes or errors. The first is called a Type I error. The probability of committing a Type I error is called alpha (α). The second is called a Type II error. The probability of committing a Type II error is called beta (β).

7.6 KEY TERMS

- **Normal probability distribution:** Among all the probability distributions, the normal probability distribution is by far the most important and frequently used continuous probability distribution
- **Normal distribution:** It has only one mode since the curve has a single peak or in other words, it is always a unimodal distribution
- **Mean μ:** It defines where the peak of the curve occurs or in other words, the ordinate at the mean is the highest ordinate
- **Standardization:** It is used where it is usually hypothesize that the theoretical distribution of a certain variable is normal, whereas the measurement of such variable may not give a normal distribution
- **Normal Probability Curve (NPC):** Simply known as normal curve, it is a symmetrical bell shaped curve based upon the law of probability discovered by French mathematician Abraham Demoivre (1667–1754) in the 18th century

7.7 ANSWERS TO 'CHECK YOUR PROGRESS'

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1. Normal distribution is the most important and frequently used continuous probability distribution among all the probability distributions. This is so because this distribution well fits in many types of problems. This distribution is of special significance in inferential statistics since it describes probabilistically the link between a statistic and a parameter.
2. The characteristics of a normal distribution are:
   (a) It is a symmetric distribution.
   (b) The curve is asymptotic to the base line, which means that it continues to approach but never touches the horizontal axis.
   (c) The variance defines the spread of the curve.
   (d) The normal distribution has only one mode since the curve has a single peak. In other words, it is always a unimodal distribution.
3. The probability distribution table shows the area under the normal curve (often termed as the standard normal probability distribution table) is organized in terms of standard variate (or Z) values. It gives the values for only half the area under the normal curve, beginning with $Z = 0$ at the mean.
4. The normal distribution curve has the following properties:
   (a) $\mu = x$
   (b) $\mu^2=\sigma^2$ = Variance
   (c) $\mu^4=3\sigma^4$
   (d) Moment Coefficient of Kurtosis = 3
5. Normal distribution can be theoretically derived as the limiting form of many discrete distributions. For instance, if in the binomial expansion of $(p + q)^n$, the value of 'n'is infinity and $p = q = \frac{1}{2}$, then a perfectly smooth symmetrical curve would be obtained.
6. The normal probability curve is a symmetrical bell-shaped curve.
7. 0.4505
8. The two important characteristics of NPC are as follows:
   (i) The curve is symmetrical around its vertical axis, i.e., ordinate.
   (ii) The highest of ordinate is maximum at mean.
9. Statistical significance is the result that is not likely to occur randomly, but rather it is likely to be attributable to a specific cause. Statistical significance can be strong or weak and is important feature of research in many mathematics and science related fields. The calculation of statistical significance (significance testing) is subject to a certain degree of error. The level at which one can accept whether an event is statistically significant is known as the significance level or P value. Hence, statistical significance is the number called a P value and defines the probability of the result being observed given that the null hypothesis is true. If this P value is sufficiently small, the experimenter can safely assume that the null hypothesis is false.

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7.8 QUESTIONS AND EXERCISES

Short-Answer Questions

1. Why the normal distribution is of special significance in inferential statistics?
2. Which two parameters define the normal distribution?
3. How the area under the normal curve is measured?
4. What is normal probability curve?
5. What is statistical significance?

Long-Answer Questions

1. State the distinctive features of the normal probability distributions.
2. Explain the circumstances when the normal probability distribution can be used.
3. Discuss the various applications of normal distribution.
4. In a distribution exactly normal, 7 per cent of the items are under 35 and 89 per cent are under 63. What are the mean and standard deviation of the distribution?
5. Fit a normal distribution to the following data:

Height in Inches	*Frequency*
60–62	5
63–65	18
66–68	42
69–71	27
72–74	8

6. Discuss the characteristic features of normal probability curve.
7. Explain the various uses of normal probability curve with the help of examples.
8. Discuss the significance of statistical significance and its various tests giving examples.

7.9 FURTHER READING

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UNIT 8 HYPOTHESIS TESTING

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Structure

8.0 INTRODUCTION

In this unit, you will study about hypothesis testing. Hypothesis is an assumption that is tested to find its logical or empirical consequences. A hypothesis should be clear and accurate. Various concepts, such as null and alternative hypotheses help to verify the testability of an assumption. You can determine whether the hypothesis is appropriate for judging the population proportion. You will also learn about Chi-square test also called Chi-squared or χ^2 test. Any statistical hypothesis test, in which the test statistic has a Chi-square distribution, when the null hypothesis is true, is termed as Chi-square test. Chi-square test is a non-parametric test of statistical significance for bivariate tabular analysis also known as cross-breaks.

In this unit, you will also learn about the basic principles of experimentation and ANOVA. In business decisions, we are often involved in determining if there are significant differences among various sample means, from which conclusions can be drawn about the differences among various population means. The methodology used for such types of determinations is known as ANalysis Of VAriance or ANOVA. This technique is one of the most powerful techniques in statistical analysis and was developed by R.A. Fisher. It is also called the *F*-Test. The basic principle of ANOVA

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is to test for differences among the means of the populations by examining the amount of varia.

8.1 UNIT OBJECTIVES

After going through this unit, you will be able to:

- Understand the concepts of hypothesis and the types of errors
- Explain the different types of hypotheses
- Identify the critical region or region of hypothesis rejection
- Explain the tests of equality of two proportions
- Understand the concepts of standard errors of statistics
- Explain the Chi-square test of significance
- Discuss the types of classification involved in analysis of variance
- Explain the steps involved in determining the differences within the factor of one way classification ANOVA

8.2 HYPOTHESIS TESTING

A hypothesis is an approximate assumption that a researcher wants to test for its logical or empirical consequences. Hypothesis refers to a provisional idea whose merit needs evaluation, but having no specific meaning. Though it is often referred to as a convenient mathematical approach for simplifying cumbersome calculation. Setting up and testing hypotheses is an integral art of statistical inference. Hypotheses are often statements about population parameters like variance and expected value. During the course of hypothesis testing, some inference about population like the mean and proportion are made. Any useful hypothesis will enable predictions by reasoning including deductive reasoning. According to Karl Popper a hypothesis must be falsifiable and that a proposition or theory cannot be called scientific if it does not admit the possibility of being shown false. Hypothesis might predict outcome of an experiment in a lab setting the observation of a phenomenon in nature. Thus, hypothesis is a explanation of a phenomenon proposal suggesting a possible correlation between multiple phenomena.

The characteristics of hypothesis are:

- **Clear and Accurate**: Hypothesis should be clear and accurate so as to draw a consistent conclusion.
- **Statement of Relationship between Variables**: If a hypothesis is relational, it should state the relationship between different variables.
- **Testability**: A hypothesis should be open to testing so that other deductions can be made from it and can be confirmed or disproved by observation. The researcher should do some prior study to make the hypothesis a testable one.
- **Specific with Limited Scope**: A hypothesis, which is specific, with limited scope, is easily testable than a hypothesis with limitless scope. Therefore, a researcher should pay more time to do research on such kind of hypotheses.

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- **Simplicity**: A hypothesis should be stated in the most simple and clear terms to make it understandable.
- **Consistency**: A hypothesis should be reliable and consistent with established and known facts.
- **Time-Limit**: A hypothesis should be capable of being tested within a reasonable time. In other words, it can be said that the excellence of a hypothesis is judged by the time taken to collect the data needed for the test.
- **Empirical Reference**: A hypothesis should explain or support all the sufficient facts needed to understand what the problem is all about.

A hypothesis is a statement or assumption concerning a population. For the purpose of decision-making, a hypothesis has to be verified and then accepted or rejected. This is done with the help of observations. We test a sample and make a decision on the basis of the result obtained. Decision-making plays significant role in different areas such as marketing, industry and management.

Statistical Decision-Making

Testing a statistical hypothesis on the basis of a sample enables us to decide whether the hypothesis should be accepted or rejected. The sample data enable us to accept or reject the hypothesis. Since the sample data give incomplete information about the population the result of the test need not be considered to be final or unchallengeable. The procedure, which, on the basis of sample results, enables us to decide whether a hypothesis is to be accepted or rejected, is called Hypothesis Testing or Test of Significance.

Note: A test provides evidence, if any, against a hypothesis, usually called a mill hypothesis. The test cannot prove the hypothesis to be correct. It can give some evidence against it.

The hypothesis makes some assumption about the density function of the random variate. The sampling distribution is fundamental to this subject.

The test of a hypothesis means a procedure to decide whether to accept or reject a hypothesis.

If a sample is found to have an untenable probability (of occurrence) level (called the significance level), we reject the hypothesis. Usually the probability levels of 0.05 and 0.01 are taken. They are called 5% and 1% significance levels.

Note: The acceptance of a hypothesis implies there is no evidence from the sample that we should believe otherwise.

The rejection of a hypothesis leads us to conclude that it is false. This way of putting the problem is convenient because of the uncertainty inherent in the problem. In view of this we must always briefly state a hypothesis that we hope to reject.

A hypothesis stated in the hope of being rejected is called a null hypothesis and is denoted by H_0.

If H_0 is rejected, it may lead to the acceptance of an alternative hypothesis denoted

by H_1.

For example, new fragrance soap is introduced in the market. The null hypothesis H_0, which may be rejected, is that the new soap is not better than the existing soap.

Example 1: A die is suspected to be loaded. Roll the die a number of times to test.

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Solution: The null hypothesis H_0: $p = 1/6$ for showing six.

The alternative hupothesis H_1: $p \neq 1/6$

8.2.1 Null and Alternative Hypotheses

Hypothesis is usually considered as the principal instrument in research. The basic concepts regarding the testability of a hypothesis are as follows:

(a) **Null Hypothesis and Alternative Hypothesis**: In the context of statistical analysis, while comparing any two methods, the following concepts or assumptions are taken into consideration:

(*i*) **Null Hypothesis**: While comparing two different methods in terms of their superiority, wherein the assumption is that both the methods are equally good is called null hypothesis. It is also known as statistical hypothesis and is symbolised as H_0.

(*ii*) **Alternate Hypothesis**: While comparing two different methods, regarding their superiority, wherein, stating a particular method to be good or bad as compared to the other one is called alternate hypothesis. It is symbolised as H_0.

(b) **Comparison of Null Hypothesis with Alternate Hypothesis**: Following are the points of comparison between null hypothesis and alternate hypothesis:

(*i*) Null hypothesis is always specific while Alternate Hypothesis gives an approximate value.

(*ii*) The rejection of Null hypothesis involves great risk, which is not in the case of Alternate hypothesis.

Null hypothesis is more frequently used in statistics than Alternate hypothesis because it is specific and is not based on probabilities.

The hypothesis to be tested is called the Null Hypothesis and is denoted by H_0.This is to be tested against other possible states of nature called alternative hypotheses. The alternative is usually denoted by H_1.

The null hypothesis implies that there is no difference between the statistic and the population parameter. To test whether there is no difference between the sample mean x and the population μ, we write the null hypothesis.

$$H_0: \bar{x} = \mu$$

The alternative hypothesis would be

$$H_0: \bar{x} \neq \mu$$

This means $\bar{x} > \mu$ or $\bar{x} < \mu$. This is called a two-tailed hypothesis.

The alternative H_0: $\bar{x} > \mu$ is right tailed.

The alternative H_0: $\bar{x} < \mu$ is left tailed.

These are one sided or one-tailed alternatives.

Notes:

1. The alternative hypothesis H_1 implies all such values of the parameter, which are not specified by the null hypothesis H_0.
2. Testing a statistical hypothesis is a rule, which leads to a decision to accept or reject a hypothesis.

A one tailed test requires rejection of the null hypothesis when the sample statistic is greater than the population value or less than the population value at a certain level of significance.

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1. We may want to test if the sample mean x exceeds the population mean p. Then the null hypothesis is,

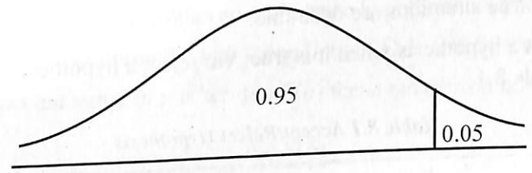
$$H_0: \bar{x} > \mu$$

2. In the other case the null hypothesis could be

$$H_0: \bar{x} < \mu$$

Each of these two situations leads to a one tailed test and has to be dealt with in the same manner as the two tailed test. Here the critical rejection is on one side only, right for $\bar{x} > \mu$ and left for $\bar{x} < \mu$. Figure 8.1 a five per cent level of test of significance.

For example, a minister in a certain government has an average life of 11 months without being involved in a scam. A new party claims to provide ministers with an average life of more than 11 months without scam. We would like to test if; on the average the new ministers last longer than 11 months. We may write the null hypothesis H_0: $\bar{x} = 11$ and alternative hypothesis H_1: $\bar{x} > 11$ or H_1: $\bar{x} < 11$.



(*a*) H_0: $> \mu$

(*b*) H_0: $< \mu$

Fig. 8.1 *Five Per Cent Level of Test of Significance*

8.2.2 Types of Errors

There are two types of errors in statistical hypothesis, which are as follows:

(a) **Type I Error**: In this type of error, you may reject a null hypothesis when it is true. It means rejection of a hypothesis, which should have been accepted. It is denoted by α (alpha) and is also known alpha error.

(b) **Type II Error**: In this type of error, you are supposed to accept a null hypothesis when it is not true. It means accepting a hypothesis, which should have been rejected. It is denoted by β (beta) and is also known as beta error.

Type I error can be controlled by fixing it at a lower level, for example, if you fix it at 2%, then the maximum probability to commit Type I error is 0.02. But reducing Type I error, has a disadvantage when the sample size is fixed as it increases the chances of Type II error. In other words, it can be said that both types of errors cannot be reduced simultaneously. The only solution of this problem is to set an appropriate level by considering the costs and penalties attached to them or to strike a proper balance between both types of errors.

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In a hypothesis test, a Type I error occurs when the null hypothesis is rejected when it is in fact true; that is, H_0 is wrongly rejected. For example, in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug; that is H_0: there is no difference between the two drugs on average. A Type I error would occur if we concluded that the two drugs produced different effects when in fact there was no difference between them.

In a hypothesis test, a Type II error occurs when the null hypothesis H_0, is not rejected when it is in fact false. For example, in a clinical trial of a new drug, the null hypothesis might be that the new drug is no better, on average, than the current drug; that is H_0: there is no difference between the two drugs on average. A Type II error would occur if it were concluded that the two drugs produced the same effect, that is, there is no difference between the two drugs on average, when in fact they produced different ones.

In how many ways can we commit errors?

We reject a hypothesis when it may be true. This is Type I error.

We accept a hypothesis when it may be false. This is Type II error.

The other true situations are desirable:

We accept a hypothesis when it is true. We reject a hypothesis when it is false as shown in Table 8.1.

***Table 8.1** Accept/Reject Hypothesis*

	Accept H_0	**Reject H_0**
H_0 True	Accept True H_0 Desirable	Reject True H_0 Type I Error
H_1 False	Accept False H_0 Type II Error	Reject False H_0 Desirable

The level of significance implies the probability of Type I error. A five per cent level implies that the probability of committing a Type I error is 0.05. A one per cent level implies 0.01 probability of committing Type I error.

Lowering the significance level and hence the probability of Type I error is good but unfortunately it would lead to the undesirable situation of committing Type II error.

Hence,

Type I Error - Rejecting H_0 when H_0 is true.

Type II Error - Accepting H_0 when H_0 is false.

Note:

The probability of making a Type I error is the level of significance of a statistical test. It is denoted by α.

Where,

α = Probability (Rejecting H_0 / H_0 true)

1–α = Probability (Accepting H_0 / H_0 true)

The probability of making a Type II error is denoted by β.

Where,

β = Probability (Accepting H_0 / H_0 false)

1–β = Probability (Rejecting H_0 / H_0 false) = Probability (The test correctly rejects H_0 when H_0 is false).

1–β is called the power of the test. It depends on the level of significance α, sample size n and the parameter value.

NOTES

8.2.3 Level of Significance

The hypothesis is examined on a pre-determined level of significance. Generally either 5 per cent level or 1 per cent level of significance is adopted for the purpose. However, it can stated here that the level of significance must be adequate keeping in view the purpose and nature of enquiry.

In this concept of hypothesis, you will formulate a rule provided both, null hypothesis and alternate hypothesis are given. Formulating a decision means either accepting null hypothesis and rejecting alternate hypothesis or rejecting null hypothesis and accepting alternate hypothesis. It can be easily understood with the help of an example, wherein you test 10 items and formulate a decision on the basis of the rule that states, a null hypothesis will be accepted if out of those 10 items, either none is defective or only 1 is defective otherwise alternate hypothesis will be accepted.

Suppose u is distributed normally with mean 0 and SD 1. Briefly we write $u \sim N$ (0, 1). If the expected value of u is written $E(u)$ the standardized normal variate is,

$$z = \frac{u - E(u)}{SE(u)}$$

The total area under the normal curve is 1 (corresponding to 100%). The area between $z = -1.96$ and $z = +1.96$ is 0.95. This is the region of acceptance with 95% confidence (see Figure 8.2). We write

$$p(-1.96 \le z \le 1.96) = 0.95$$

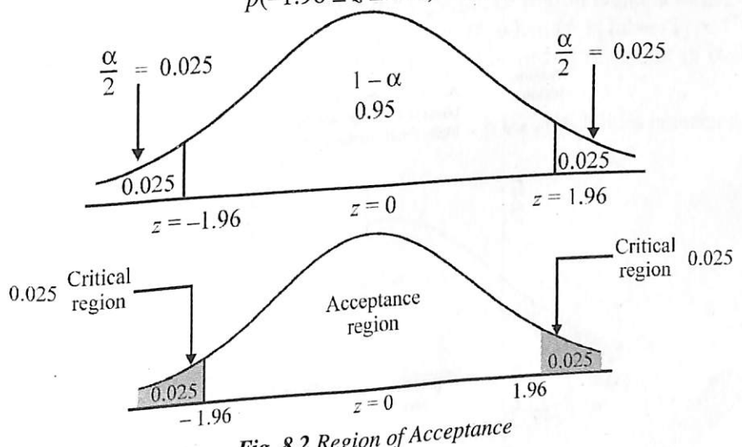

***Fig. 8.2** Region of Acceptance*

The size of the critical region is 0.05 (shaded area 0.025 on the left and 0.025 on the right). This is a two-tailed test.

If $|z|$ remains between the range $\pm$ 1.96, we are in the hypothesis acceptance region. The two values – 1.96 and 1.96 are the 5% critical values.

If $|z| > 1.96$ we are in the critical region, *i.e.*, the region of rejection of the hypothesis.

NOTES

8.2.4 Critical Region

The Critical Region (CR), or Rejection Region (RR), is a set of values for testing statistic for which the null hypothesis is rejected in a hypothesis test. It means, the sample space for the test statistic is partitioned into two regions; one region as the critical region will lead us to reject the null hypothesis H_0, the other not. So, if the observed value of the test statistic is a member of the critical region, we conclude that 'reject H_0'; if it is not a member of the critical region then we conclude that 'do not reject H_0'.

We shall consider test problems arising out of Type I error.

The level of significance of a test is the maximum probability with which we are willing to take a risk of Type I error.

If we take a 5% significance level ($p = 0.05$) we are 95% confident ($p = 0.95$) that a right decision has been made.

A 1% significance level ($p = 0.01$) makes us 99% confident ($p = 0.99$) about the correctness of the decision.

The critical region is the area of the sampling distribution in which the test statistic must fall for the null hypothesis to be rejected.

We can say that the critical region corresponds to the range of values of the statistic, which according to the test requires the hypothesis to be rejected.

8.2.5 One-Tailed and Two-Tailed Tests

A two-tailed test rejects the null hypothesis if the sample mean is either more or less than the hypothesised value of the mean of the population. It is considered to be apt when null hypothesis is of some specific value whereas alternate hypothesis is not equal to the value of null hypothesis. In a two-tailed curve there are two rejection regions, also called critical regions (see Figure 8.3).

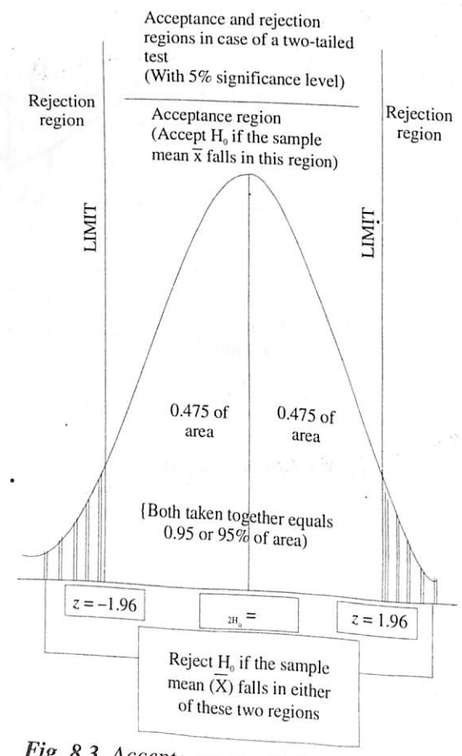

Fig. 8.3 Acceptance and Rejection Regions

NOTES

Conditions for the Occurrence of One-Tailed Test: When the population mean is either lower or higher than some hypothesised value, one-tailed test is considered to be appropriate where the rejection is only on the left tail of the curve. This is also known as left-tailed test (see Figure 8.4).

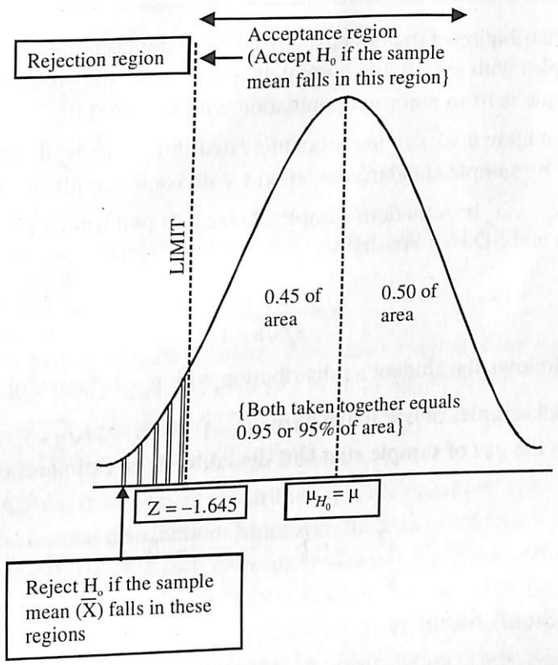

Fig. 8.4 Left-Tailed Test

For example, what will happen if the acceptance region is made larger? α will decrease. It will be more easily possible to accept H_0 when H_0 is false (Type II error), i.e., it will lower the probability of making a Type I error but raise that of α, Type II error.

Note: α, β are probabilities of making an error; 1 – α, 1 – β are probabilities of making correct decisions (see Figure 8.5).

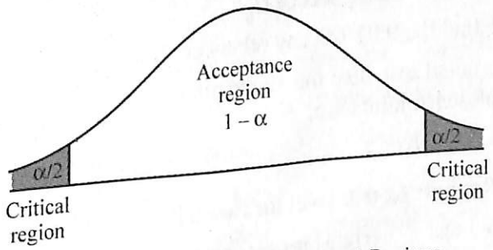

Fig. 8.5 Acceptance and Critical Regions

Example 2: Can we say $\alpha + \beta = 1$?

Solution: No. Each is concerned with a different type of error. But both are not independent of each other.

Check Your Progress

1. Define the term hypothesis.
2. What do you understand by 'Type-I' and 'Type-II' errors?
3. Explain null and alternate hypothesis.
4. Define the term critical region.
5. Explain decision rule briefly.

NOTES

8.3 TESTING THE SIGNIFICANCE OF DIFFERENCE BETWEEN MEAN AND PROPORTION

The sampling distribution of many statistics for large samples is approximately normal. For small samples with $n < 30$, the normal distribution, as shown above, can be used only if the sample is from a normal population with known σ.

If σ is not known we can use student's t distribution instead of the normal. We then replace σ by sample standard deviation s with some modification, given below.

Let $x_1, x_2, ..., x_n$ be a random sample of size n drawn from a normal population with mean (μ) and SD (σ). We define,

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

Here t follows the student's t distribution with $n - 1$ degree of freedom.

Note: For small samples of $n < 30$ the term $\sqrt{n-1}$ in $SE = s/\sqrt{n-1}$ corrects the bias resulting from the use of sample standard deviation as an estimator of σ.

Also,

$$\frac{s^2}{S^2} = \frac{n-1}{n} \text{ or } s = S\sqrt{\frac{n-1}{n}}$$

Procedure: Small Samples

To test the null hypothesis $H_0 : \mu = \mu_0$

Against the alternative $H_1 : \mu \neq \mu_0$

Calculate $|t| = \frac{\bar{x} - \mu}{SE(\bar{x})}$ and compare it with the table value with $n - 1$ degrees of freedom at level of significance $\alpha\%$

If this value > table value, reject H_0

If this value < table value, accept H_0

We can also find the 95% (or any other) confidence limits for μ.

For the two-tailed test (use the same rules as for large samples; substitute t for z) the 95% confidence limits are,

$\bar{x} \pm t_0 s/\sqrt{n-1}$ $\alpha = 0.025$

Rejection Region: At $\alpha\%$ level for two-tailed test. if $|t| > t_{\alpha/2}$ reject

For one-tailed test (right) if $t > t_\alpha$ reject

(left) if $t > t_\alpha$ reject

At 5% level the three cases are

If $\|t\| > t_{0.025}$ reject	two-tailed
if $t > t_{0.05}$ reject	one-tailed right
if $t < -t_{0.05}$	reject one-tailed left

For proportions, the same procedure is to be followed.

NOTES

Example 3: A firm produces tubes of diameter 2 cm. A sample of 10 tubes is found to have a diameter of 2.01 cm and variance 0.004. Is the difference significant? Given $t_{0.05,9} = 2.26$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$
$$= \frac{2.01 - 2}{\sqrt{0.004/10 - 1}}$$
$$= \frac{0.01}{0.021}$$
$$= 0.48$$

Since $|t| < 2.26$ the difference is not significant at 5% level.

8.3.1 *t*-Test for Single Mean

Sir William S. Gosset (pen name Student) developed a significance test and through it made significant contribution in the theory of sampling applicable in case of small samples. When population variance is not known, the test is commonly known as Student's t-test and is based on the t distribution.

Like the normal distribution, t distribution is also symmetrical but happens to be flatter than the normal distribution. Moreover, there is a different t distribution for every possible sample size. As the sample size gets larger, the shape of the t distribution loses its flatness and becomes approximately equal to the normal distribution. In fact, for sample sizes of more than 30, the t distribution is so close to the normal distribution that we will use the normal to approximate the t distribution. Thus, when n is small, the t distribution is far from normal, but when n as infinite, it is identical with normal distribution.

For applying t-test in context of small samples, the t value is calculated first of all and then calculated value is compared with the table value of t at certain level of significance for given degrees of freedom. If the calculated value of t exceeds the table value (say $t_{0.05}$), we infer that the difference is significant at 5% level, but if calculated value is t is less than its concerning table value, the difference is not treated as significant.

The t-test is used when the following two conditions are fullfilled:

(*i*) The sample size is less than 30, i.e., when $n \leq 30$.

(*ii*) The population standard deviation (σ_p) must be unknown.

In using the t-test we assume the following:

(*i*) That the population is normal or approximately normal;

(*ii*) That the observations are independent and the samples are randomly drawn samples;

(*iii*) That there is no measurement error;

(*iv*) That in the case of two samples, population variances are regarded as equal if equality of the two population means is to be tested.

NOTES

The following formulae are commonly used to calculate the t value:

(i) To test the significance of the mean of a random sample

$$t = \frac{|\bar{X} - \mu|}{S \,|\, SE_{\bar{x}}\bar{X}}$$

where $\bar{X}$ = Mean of the sample

μ = Mean of the universe

$SE_{\bar{x}}$ = S.E. of mean in case of small sample and is worked out as follows:

$$SE_{\bar{x}} = \frac{\sigma_s}{\sqrt{n}} = \frac{\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{\sqrt{n}}}}{\sqrt{n}}$$

and the degrees of freedom = $(n - 1)$

The above stated formula for t can as well be stated as under:

$$t = \frac{|\bar{x} - \mu|}{SE_{\bar{x}}}$$

$$= \frac{|\bar{x} - \mu|}{\frac{\sqrt{\Sigma(x - \bar{x})^2}}{n-1}}$$

$$= \frac{|\bar{x} - \mu|}{\sqrt{\frac{\Sigma(x - \bar{x})^2}{n-1}}} \times \sqrt{n}$$

If we want to work out the probable or fiducial limits of population mean (μ) in case of small samples, we can use either of the following:

(a) Probable limits with 95% confidence level,

$$\mu = \bar{X} \pm SE_{\bar{x}}\ (t_{0.05})$$

(b) Probable limits with 99% confidence level,

$$\mu = \bar{X} \pm SE_{\bar{x}}\ (t_{0.01})$$

At other confidence levels, the limits can be worked out in a similar manner, taking the concerning table value of t just as we have taken $t_{0.05}$ in (*i*) and $t_{0.01}$ in (*ii*) above.

(ii) To test the difference between the means of the two samples

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{\mathrm{SE}_{\bar{x}_1 - \bar{x}_2}}$$

where $\bar{X}_1$ = Mean of the sample 1

$\bar{X}_2$ = Mean of the sample 2

$SE_{\bar{x}_1 - \bar{x}_2}$ = Standard Error of difference between two sample means and is worked out as follows,

NOTES

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sum(X_{1i} - \bar{x}_1)^2 + \sum(X_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

and the degrees of freedom = $(n_1 + n_2 - 2)$

When the actual means are in fraction, then use of assumed means is convenient. In such a case, the standard deviation of difference, i.e.,

$$\sqrt{\frac{\Sigma(x_{1i} + x_1)^2 + \Sigma(x_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}}$$

can be worked out by the following short-cut formula:

$$= \frac{\sqrt{\Sigma(x_{1i} - A_1)^2 + \Sigma(x_{2i} - A_1)^2 - n_1(x_{1i} - A_2)^2 - n_2(x_{2i} - A_2)^2}}{n_1 + n_2 - 2}$$

where A_1 = Assumed mean of sample 1

A_2 = Assumed mean of sample 2

X_1 = True mean of sample 1

X_2 = True mean of sample 2

(iii) To test the significance of an observed correlation coefficient

$$t = \frac{r}{\sqrt{1 - r^2}} \times \sqrt{n-2}$$

Here t is based on $(n - 2)$ degrees of freedom.

(iv) In context of the 'difference test'

Difference test is applied in the case of paired data and in this context t is calculated as under:

$$t = \frac{\bar{x}_{Diff} - 0}{6_{Diff}\sqrt{n}} = \frac{\bar{x}_{Diff} - 0}{6_{Diff}} \sqrt{n}$$

where $\bar{X}_{Diff}$ or $\bar{D}$ = Mean of the differences of sample items.

0 = The value zero on the hypothesis that there is no difference

$\sigma_{\text{Diff.}}$ = Standard deviation of difference and is worked out as,

$$\sqrt{\frac{\sum D - \bar{X}_{Diff})^2}{(n-1)}}$$

or

$$\sqrt{\frac{\Sigma D^2 - (\bar{D})^2 n}{(n-1)}}$$

NOTES

D = Differences

n = Number of pairs in two samples and is based on $(n-1)$ degrees of freedom.

The following examples would illustrate the application of t-test using the above stated formulae.

Example 4: A sample of 10 measurements of the diameter of a sphere gave a mean $X = 4.38$ inches ad a standard deviation, $\sigma = 0.06$ inches. Find (a) 95% and (b) 99% confidence limits for the actual diameter.

Solution: On the basis of the given data the standard error of mean

$$= \frac{\sigma_s}{\sqrt{n-1}} = \frac{0.06}{\sqrt{10-1}} = \frac{0.06}{3} = 0.02$$

Assuming the sample mean 4.38 inches to be the population mean, the required limits are as follows:

(a) 95% confidence limits = $\bar{X} \pm SE_{\bar{x}}(t_{0.05})$ with degrees of freedom

$= 4.38 \pm .02(2.262)$

$= 4.38 \pm .04524$

i.e., 4.335 to 4.425

(b) 99% confidence limits = $\bar{X} \pm SE_{\bar{x}}(t_{0.01})$ with 9 degrees of freedom

$= 4.38 \pm .02(3.25) = 4.38 \pm .0650$

i.e., 4.3150 to 4.4450.

Example 5: The specimen of copper wires drawn from a large lot have the following breaking strength (in kg. wt.):

578, 572, 570, 568, 572, 578, 570, 572, 596, 544

Tests whether the mean breaking strength of the lot may be taken to be 578 kg. wt.

Solution: We take the hypothesis that there is no difference between the mean height of the sample and the given height of universe. In other words we can write, $H_0 : \mu = \bar{X}$, $H_0 : \mu \neq \bar{X}$. Then on the basis of the sample data the mean and standard deviation has been worked out as under:

S. No.	X	$(X - \bar{X})$	$(X_i - \bar{X})^2$
1	578	6	36
2	572	0	0
3	570	–2	4
4	568	–4	16
5	572	0	0
6	578	6	36
7	570	–2	4
8	572	0	0
9	596	24	576
10	544	–28	784
$n = 10$	$\Sigma X_i = 5720$		$\Sigma(X_i - \bar{X})^2 = 1456$

NOTES

$$\bar{X} = \frac{\Sigma x}{n} = \frac{5720}{10}$$

$$= 572$$

$$\sigma_s = \sqrt{\frac{\Sigma(x - \bar{x}_s)^2}{n-1}}$$

$$= \sqrt{\frac{1456}{10-1}} = \sqrt{\frac{1456}{9}}$$

$$= 12.72$$

$$SE_x = \frac{\sigma_s}{\sqrt{n}} = \frac{12.72}{\sqrt{10}}$$

$$= \frac{12.72}{3.16} = 4.03$$

$$t = \frac{|\bar{x} - \mu|}{SE_x} = \frac{|572 - 578|}{4.03}$$

$$= 1.488$$

Degrees of freedom = $n - 1 = 9$

At 5% level of significance for 9 degrees of freedom the table value of t = 2.262 for a two-tailed test.

The calculated value of t is less than its table value and hence the difference is insignificant. The mean breaking strength of the lot may be taken to be 578 kg. wt. with 95% confidence level.

Example 6: Sample of sales in similar shops in two towns are taken for a new product with the following results:

	Mean Sales	Variance	Size of Sample
Town *A*	57	5.3	5
Town *B*	61	4.8	7

Is there any evidence of difference in sales in the two towns?

Solution: We take the hypothesis that there is no difference between the two sample means concerning sales in the two towns. In other words, $H_0 : \bar{X}_1 = \bar{X}_2$, $H_0 : \bar{X}_1 \neq \bar{X}_2$. Then we work out the concerning t value as follows:

$$t = \frac{|\bar{X}_1 - \bar{X}_2|}{SE_{\bar{x}_1 - \bar{x}_2}}$$

Where, $\bar{x}_1$ = Mean of the sample concerning Town *A*

$\bar{x}_2$ = Mean of the sample concerning Town *B*

$SE_{\bar{x}_1 - \bar{x}_2}$ = Standard Error of the difference between two means

$$SE_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\Sigma(x_{1i} - \bar{x}_1)^2 + \Sigma(x_{2i} - \bar{x}_2)^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

NOTES

Hence,

$$t = \frac{|57 - 611|}{1.421} = \frac{4}{1.421}$$

$$= 2.82$$

Degrees of freedom = $(n_1 + n_2 - 2) = (5 + 7 - 2) = 10$

Table value of t at 5% level of significance for 10 degrees of freedom is 2.228, for a two-tailed test.

The calculated value of t is greater than its table value. Hence the hypothesis is wrong and the difference is significant.

Example 7: The sales data of an item in six shops before and after a special promotional campaign are:

Shops	*A*	*B*	*C*	*D*	*E*	*F*
Before the Promotional Campaign	53	28	31	48	50	42
After the campaign	58	29	30	55	56	45

Can the campaign be judged to be a success? Test at 5% level of significance.

Solution: We take the hypothesis that the campaign does not bring any improvement in sales. We can thus write:

In order to judge this, we apply the 'difference test'. For this purpose we calculate the mean and standard deviation of differences in two sample items as follows:

Shops	*Sales Before Campaign* X_{Bi}	*Sales After Campaign* X_{Ai}	*Difference = D (i.e., Increase Or Decrease After the Campaign)*	$(D - \bar{D})$	$(D - \bar{D})^2$
A	53	58	+5	+1.5	2.25
B	28	29	+1	–2.5	6.25
C	31	30	–1	–4.5	20.25
D	48	55	+7	+3.5	12.25
E	50	56	+6	+2.5	6.25
F	42	45	+3	–0.5	0.25
$n = 6$			$\Sigma D = 21$		$\Sigma(D - \bar{D})^2 = 47.50$

Mean of difference or $\bar{X}_{Diff} = \frac{\Sigma D}{n} = \frac{21}{6} = 3.5$

Standard deviation of difference,

$$\sigma_{Diff} = \sqrt{\frac{\Sigma(D - \bar{D})^2}{n - 1}} = \sqrt{\frac{47.50}{6 - 1}} = 3.08$$

$$t = \frac{\bar{X}_{Diff} - 0}{\sigma_{Diff}} = \sqrt{n}$$

$$= 1.14 \times 2.45 = 2.793$$

NOTES

Degrees of freedom = $(n - 1) = (6 - 1) = 5$

Table value of t at 5% level of significance for 5 degrees of freedom = 2.015 for one-tailed test.

Since the calculated value of t is greater than its table value, the difference is significant. Thus the hypothesis is wrong and the special promotional campaign can be taken as a success.

Example 8: Memory capacity of 9 students was tested before and after training. From the following scores, state whether the training was effective or not.

Student	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Before (X_{Bi})	10	15	9	3	7	12	16	17	4
After (X_{Ai})	12	17	8	5	6	11	18	20	3

Solution: We take the hypothesis that training was not effective. We can write, $H_0 : \bar{X}_A = \bar{X}_B$, $H_0 : \bar{X} > \bar{X}_B$. We apply the difference test for which purpose first of all we calculate the mean and standard deviation of difference as follows:

Students	*Before* X_{Bi}	*After* X_{Ai}	*Difference = D*	D^2
1	10	12	2	4
2	15	17	2	4
3	9	8	–1	1
4	3	5	2	4
5	7	6	–1	1
6	12	11	–1	1
7	16	18	2	4
8	17	20	3	9
9	4	3	–1	1
$n = 9$			$\Sigma D = 7$	$\Sigma D^2 = 29$

$$\bar{D} = \frac{\Sigma D}{n} = \frac{7}{9} = 0.78$$

$$\sigma_{Diff} = \sqrt{\frac{\Sigma D^2 - (\bar{D})^2 n}{n - 1}} = \sqrt{\frac{29 - (0.78)^2 \times 9}{9 - 1}} = 1.71$$

$$\therefore\ t = \frac{0.78}{1 - 71} = 1.369$$

Degrees of freedom = $(n - 1) = (9 - 1) = 8$

Table value of t at 5% level of significance for 8 degrees of freedom

= 1.860 for one-tailed test.

Since the calculated value of t is less than its table value, the difference is insignificant and the hypothesis is true. Hence it can be inferred that the training was not effective.

Example 9: It was found that the coefficient of correlation between two variables calculated from a sample of 25 items was 0.37. Test the significance of r at 5% level with the help of t-test.

NOTES

Solution: To test the significance of r through t-test, we use the following formula for calculating t value:

$$t = \frac{r}{\sqrt{1-r^2}} \times \sqrt{n-2}$$

$$= \frac{0.37}{1-(0.37)^2} \times \sqrt{25-2}$$

$$= 1.903$$

Degrees of freedom = $(n-2) = (25-2) = 23$

The table value of at 5% level of significance for 23 degrees of freedom is = 2.069 for a two-tailed test.

The calculated value of t is less than its table value, hence r *is* insignificant.

Example 10: A group of seven week old chickens reared on high protein diet weigh 12, 15, 11, 16, 14, 14 and 16 ounces; a second group of five chickens similarly treated except that they receive a low protein diet weigh 8, 10, 14, 10 and 13 ounces. Test at 5% level whether there is significant evidence that additional protein has increased the weight of chickens. (Use assumed mean (or A_1) = 10 for the sample of 7 and assumed mean (or A_2) = 8 for the sample of 5 chickens in your calculation).

Solution: We take the hypothesis that additional protein has not increased the weight of the chickens. We can write, $H_0: X_1 > X_2$ $H_0: X_1 > X_2$.

Applying t-test we work out the value of t for measuring the significance of two sample means as follows:

$$t = \frac{X_1 - X_2}{SE_{x_1 - x_2}}$$

Calculation can be done as under:

X_1	$(X_{i1}-A_1)$ $A_1=10$	$(X_{i1}-A_1)^2$	X_2	$(X_{i2}-A_2)$ $A_2=8$	$(X_{i2}-A_2)^2$
12	2	4	8	0	0
15	5	25	10	2	4
11	1	1	14	6	36
16	6	36	10	2	4
14	4	16	13	5	25
14	4	16			
16	6	36			
$n_1=7$	$\Sigma(X_{1i}-A_1)$ =28	$\Sigma(X_{1i}-A_1)^2$ =134	$n_2=5$	$\Sigma(X_{2i}-A_2)$ =15	$\Sigma(X_{2i}-A_2)^2$ = 69

$$X_1 = A_1 + \Sigma(x_{1i} - A_1)/n_1$$

$\therefore$ $$= 10 + \frac{28}{7} = 14$$

Similarly, $$X_2 = A_1 + \frac{\Sigma(x_{2i} - A_2)}{n_2}$$

$$= 8 + \frac{15}{5} = 11$$

NOTES

Hence,

$$SE_{X_1 - X_2} = \sqrt{\frac{\Sigma(X_{1i}-A_1)^2 + \Sigma(X_{2i}-A_2)^2 - n_1(\bar{X}_1 - A_1)^2 - n_2(\bar{X}_2 - A_2)^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$= \sqrt{\frac{134 + 69 - 7\,(14 - 10)^2 - 5(11-8)^2}{7+5-2}} \times \sqrt{\frac{1}{7} + \frac{1}{5}}$$

$$= (2.14)\ (0.59) = 1.2626$$

We now calculate the value under t,

$$t = \frac{X_1 - X_2}{SE_{X_1 - X_2}} = \frac{14-11}{1.2626} = 2.397$$

Degree of freedom = $(n_1 + n_2 - 2) = (7 + 5 - 2) = 10$

The table value of t at 5% level of significance for 10 degrees of freedom = 1.812 for one-tailed test.

The calculated value of t is higher than its table value and hence the difference is significant which means the hypothesis is wrong. It can therefore be concluded that additional protein has increased the weight of chickens.

8.3.2 Paired *t*-Test: Difference of Means

Let (x_1, y_1), $(x_2, y_2,)$, ...,(x_n, y_n) be the pairs of values for the same subjects, e.g., Sales data before (x) and after an advertisement campaign (y).

Performance of candidates before (X) and after training (y).

We have to test the significance of the difference between x, y values.

For each pair (x_i, y_i) find $d_i = x_i - y_i$

$H_0: \mu_1 = \mu_2$, i.e., no difference before and after and $H_0: \mu_1 \neq \mu_2$

We find the mean $\bar{d}$ of d values and use the statistic,

$$t = \frac{\bar{d}}{S/\sqrt{n}}$$

$$S = \sqrt{\frac{\sum (d - \bar{d})^2}{n-1}}$$

8.3.3 *F*-Test

An F-test is any statistical test in which if the null hypothesis is true, the test statistic has an F-distribution. A great variety of hypotheses in applied statistics are tested by F-tests. Among these are given below:

(a) The hypothesis that the means of multiple normally distributed populations, all having the same standard deviation, are equal. This is perhaps the most well-known of hypothesis tested by means of an F-test, and the simplest problem in the ANalysis Of Variance (ANOVA).

(b) The hypothesis that the standard deviations of two normally distributed populations are equal, and thus that they are of comparable origin.

NOTES

If there are two independent random samples from normal populations we have to test the hypothesis that the population variances σ_1^2, σ_2^2 are the same

$H_0 : \sigma_1^2 = \sigma_2^2$ and $H_1 : \sigma_1^2 = \sigma_2^2$

We find

$$S_1^2 = \frac{\sum (x_1 - \bar{x}_1)^2}{n_1 - 1}, \text{ estimate of } \sigma_1^2$$

$$S_2^2 = \frac{\sum (x_2 - \bar{x}_2)^2}{n_2 - 1}, \text{ estimate of } \sigma_2^2$$

To carry out the test of significance, find

$$F = \frac{S_1^2}{S_2^2} \text{ if } S_1^2 > S_2^2$$

or $$F = \frac{S_2^2}{S_1^2} \text{ if } S_2^2 > S_1^2$$

See the F tables for $n_1 - 1$, $n_2 - 1$ degree of freedom ($n_1 - 1$ corresponds to the numerator of F with the greater variance, $n_2 - 1$ for the denominator) at 5% level of significance.

If the observed F is less than the table value we assume the two populations have a common variance.

At 1% level of significance, the same procedure is to be followed.

Example 11: One sample of 10 bulbs gives a S.D. of 9 hours of life and another sample of 11 bulbs gives a S.D. of 10 hours of life. Can you say the variances are different at 1% level of significance? (Note: here $S_1 = 9$, $S_2 = 10$)

Solution:

$$S_1^2 = \frac{n_1}{n_1 - 1} S_1^2 = \frac{10}{10 - 1} \times (9)^2 = 90$$

$$S_2^2 = \frac{n_2}{n_2 - 1} S_2^2 = \frac{11}{11 - 1} \times (10)^2 = 110$$

$$F = \frac{S_2^2}{S_1^2} = \frac{110}{90} = 1.22 < \text{table value}$$

(Table value $F_{9,10,00.1} = 4.94$)

We accept the null hypothesis. The population variances may not be different.

8.3.4 Test for Equality of Two Population Variances

If p_1, p_2 are proportions of some characteristic of two samples of sizes n_1, n_2 drawn from populations with proportions P_1, P_2 then we have H_0: $P_1 = P_2$ vs H_1: $P_1 \neq P_2$

Case (a): If H_0 is true then let $P_1 = P_2 = p$

Where p can be found from the data,

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$$

$$q = 1 - p$$

p is the mean of the two proportions,

NOTES

$$SE(p_1 - p_2) = \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

$$z = \frac{P_1 - P_2}{SE(p_1 - p_2)}$$

We write z ~ $N(0, 1)$

The usual rules for rejection or acceptance are applicable here.

Case (b): If it is assumed that the proportion under question is not the same in the two populations from which the samples are drawn and that p_1, p_2 are the true proportions, we write,

$$SE(p_1 - p_2) = \sqrt{\left(\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}\right)}$$

We can also write the confidence interval *for* $p_1 - p_2$

For 2 independent samples of sizes $n_1 - n_2$ selected from two binomial populations, the 100 $(1 - a)$ % confidence limits *for* $p_1 - p_2$ are,

$$(p_1 - p_2) \pm z_{\alpha/2} \sqrt{\left(\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}\right)}$$

The 90%, confidence limits would be [with $\alpha = 0.1$, $100\ (1 - \alpha) = 0.90$]

$$(p_1 - p_2) \pm 1.645 \sqrt{\left(\frac{p_1 q_1}{n_1} + \frac{p_2 q_2}{n_2}\right)}$$

For example, out of 5000 interviewees, 2400 are in favour of a proposal; out of another set of 2000 interviewees 1200 are in favour. Is the difference significant?

$$p_1 = \frac{2400}{5000} = 0.48$$

$$p_2 = \frac{1200}{2000} = 0.6$$

$$n_1 = 5000$$

$$n_2 = 2000$$

$$SE = \sqrt{\left(\frac{.48 \times 52}{5000} + \frac{.6 \times .4}{2000}\right)} = 0.013 \text{ Case } (b)$$

$$z = \left|\frac{P_1 - P_2}{SE}\right|$$

$$= \frac{0.12}{0.013}$$

$$= 9.2 > 1$$

The difference is highly significant at 0.27% level.

NOTES

8.3.5 Test for Single Mean

We have to test the null hypothesis that the population mean has a specified value μ, i.e., H_0: $\bar{x} = \mu$. For large n, if H_0 is true then,

$z = \left|\frac{\bar{x} - \mu}{SE(\bar{x})}\right|$ is approximately nominal. The theoretical region for z depending on the desired level of significance can be found out.

Example 12: A factory produces items, each weighing 5 kg with variance 4. Can a random sample of size 900 with mean weight 4.45 kg. be justified as having been taken from this factory?

Solution:

$n = 900$

$\bar{x} = 4.45$

$\mu = 5$

$\sigma = \sqrt{4} = 2$

$$z = \left|\frac{\bar{x} - \mu}{SE(\bar{x})}\right| = \left|\frac{\bar{x} - \mu}{\sigma/\sqrt{4}}\right| = \left|\frac{4.45 - 5}{2/30}\right| = 8.25$$

We have $z > 3$. The null hypothesis is rejected. The sample may not be regarded as originally from the factory at 0.27% level of significance (corresponding to 99.73% acceptance region).

8.3.6 Test for Difference of Means

Suppose two samples of sizes n_1, n_2 are drawn from populations having means μ_1, μ_2 and standard deviations σ_1, σ_2

To test the equality of means $\bar{x}_1, \bar{x}_2$ we write,

$H_0 : \mu_1 = \mu_2$

$H_0 : \mu_1 \neq \mu_2$

If we assume H_0 is true then,

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Approximately normally distributed with mean -0, S.D.-I.

We write $z \sim N(0,1)$

As usual, if $|z| > 2$, we reject H_0 at 4.55% level of significance, and so on.

Example 13: Two groups of sizes 121 and 81 are subjected to tests. Their means are found to be 84 and 81 and standard directions 10 and 12.

Solution:

Test for the significance of difference between the groups,

$\bar{x}_1 = 84$

$\bar{x}_2 = 81$

NOTES

$n_1 = 121$

$n_2 = 81$

$\sigma_1 = 10$

$\sigma_2 = 12$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{84 - 81}{\sqrt{\frac{100}{121} + \frac{144}{81}}}$$

$= 1.86 < 1.96$

The difference is not significant at the 5% level of confidence.

8.4 CHI-SQUARE (χ^2) TEST

Chi-square test is a non-parametric test of statistical significance for bivariate tabular analysis (also known as cross-breaks). Any appropriate test of statistical significance lets you know the degree of confidence you can have in accepting or rejecting a hypothesis. Typically, the Chi-square test is any statistical hypothesis test, in which the test statistics has a Chi-square distribution when the null hypothesis is true. It is performed on different samples (of people) who are different enough in some characteristic or aspect of their behaviour that we can generalize from the samples selected. The population from which our samples are drawn should also be different in the behaviour or characteristic. Amongst the several tests used in statistics for judging the significance of the sampling data, Chi-square test, developed by Ronald A. Fisher, is considered as an important test. Chi-square, symbolically written as χ^2 (pronounced as Ki-square), is a statistical measure with the help of which, it is possible to assess the significance of the difference between the observed frequencies and the expected frequencies obtained from some hypothetical universe. Chi-square tests enable us to test whether more than two population proportions can be considered equal. In order that Chi-square test may be applicable, both the frequencies must be grouped in the same way and the theoretical distribution must be adjusted to give the same total frequency which is equal to that of observed frequencies. χ^2 is calculated with the help of the following formula:

$$\chi^2 = \sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\}$$

Where, f_o = The occurrence of observed or experimentally determined facts

f_e = The expected frequency of occurrence

Whether or not a calculated value of χ^2 is significant, it can be ascertained by looking at the tabulated values of χ^2 (given at the end of this book in appendix part) for given degrees of freedom at a certain level of confidence (generally a 5% level is taken). If the calculated value of χ^2 exceeds the table value, the difference between the observed and expected frequencies is taken as significant but if the table value is more than the calculated value of χ^2, then the difference between the observed and expected frequencies is considered as insignificant, i.e., considered to have arisen as a result of chance and as such can be ignored.

Check Your Progress

6. Define properties of the test for independence.
7. What is *F*-test?
8. What is statistical decision-making?

NOTES

8.4.1 Degrees of Freedom

The number of independent constraints determines the number of degrees of freedom (or *df*). If there are 10 frequency classes and there is one independent constraint, then there are (10 – 1) = 9 degrees of freedom. Thus, if n is the number of groups and one constraint is placed by making the totals of observed and expected frequencies equal, $df = (n - 1)$; when two constraints are placed by making the totals as well as the arithmetic means equal then $df = (n - 2)$, and so on. In the case of a contingency table, i.e., a table with two columns and more than two rows or table with two rows but more than two columns or a table with more than two rows and more than two columns or in the case of a 2 × 2 table the degrees of freedom is worked out as follows:

$$df = (c - 1)(r - 1)$$

Where, c = Number of columns

r = Number of rows

Conditions for the Application of Test

The following conditions should be satisfied before the test can be applied:

(a) Observations recorded and used are collected on a random basis.

(b) All the members (or items) in the sample must be independent.

(c) No group should contain very few items say less than 10. In cases where the frequencies are less than 10, regrouping is done by combining the frequencies of adjoining groups so that the new frequencies become greater than 10. Some statisticians take this number as 5, but 10 is regarded as better by most of the statisticians.

(d) The overall number of items (i.e., N) must be reasonably large. It should at least be 50, howsoever small the number of groups may be.

(e) The constraints must be linear. Constraints which involve linear equations in the cell frequencies of a contingency table (i.e., equations containing no squares or higher powers of the frequencies) are known as linear constraints.

8.4.2 Areas of Application of Chi-Square Test

Chi-square test is applicable in large number of problems. The test is, in fact, a technique through the use of which it is possible for us to (a) Test the goodness of fit; (b) Test the homogeneity of a number of frequency distributions; and (c) Test the significance of association between two attributes. In other words, Chi-square test is a test of independence, goodness of fit and homogeneity. At times Chi-square test is used as a test of population variance also.

***As a Test of Goodness of Fit*:** χ^2 test enables us to see how well the distribution of observe data fits the assumed theoretical distribution, such as Binomial distribution, Poisson distribution or the Normal distribution.

***As a Test of Independence*:** χ^2 test helps explain whether or not two attributes are associated. For instance, we may be interested in knowing whether a new medicine is effective in controlling fever or not then χ^2 test will help us in deciding this issue. In such a situation, we proceed on the null hypothesis that the two attributes (viz., new medicine and control of fever) are independent. Which means that new medicine is not effective in controlling fever. It may, however, be stated here that χ^2 is not a measure of the degree of relationship or the form of relationship between two attributes but it simply is a technique of judging the significance of such association or relationship between two attributes.

***As a Test of Homogeneity*:** χ^2 test helps us in stating whether different samples come from the same universe. Through this test, we can also explain whether the results worked out on the basis of sample/samples are in conformity with well defined hypothesis or the results fail to support the given hypothesis. As such the test can be taken as an important decision-making technique.

***As a Test of Population Variance*:** Chi-square is also used to test the significance of population variance through confidence intervals, specially in case of small samples.

8.4.3 Steps Involved in Finding the Value of Chi-Square

The various steps involved are as follows:

(a) First of all calculate the expected frequencies.

(b) Obtain the difference between observed and expected frequencies and find out the squares of these differences, i.e., calculate $(f_o - f_e)^2$.

(c) Divide the quantity $(f_o - f_e)^2$ obtained, as stated above by the corresponding expected frequency to get $\frac{(f_o - f_e)^2}{f_e}$.

(d) Then find summation of $\frac{(f_o - f_e)^2}{f_e}$ values or what we call $\sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\}$.

This is the required χ^2 value.

The χ^2 value obtained as such should be compared with relevant table value of χ^2 and inference may be drawn as stated above.

The following examples illustrate the use of Chi-square test.

Example 14: A dice is thrown 132 times with the following results:

Number Turned Up	1	2	3	4	5	6
Frequency	16	20	25	14	29	28

Test the hypothesis that the dice is unbiased.

Solution: Let us take the hypothesis that the dice is unbiased. If that is so, the probability of obtaining any one of the six numbers is 1/6 and as such the expected frequency of any one number coming upward is $132 \times \frac{1}{6} = 22$. Now, we can write the observed frequencies along with expected frequencies and work out the value of χ^2 as follows:

No. Turned Up	Observed Frequency (or f_o)	Expected Frequency (or f_e)	$(f_o - f_e)$	$(f_o - f_e)^2$	$\frac{(f_o - f_e)^2}{f_e}$
1	16	22	–6	36	36/22
2	20	22	–2	4	4/22
3	25	22	3	9	9/22
4	14	22	–8	64	64/22
5	29	22	7	49	49/22
6	28	22	6	36	36/22

NOTES

NOTES

$$\therefore \quad \sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\} = 9$$

Hence, the calculated value of $\chi^2 = 9$

$\because$ Degrees of freedom in the given problem is $(n - 1) = (6 - 1) = 5$

The table value of χ^2 for 5 degrees of freedom at 5% level of significance is 10.071. If we compare the calculated and table values of χ^2 we find that calculated value is less than the table value and as such could have arisen due to fluctuations of sampling. The result thus supports the hypothesis and it can be concluded that the dice is unbiased.

Example 15: Find the value of χ^2 for the following information:

Class Observed	A	B	C	D	E
Frequency	8	29	44	15	4
Theoretical (or Expected) Frequency	7	24	38	24	7

Solution: Since some of the frequencies are less than 10, we shall first regroup the given data as follows and then work out the value of χ^2:

Class	*Observed Frequency* (f_o)	*Expected Frequency* (f_e)	$(f_o - f_e)$	$\frac{(f_o - f_e)^2}{f_e}$
A and B	(8+29) = 37	(7+24) = 31	6	36/31
C	44	38	6	36/38
D and E	(15+4) = 19	(24+7) = 31	−12	144/31

$$\therefore \quad \chi^2 = \sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\} = 6.76 \text{ approx.}$$

The table value of χ^2 for two degrees of freedom at 5% level of significance is 5.991. The calculated value of χ^2 is much higher than this table value which means that the calculated value cannot be said to have arisen just because of chance. It is significant. Hence, the hypothesis does not hold good. This means that the sampling techniques adopted by the two investigators differ and are not similar. Naturally, then the technique of one must be superior than that of the other.

Calculation of Chi-Square

In a situation where the numbers of a random sample are classified into mutually exclusive categories, we can evaluate whether the observed frequencies (i.e., number of subjects in different categories on the basis of our observation) in these categories are consistent with some hypothesis concerning the relative frequencies.

The first step in computing the Chi-square test of independence is to compute the expected frequency for each cell under the assumption that the null hypothesis is true. This can be further explained with the help of following examples.

Example 16: The table below shows that of the 167 subjects in the experiment, 116 graduated. Use the Chi-square test of independence to compute the expected frequency for each cell under the assumption that the null hypothesis is true.

NOTES

	Graduated	Failed to Graduate	Total
Experimental	73	12	85
Control	43	39	82
Total	116	51	167

Solution: To calculate the expected frequency of the first cell in the table, i.e., experimental condition, graduated, first calculate the proportion of subjects that graduated without considering their condition. The table above shows that of the 167 subjects in the experiment, 116 graduated.

As per the question, if the null hypothesis is taken as true then the expected frequency for the first cell would be calculated as follows:

Expected frequency = Number of people in the total experimental condition (85) × Proportion of people graduating (116/167)

$$= \frac{(85)\times(116)}{(167)} = 59.042$$

Therefore, the expected frequency for the first cell is 59.042.

The general formula for calculating the expected cell frequencies is as follows:

$$E_{ij} = \frac{T_i \times T_j}{N}$$

Where,

E_{ij} = Expected frequency for the cell in the ith row and the jth column

T_i = Total number of subjects in the ith row

T_j = Total number of subjects in the jth column

N = Total number of subjects in the whole table

The first cell is named as E_{11} which means first row and first column. Similarly E_{21} means second row and first column, E_{12} means first row and second column and E_{22} means second row and second column. Now we will calculate the expected frequency for each cell as follows:

$$E_{11} = \frac{(85)\times(116)}{(167)} = 59.042 \qquad E_{12} = \frac{(85)\times(51)}{(167)} = 25.958$$

$$E_{21} = \frac{(82)\times(116)}{(167)} = 56.958 \qquad E_{22} = \frac{(82)\times(51)}{(167)} = 25.042$$

When the expected cell frequencies are computed then it is entered into the original table as shown below. The expected frequencies are in parentheses.

	Graduated	Failed to Graduate	Total
Experimental	73 (59.042)	12 (25.958)	85
Control	43 (56.958)	39 (25.042)	82
Total	116	51	167

NOTES

We use the formula of Chi-square test for independence which is as follows:

$$\chi^2 = \sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\}$$

For this example, $\chi^2 = 22.01$

Example 17: Suppose we take a very limited opinion poll with a small sample of 72 students of class *X*, regarding their plan of opting for arts, science or commerce at a later stage. The opinion of the students and their number (frequencies) are given below. Which is the most appropriate measure to test the agreement between these observed and expected results?

Arts	Science	Commerce
36	12	24

Solution: From the information given above in the table, can we assume that the difference in frequencies of these three exclusive categories is only due to chance and not due to the fact that the three streams are equally popular among the students. Here, Chi-square test is the most appropriate measure to test the agreement between these observed and expected results.

The formula for Chi-square (χ^2) is: $\chi^2 = \sum\left\{\frac{(f_o - f_e)^2}{f_e}\right\}$

Where,

f_o = Frequency of the occurrence of observed or experimentally determined facts

f_e = Expected frequency of occurrence

Row	Columns			
	Arts	Science	Commerce	Total
Observed Frequencies (f_o)	36	12	24	72
Expected Frequencies (f_e)	24	24	24	72
$(f_o - f_e)$	12	−12	0	
$(f_o - f_e)^2$	144	144	0	
$\frac{(f_o - f_e)}{f_e}$	6	6	0	

$\chi^2 = 6 + 6 + 0 = 12$

$df = 2$

The significance of χ^2 is ascertained by χ^2 table values with 'Degrees of Freedom' or '*df* 2.

Degrees of freedom or '$df = (r-1)\,(c-1)$

Where,

r = Number of rows

c = Number of columns

$df = (2-1)\,(3-1) = 2$

NOTES

Table value of χ^2 with *df* 2 at 0.05 and 0.01 level of significance is as follows:

0.05	0.01
5.99	9.21
5.100	

The obtained χ^2 value 12 with *df* 2 is greater than the table value even at 0.01 level of significance. The null hypothesis is rejected as the difference is highly significant. Hence, it may be concluded that all three groups differ significantly.

Alternative Formula for Finding the Value of Chi-Square in a (2 × 2) Table

There is an alternative method of calculating the value of χ^2 in the case of a (2 × 2) table. Let us write the cell frequencies and marginal totals in case of a (2 × 2) table as follows:

a	b	$(a + b)$
c	d	$(c + d)$
$(a + c)$	$(b + d)$	N

Then the formula for calculating the value of χ^2 will be stated as follows:

$$\chi^2 = \frac{(ad - bc)^2 N}{(a+c)(b+d)(a+b)(c+d)}$$

Where, N means the total frequency, ad means the larger cross product, bc means the smaller cross product and $(a + c)$, $(b + d)$, $(a + b)$ and $(c + d)$ are the marginal totals. The alternative formula is rarely used in finding out the value of Chi-square as it is not applicable uniformly in all cases but can be used only in a (2 × 2) contingency table.

8.4.4 Chi-Square as a Test of Population Variance

χ^2 is used, at times, to test the significance of population variance $(\sigma_p)^2$ through confidence intervals. This, in other words, means that we can use χ^2 test to judge if a random sample has been drawn from a normal population with mean (μ) and with specified variance $(\sigma_p)^2$. In such a situation, the test statistic for a null hypothesis will be as under:

$$\chi^2 = \Sigma\,\frac{(X_i - \bar{X}_s)^2}{(\sigma_p)^2} = \frac{n(\sigma_s)^2}{(\sigma_p)^2} \text{ with } (n-1) \text{ degrees of freedom.}$$

By comparing the calculated value (with the help of the above formula) with the table value of χ^2 for $(n-1)$ *df* at a certain level of significance, we may accept or reject the null hypothesis. If the calculated value is equal or less than the table value, the null hypothesis is to be accepted but if the calculated value is greater than the table value, the hypothesis is rejected. All this can be made clear by an example.

Example 18: Weight of 10 students is as follows:

Sl. No.	1	2	3	4	5	6	7	8	9	10
Weight in kg	38	40	45	53	47	43	55	48	52	49

NOTES

Can we say that the variance of the distribution of weights of all students from which the above sample of 10 students was drawn is equal to 20 square kg? Test this at 5% and 1% level of significance.

Solution:

First of all, we should work out the standard deviation of the sample (σ_s).

Calculation of the sample standard deviation:

Sl. No.	X_i *Weight in kg*	$X_i - \bar{X}_s$	$(X_i - \bar{X}_s)^2$
1	38	− 9	81
2	40	− 7	49
3	45	− 2	04
4	53	+ 6	36
5	47	+ 0	00
6	43	− 4	16
7	55	+ 8	64
8	48	+ 1	01
9	52	+ 5	25
10	49	+ 2	04
$n = 10$	$\Sigma X_i = 470$		$\Sigma (X_i - \bar{X}_s)^2 = 280$

$$\bar{X}_s = \frac{\Sigma X_i}{n} = \frac{470}{10} = 47 \text{ kg}$$

$$\therefore \quad \sigma_s = \sqrt{\frac{\Sigma (X_i - \bar{X}_s)^2}{n}} = \sqrt{\frac{280}{10}} = \sqrt{28} = 5.3 \text{ kg}$$

$$\therefore (\sigma_s)2 = 28$$

Taking the null hypothesis as H_0: $(\sigma_p)^2 = (\sigma_s)^2$

The test statistic $\chi^2 = \frac{n(\sigma_s)^2}{(\sigma_p)^2} = \frac{10 \times 28}{20} = \frac{280}{20} = 14$

Degrees of freedom in this case is $(n - 1) = 10 - 1 = 9$

At 5% level of significance, the table value of $\chi^2 = 16.92$, and at 1% level of significance it is 21.67 for 9 *df*, and both these values are greater than the calculated value of χ^2 which is 14. Hence, we accept the null hypothesis and conclude that the variance of the given distribution can be taken as 20 square kg at 5% as well as at 1% level of significance.

8.4.5 Additive Property of Chi-Square (χ^2)

An important property of χ^2 is its additive nature. This means that several values of χ^2 can be added together and if the degrees of freedom are also added, this number gives the degrees of freedom of the total value of χ^2. Thus, if a number of χ^2 values have been obtained from a number of samples of similar data, then, because of the additive nature of χ^2, we can combine the various values of χ^2 by just simply adding them. Such addition of various values of χ^2 gives one value of χ^2 which helps in forming a better idea about the significance of the problem under consideration. The following example illustrates the additive property of the χ^2.

NOTES

Example 19: The following values of χ^2 are obtained from different investigations carried to examine the effectiveness of a recently invented medicine for checking malaria.

Investigation	χ^2	*df*
1	2.5	1
2	3.2	1
3	4.1	1
4	3.7	1
5	4.5	1

What conclusion would you draw about the effectiveness of the new medicine on the basis of the five investigations taken together?

Solution: By adding all the values of χ^2, we obtain a value equal to 18.0. Also by adding the various *df* as given in the question, we obtain a figure 5. We can now state that the value of χ^2 for 5 degrees of freedom (when all the five investigations are taken together) is 18.0.

Let us take the hypothesis that the new medicine is not effective. The table value of χ^2 for 5 degrees of freedom at 5% level of significance is 10.070. But our calculated value is higher than this table value which means that the difference is significant and is not due to chance. As such the hypothesis is wrong and it can be concluded that the new medicine is effective in checking malaria.

Important Characteristics of Chi-Square (χ^2) Test

(a) This test is based on frequencies and not on the parameters like mean and standard deviation.

(b) This test is used for testing the hypothesis and is not useful for estimation.

(c) This test possesses the additive property.

(d) This test can also be applied to a complex contingency table with several classes and as such is a very useful test in research work.

(e) This test is an important non-parametric (or a distribution free) test as no rigid assumptions are necessary in regard to the type of population and no need of the parameter values. It involves less mathematical details.

A Word of Caution in Using χ^2 Test

Chi-square test is no doubt a most frequently used test but its correct application is equally an uphill task. It should be borne in mind that the test is to be applied only when the individual observations of sample are independent which means that the occurrence of one individual observation (event) has no effect upon the occurrence of any other observation (event) in the sample under consideration. The researcher, while applying this test, must remain careful about all these things and must thoroughly understand the rationale of this important test before using it and drawing inferences concerning his hypothesis.

Check Your Progress

9. Define the term Chi-square.
10. What is the use of Chi-square test?
11. Write the rule for correction using Yate's correction.
12. How a null hypothesis is accepted or rejected?
13. Write one important characteristics of Chi-square test.

NOTES

Can we say that the variance of the distribution of weights of all students from which the above sample of 10 students was drawn is equal to 20 square kg? Test this at 5% and 1% level of significance.

Solution:

First of all, we should work out the standard deviation of the sample (σ_s).

Calculation of the sample standard deviation:

Sl. No.	X_i *Weight in kg*	$X_i - \bar{X}_s$	$(X_i - \bar{X}_s)^2$
1	38	– 9	81
2	40	– 7	49
3	45	– 2	04
4	53	+ 6	36
5	47	+ 0	00
6	43	– 4	16
7	55	+ 8	64
8	48	+ 1	01
9	52	+ 5	25
10	49	+ 2	04
$n = 10$	$\Sigma X_i = 470$		$\Sigma (X_i - \bar{X}_s)^2 = 280$

$$\bar{X}_s = \frac{\Sigma X_i}{n} = \frac{470}{10} = 47 \text{ kg}$$

$$\therefore \quad \sigma_s = \sqrt{\frac{\Sigma (X_i - \bar{X}_s)^2}{n}} = \sqrt{\frac{280}{10}} = \sqrt{28} = 5.3 \text{ kg}$$

$\therefore (\sigma_s)2 = 28$

Taking the null hypothesis as H_0: $(\sigma_p)^2 = (\sigma_s)^2$

The test statistic $\chi^2 = \dfrac{n(\sigma_s)^2}{(\sigma_p)^2} = \dfrac{10 \times 28}{20} = \dfrac{280}{20} = 14$

Degrees of freedom in this case is $(n - 1) = 10 - 1 = 9$

At 5% level of significance, the table value of $\chi^2 = 16.92$, and at 1% level of significance it is 21.67 for 9 *df*, and both these values are greater than the calculated value of χ^2 which is 14. Hence, we accept the null hypothesis and conclude that the variance of the given distribution can be taken as 20 square kg at 5% as well as at 1% level of significance.

8.4.5 Additive Property of Chi-Square (χ^2)

An important property of χ^2 is its additive nature. This means that several values of χ^2 can be added together and if the degrees of freedom are also added, this number gives the degrees of freedom of the total value of χ^2. Thus, if a number of χ^2 values have been obtained from a number of samples of similar data, then, because of the additive nature of χ^2, we can combine the various values of χ^2 by just simply adding them. Such addition of various values of χ^2 gives one value of χ^2 which helps in forming a better idea about the significance of the problem under consideration. The following example illustrates the additive property of the χ^2.

NOTES

Example 19: The following values of χ^2 are obtained from different investigations carried to examine the effectiveness of a recently invented medicine for checking malaria.

Investigation	χ^2	*df*
1	2.5	1
2	3.2	1
3	4.1	1
4	3.7	1
5	4.5	1

What conclusion would you draw about the effectiveness of the new medicine on the basis of the five investigations taken together?

Solution: By adding all the values of χ^2, we obtain a value equal to 18.0. Also by adding the various *df* as given in the question, we obtain a figure 5. We can now state that the value of χ^2 for 5 degrees of freedom (when all the five investigations are taken together) is 18.0.

Let us take the hypothesis that the new medicine is not effective. The table value of χ^2 for 5 degrees of freedom at 5% level of significance is 10.070. But our calculated value is higher than this table value which means that the difference is significant and is not due to chance. As such the hypothesis is wrong and it can be concluded that the new medicine is effective in checking malaria.

Important Characteristics of Chi-Square (χ^2) Test

(a) This test is based on frequencies and not on the parameters like mean and standard deviation.

(b) This test is used for testing the hypothesis and is not useful for estimation.

(c) This test possesses the additive property.

(d) This test can also be applied to a complex contingency table with several classes and as such is a very useful test in research work.

(e) This test is an important non-parametric (or a distribution free) test as no rigid assumptions are necessary in regard to the type of population and no need of the parameter values. It involves less mathematical details.

A Word of Caution in Using χ^2 Test

Chi-square test is no doubt a most frequently used test but its correct application is equally an uphill task. It should be borne in mind that the test is to be applied only when the individual observations of sample are independent which means that the occurrence of one individual observation (event) has no effect upon the occurrence of any other observation (event) in the sample under consideration. The researcher, while applying this test, must remain careful about all these things and must thoroughly understand the rationale of this important test before using it and drawing inferences concerning his hypothesis.

Check Your Progress

9. Define the term Chi-square.
10. What is the use of Chi-square test?
11. Write the rule for correction using Yate's correction.
12. How a null hypothesis is accepted or rejected?
13. Write one important characteristics of Chi-square test.

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8.5 ANALYSIS OF VARIANCE

In business decisions, we are often involved in determining if there are significant differences among various sample means, from which conclusions can be drawn about the differences among various population means. For example, we may be interested to find out if there are any significant differences in the average sales figures of 4 different salesman employed by the same company, or we may be interested to find out if the average monthly expenditures of a family of 4 in 5 different localities are similar or not, or the telephone company may be interested in checking, whether there are any significant differences in the average number of requests for information received in a given day among the 5 areas of City (Under Study), and so on. The methodology used for such types of determinations is known as ANalysis Of VAriance or ANOVA. This technique is one of the most powerful techniques in statistical analysis and was developed by R.A. Fisher. It is also called the *F*-Test.

There are two types of classifications involved in the analysis of variance. The one-way analysis of variance refers to the situations when only one fact or variable is considered. For example, in testing for differences in sales for three salesman, we are considering only one factor, which is the salesman's selling ability. In the second type of classification, the response variable of interest may be affected by more than one factor. For example, the sales may be affected not only by the salesman's selling ability, but also by the price charged or the extent of advertising in a given area.

The Basic Principle of ANOVA

The basic principle of ANOVA is to test for differences among the means of the populations by examining the amount of variation within each of these samples, relative to the amount of variation between the samples. In terms of variation within the given population it is assumed that the values of (x_{ij}) differ from the mean of this population only because of random effects i.e., there are influences on (x_{ij}) which are unexplainable, whereas in examining differences between populations we assume that the difference between the mean of the *j*th population and the grand mean is attributable to what is called a 'specific factor' or what is technically described as treatment effect. Thus, while using ANOVA, we assume that each of the samples is drawn from a normal population and that each of these populations has the same variance. We also assume that all factors other than the one or more being tested are effectively controlled. This, in other words, means that we assume the absence of many factors that might affect our conclusions concerning the factor(s) to be studied.

8.5.1 One-Way Classification

Under the one-way ANOVA, we consider only one factor and then observe that the reason for the said factor to be important is that several possible types of samples can occur within that factor. We then determine if there are differences within that factor. The technique involves the following steps:

(a) Obtain the mean of each sample ie, obtain $\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_K$, i.e., when there are K samples.

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(b) Work out the mean of the sample means as follows:

$$\bar{\bar{x}} = \left(\frac{\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \cdots + \bar{x}_K}{K}\right)$$

where K = Number of samples.

(c) Take the deviations of the sample means from the mean of the sample means and calculate the square of such deviations which may be multiplied by the no. of items in the corresponding sample, and then obtain their total. This is known as the sum of squares for variance between the samples (or SS between).

Symbolically, this can be written as:

$$SS \text{ between} = \left[n_1\left(\bar{x}_1 - \bar{\bar{x}}\right)^2 + n_2\left(\bar{x}_2 - \bar{\bar{x}}\right)^2 + \cdots + n_k\left(\bar{x}_k - \bar{\bar{x}}\right)^2\right]$$

(d) Divide the result of the Step (c) by the (no. of) degrees of freedom between the samples to obtain variance or Mean Square (MS) between samples.

Symbolically, this can be written as:

$$MS \text{ between} = \left[\frac{SS \text{ between}}{(k-1)}\right]$$

where $(k-1)$ represents degrees of freedom (d.f.) between samples.

(e) Obtain the deviations of the values of the sample items for all the samples from corresponding means of the samples and calculate the squares of such deviations and then obtain their total. This total is known as the sum of squares for variance within samples (or *SS* within). Symbolically, this can be written as:

$$SS \text{ within} = \left[\Sigma\left(x_{1i} - \bar{x}_1\right)^2 + \Sigma\left(x_{2i} - \bar{x}_2\right)^2 + \cdots + \Sigma\left(x_{ki} - \bar{x}_k\right)^2\right]$$

where $i = 1, 2, 3, \ldots$

(f) Divide the result of Step (e) by the degrees of freedom within samples to obtain the variance or Mean Square (*MS*) within samples. Symbolically, this can be written as:

$$MS \text{ within} = \left[\frac{SS \text{ within}}{(n-k)}\right]$$

where $(n-k)$ represents the degrees of freedom within samples, (n = total no. of items in all the samples i.e., $(n_1 + n_2 + \ldots n_k)$ and k = no. of samples.

(g) For a check, the sum of squares of deviations for total variance can also be worked out by adding the squares of deviations when the deviations for the individual items in all the samples have been taken from the mean of the sample means. Symbolically, this can be written as:

$$SS \text{ for total variance} = \Sigma\left(x_{ij} - \bar{\bar{x}}\right)^2$$

$i = 1, 2, 3, \cdots$

$j = 1, 2, 3, \cdots$

This should be equal to the total of the results of Step (c) and Step (e) explained before, i.e.,

SS for total variance = *SS* between + *SS* within

NOTES

The degrees of freedom for total variance will be equal to the no. of items in all samples minus unity i.e., $(n-1)$. The degrees of freedom for 'between' and 'within' must add up to the degrees of freedom for total variance, i.e.,

$$(n-1) = [(k-1) + (n-k)]$$

This fact explains the additive property of the ANOVA technique.

(h) Finally, *F*-ratio may be worked out as under:

$$F\text{-ratio} = \frac{MS \text{ between}}{MS \text{ within}}$$

This ratio is used to judge whether the difference among several sample means is significant or is just a matter of sampling fluctuations. For this purpose we look into the table giving the values of F for given degrees of freedom at different levels of significance. If the worked out value of F, as stated above, is less than the table value of F, the difference is taken as insignificant, i.e., due to chance and the null hypothesis of no difference between sample means stands. In case the calculated value of F happens to be either equal or more than its table value, the difference is considered as significant (which means the samples could not have come from the same universe) and accordingly the conclusion may be drawn. The higher the calculated value of F is above the table value, the more definite and sure one can be about his conclusions/ inferences.

For the sake of convenience the information obtained through various steps stated above can be put as under:

Analysis of Variance Table for One-Way ANOVA
(There are K samples having in all *n* items)

Source of Variation	*Sum of Squares (SS)*	*Degrees of Freedom (d.f)*	*Mean Square (MS)* $\left(MS = \frac{SS}{d.f.}\right)$	*F-Ratio* $\left(\frac{MS \text{ between}}{MS \text{ within}}\right)$
Between Samples or Categories	$\left[n_1(\bar{x}_1 - \bar{\bar{x}})^2 + \ldots + n_k(\bar{x}_k - \bar{\bar{x}})^2\right]$ (= *SSC*)	$(k-1)$	$\frac{SS \text{ between}}{(k-1)}$ i.e., $MSC = \left\{\frac{SSC}{(k-1)}\right\}$	
Within Samples or Categories	$\left[\Sigma(x_{1i} - \bar{x}_1)^2 + \ldots + \Sigma(x_{ki} - \bar{x}_k)^2\right]$ (= *SSR*) $(i = 1, 2, 3, \ldots)$	$(n-k)$	$\frac{SS \text{ within}}{(n-k)}$ i.e., $MSR = \frac{SSR}{(n-k)}$	i.e., $\frac{MSC}{MSR}$
Total	$\Sigma(x_{ij} - \bar{\bar{x}})^2$ $i = 1, 2, \ldots$ (= *SST*) $j = 1, 2, \ldots$	$(n-1)$		

NOTES

Short-Cut Method for One-Way ANOVA

ANOVA can be performed by following the short-cut method which is usually used in practice since the same happens to be a very convenient method, particularly when means of the samples and/or mean of the sample means happen to be non-integer values. The various steps involved in the short-cut method are as under:

(a) Take the total of the values of individual items in all the samples, i.e., work out Sx_{ij} (where $i = 1, 2, 3, \cdots$ and $j = 1, 2, 3, \cdots$) and call it as T.

(b) Work out the correction factor as under:

$$\text{Correction factor} = \frac{T^2}{n}$$

(c) Find out the squares of all the item values one by one and then take their total. Subtract the correction factor from this total and the result is the sum of squares for total variance. Symbolically, this can be written as:

$$SS \text{ total, } SST = \left[\Sigma x_{ij}^2 - \frac{T^2}{n}\right]$$

where $i = \quad 1, 2, 3, \cdots$

and $j = 1, 2, 3, \cdots$

(d) Obtain the square of each sample total $\left(T_{ij}^2\right)$ and divide such square value by each sample by the number of items in the concerning sample and take the total of the result thus obtained. Subtract the correction factor from this total and the result is the sum of squares for variance between the samples. Symbolically, we can write:

$$SS \text{ between, } SSC = \left[\Sigma \frac{T_j^2}{nj} - \frac{T^2}{n}\right]$$

$(j = 1, 2, 3, \cdots)$

where subscript j represents different samples or categories.

(e) The sum of squares within the samples can be found out by subtracting the result of Step (d) from the result of Step (c) stated above and can be written as under:

$$SS \text{ within, } SSC = \left[\left\{\Sigma x_{ij}^2 - \frac{T^2}{n}\right\} - \left\{\Sigma \frac{Tj^2}{nj} - \frac{T^2}{n}\right\}\right]$$

$$= \left[\Sigma x_{ij}^2 - \Sigma \frac{Tj^2}{nj}\right]$$

After doing all this, the ANOVA table can be set up in the same way as explained earlier.

NOTES

ACTIVITY

1. A dice is rolled 5000 times. Of these, 2250 times it shows 3 or 4. Test the hypothesis that the die is unbiased.
2. Set up an ANOVA table for the following per acre production data for three kinds or varieties of wheat, each grown on 4 plots and state if the variety differences are significant.

Plot of land	*Per acre production data (variety of wheat)*		
	A	*B*	*C*
1	6	5	5
2	7	5	4
3	3	3	3
4	8	7	4

Did You Know

A working hypothesis is a hypothesis that is provisionally accepted as a basis for further research in the anticipation that a acceptable theory will be produced, even if the hypothesis ultimately fails. Like all hypotheses, a working hypothesis is constructed as a statement of expectations, which can be linked to the exploratory research purpose in empirical investigation and are often used as a conceptual framework in qualitative research.

8.6 SUMMARY

- A hypothesis is an approximate assumption that a researcher wants to test for its logical or empirical consequences. Hypothesis refers to a provisional idea whose merit needs evaluation, but having no specific meaning.
- Hypothesis might predict outcome of an experiment in a lab setting the observation of a phenomenon in nature. Thus, hypothesis is an explanation of a phenomenon proposal suggesting a possible correlation between multiple phenomena.
- While comparing two different methods in terms of their superiority, wherein the assumption is that both the methods are equally good is called null hypothesis. While comparing two different methods, regarding their superiority, wherein, stating a particular method to be good or bad as compared to the other one is called alternate hypothesis.
- The Type I and Type II errors cannot be reduced simultaneously. The only solution of this problem is to set an appropriate level by considering the costs and penalties attached to them or to strike a proper balance between both types of errors.
- The hypothesis is examined on a pre-determined level of significance. Generally either 5 per cent level or 1 per cent level of significance is adopted for the purpose. However, it can be stated here that the level of significance must be adequate keeping in view the purpose and nature of enquiry.
- Formulating a decision means either accepting null hypothesis and rejecting alternate hypothesis or rejecting null hypothesis and accepting alternate hypothesis.

Check Your Progress

14. Write the basic principle of ANOVA.
15. Define the term *F*-ratio.
16. Why *F*-ratio is used in one-way ANOVA?

NOTES

- The Critical Region (CR), or Rejection Region (RR), is a set of values for testing statistic for which the null hypothesis is rejected in a hypothesis test. It means that the sample space for the test statistic is partitioned into two regions; one region as the critical region will lead us to reject the null hypothesis H_0, the other not.
- A two-tailed test rejects the null hypothesis if the sample mean is either more or less than the hypothesized value of the mean of the population. It is considered to be apt when null hypothesis is of some specific value whereas alternate hypothesis is not equal to the value of null hypothesis.
- The sampling distribution of many statistics for large samples is approximately normal. For small samples with $n < 30$, the normal distribution, as shown above, can be used only if the sample is from a normal population with known σ.
- Sir William S. Gosset (pen name Student) developed a significance test and through it made significant contribution in the theory of sampling applicable in case of small samples. When population variance is not known, the test is commonly known as Student's *t*-test and is based on the *t* distribution.
- Like the normal distribution, *t* distribution is also symmetrical but happens to be flatter than the normal distribution. Moreover, there is a different *t* distribution for every possible sample size. As the sample size gets larger, the shape of the *t* distribution loses its flatness and becomes approximately equal to the normal distribution.
- In the test for given population variance, the variance is the square of the standard deviation, whatever you say about a variance can be, for all practical purposes, extended to a population standard deviation.
- An *F*-test is any statistical test in which if the null hypothesis is true, the test statistic has an *F*-distribution. A great variety of hypotheses in applied statistics are tested by *F*-tests.
- The hypothesis that the means of multiple normally distributed populations, all having the same standard deviation, are equal. This is perhaps the most well-known of hypotheses tested by means of an *F*-test, and the simplest problem in the ANalysis Of VAriance (ANOVA).
- Chi-square test is a non-parametric test of statistical significance for bivariate tabular analysis (also known as cross-breaks).
- The Chi-square test is any statistical hypothesis test, in which the test statistics has a chi-square distribution when the null hypothesis is true.
- Chi-square, symbolically written as χ^2 (pronounced as Ki-square), is a statistical measure with the help of which, it is possible to assess the significance of the difference between the observed frequencies and the expected frequencies obtained from some hypothetical universe.
- The correction suggested by Yates is popularly known as Yates' correction. It involves the reduction of the deviation of observed, from expected frequencies which of course reduces the value of χ^2.
- χ^2 is used to test the significance of population variance $(\sigma_p)^2$ through confidence intervals.
- An important property of χ^2 is its additive nature. This means that several values of χ^2 can be added together and if the degrees of freedom are also added, this number gives the degrees of freedom of the total value of χ^2.
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- An important property of χ^2 is its additive nature. This means that several values of χ^2 can be added together and if the degrees of freedom are also added, this number gives the degrees of freedom of the total value of χ^2.
- ANalysis Of VAriance or ANOVA technique is one of the most powerful techniques in statistical analysis and was developed by R.A. Fisher. It is also called the *F*-Test.

NOTES

- The basic principle of ANOVA is to test for differences among the means of the populations by examining the amount of variation within each of these samples, relative to the amount of variation between the samples.

8.13 KEY TERMS

- **Two-tailed test:** The statistical test used in hypothesis testing and named after the 'tail' of data falling under the far left and far right of a bell-shaped normal distribution or normal bell curve
- **One-tailed test:** The statistical test used in hypothesis testing and named after the 'tail' of data falling either on the left or on the right side of a bell-shaped normal distribution or normal bell curve
- ***t*-test:** This test is commonly known as students *t*-test and is based on the *t*-distribution. This is also used for small samples when population variance is not known
- ***F*-test:** This test is generally known as variance ratio test and is mostly used in the context of analysis of variance, a technique developed by the famous statistician Prof. R. A. Fisher
- ***F*-ratio:** It is used to judge whether the difference among several sample means is significant or is just a matter of sampling fluctuations
- **Chi-square test:** A non-parametric test of statistical significance used to compare observed data with expected data. It also tests the validity of null hypothesis
- **Degrees of freedom:** The number of independent observations in a sample of data to estimate a parameter of the population from which that sample is drawn

8.14 ANSWERS TO 'CHECK YOUR PROGRESS'

1. Hypothesis is an assumption that is tested to find its logical or empirical consequence.
2. Type I Error: In this type of error, you may reject a null hypothesis when it is true. It means rejection of a hypothesis which should have been accepted. It is denoted by α (alpha) and is also known as alpha error.

   Type II Error: In this type of error, you are supposed to accept a null hypothesis when it is not true. It means accepting a hypothesis which should have been rejected. It is denoted by β (beta) and is also known as beta error.
3. Null Hypothesis: While comparing two different methods in terms of their superiority, wherein the assumption is that both the methods are equally good is called null hypothesis. It is also known as statistical hypothesis and is symbolized as H_0.

   Alternate Hypothesis: While comparing two different methods, regarding their superiority, wherein, stating a particular method to be good or bad as compared to the other one is called alternate hypothesis. It is symbolized as H_a.
4. The Critical Region (CR), or Rejection Region (RR) is a set of values for testing statistics for which the null hypothesis is rejected in a hypothesis test.
5. In this concept of hypothesis, you will formulate a rule provided both null hypothesis and alternate hypothesis are given. Formulating a decision means either accepting null hypothesis and rejecting alternate hypothesis or rejecting null hypothesis and accepting alternate hypothesis.

NOTES

6. The following are the properties of the test for independence:
   - The data are the observed frequencies.
   - The data is arranged into a contingency table.
   - The degrees of freedom are the degrees of freedom for the row variable times the degrees of freedom for the column variable. It is not one less than the sample size, it is the product of the two degrees of freedom.
   - It is always a right tail test.
   - It has a chi-square distribution.
   - The expected value is computed by taking the row total times the column total and dividing by the grand total.
   - The value of the test statistic doesnot change if the order of the rows or columns are switched.
   - The value of the test statistic doesnot change if the rows and columns are interchanged (Transpose of the matrix).
7. An *F*-test is any statistical test in which if the null hypothesis is true, the test statistic has an *F*-distribution. A great variety of hypotheses in applied statistics are tested by *F*-tests.
8. Statistical decisions have to be made in the presence of uncertainty. In the testing of the hypothesis, the choice is between H_0 and H_1. In estimation, there are several choices available. The design of experiments requires one to choose between the nature and extent of observations.
9. Chi-square, symbolically written as χ^2 (pronounced as Ki-square), is a statistical measure with the help of which, it is possible to assess the significance of the difference between the observed frequencies and the expected frequencies obtained from some hypothetical universe.
10. Chi-square test is applicable in large number of problems. The test is, in fact, a technique through the use of which it is possible for us to (a) Test the goodness of fit; (b) Test the homogeneity of a number of frequency distributions; and (c) Test the significance of association between two attributes.
11. The rule for correction is to adjust the observed frequency in each cell of a (2×2) table in such a way as to reduce the deviation of the observed from the expected frequency for that cell by 0.5, and this adjustment is made in all the cells without disturbing the marginal totals.
12. By comparing the calculated value with the table value of χ^2 for $(n-1)$ *df* at a certain level of significance, we may accept or reject the null hypothesis. If the calculated value is equal or less than the table value, the null hypothesis is to be accepted but if the calculated value is greater than the table value, the hypothesis is rejected.
13. Chi-square test is an important non-parametric (or a distribution free) test as no rigid assumptions are necessary in regard to the type of population and no need of the parameter values. It involves less mathematical details.
14. The basic principle of ANOVA is to test for differences among the means of the populations by examining the amount of variation within each of these samples, relative to the amount of variation between the samples.
15. *F*-ratio is used to judge whether the difference among several sample means is significant or is just a matter of sampling fluctuations.
16. *F*-ratio is used in One-way ANOVA to judge whether the difference among several sample mean is significant or is just a matter of sampling function.

NOTES

8.15 QUESTIONS AND EXERCISES

Short-Answer Questions

1. What is hypothesis?
2. What are the importance of statistical decision-making?
3. Define the terms null and alternate hypothesis.
4. What are the various types of errors that occur in statistical hypothesis?
5. What is standard error?
6. What do you mean by the level of significance?
7. What is critical region?
8. Define the term one-tailed test.
9. What is the importance of a two-tailed test in statistics?
10. Write the importance of a small sample test.
11. What is the use of *t*-test?
12. Define the term *F*-test.
13. Differentiate between a small sample and a large sample.
14. Define the necessary conditions required for the application of test.
15. What are the areas of application of Chi-square test?
16. How will you find the value of Chi-square?
17. How Chi-square test is used as a test of population variance?
18. Write the basic principle of ANOVA.
19. What are the two types of classification involved in the analysis of variance?
20. What do you mean by term *F*-ratio?
21. Write the steps involved in the short-cut method for one-way ANOVA.
22. What happens if *F*-ratio is less than the table value of *F* in one-way ANOVA?

Long-Answer Questions

1. The normal rate of infection for a certain disease in cattle is known to be 50%. In an experiment with seven animals injected with a new vaccine it was found that none of the animals caught infection. Can the evidence be regarded as conclusive (at 1% level of significance) to prove the value of the new vaccine?
2. Is a particular significance level important? Explain.
3. A dice is rolled 9000 times. Of these, 3220 times it shows 3 or 4. Test the hypothesis that the die is unbiased.
4. A coin is tossed 200 times. It shows heads 116 times. Can you say the coin is biased?
5. A die was rolled 400 times. It showed a 6 coming up 80 times. Can you say the die is unbiased? p = 80/400=1/5 P=1/6
6. A company supplied 500 units of an item. The number of defectives was found to be 42 as against the company's conviction that 6 per cent items could be defective. Examine the tenability of the company claim.

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7. The average score of two groups A B were found to be 25 and 22 with S.D. 4 and 5.5, respectively. Test for the equality of the two group scores. Given

$$n_1 = n_2 = 400.$$

8. 800 ore pieces from a mine were found to contain an average of 74.5 gm of gold. From a nearby mine, 1600 pieces had 75 gm gold. Test the equality of the averages from the two mines, each having an S.D. of 2.4.
9. To test the goodness of a coin, it is tossed 5 times. It is considered a bad coin if more than 4 heads show up. (a) What is the probability of Type I error? (b) If the probability of a head is 0.2, what is the probability of Type II error?
10. In a sample of 500 people, 280 are tea drinkers and the rest coffee drinkers. Are tea and coffee equally popular?
11. 10 persons randomly selected are found to have heights 63, 63, 66, 67, 68, 69, 70, 71, 71, 71 inches. Discuss the suggestion that the mean height in the population is 66 inches.
12. 360 persons out of 600 are found to suffer from pollution induced bronchitis in one city. In another, 400 out of 500 are found to suffer from bronchitis. Is there any significant difference in the incidence of bronchitis?
13. In two large populations there are 30 per cent and 25 per cent smokers. Is this difference likely to be hidden in samples of 1200 and 900 from the populations?
14. A random sample of size 20 from a normal population gives a sample mean of 42 and a sample standard deviation of 6. Test the hypothesis that the population standard deviation is 9 at 5% level of significance.
15. Explain the significance of Chi-square test. Why is it considered an important test in statistical analysis?
16. Explain the important characteristics of Chi-square test.
17. Describe the significance of 'Degrees of Freedom'.
18. Chi-square can be used as a test of population variance. Explain with the help of example.
19. Describe the additive properties of Chi-square test.
20. Explain the test to determine the differences within the factor under the one-way ANOVA.
21. Discuss the various steps involved in the short-cut method for one-way ANOVA.
22. Describe the significance of randomized designs with the help of examples.
23. Two random samples were drawn from two normal populations and their values are:

A	66	67	75	76	82	84	88	90	92		
B	64	66	74	78	82	85	87	92	93	95	97

Test whether the two populations have the same variance at the 5% level of significance. $F = 3.36$ at 5% level for $n_1 = 10$ and $n_2 = 8$.

24. A manufacturing company has purchased three new machines of different makes and wishes to determine whether one of them is faster than the others in producing a certain output. Five hourly production figures are observed at random from each machine and the results are given below:

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Observations	A_1	A_2	A_3
1	25	31	24
2	30	39	30
3	36	38	28
4	38	42	25
5	31	35	28

Use ANOVA and determine whether the machines are significantly different in their mean speed. (Given at 5% level, $F_{2,12} = 3.89$)

8.16 FURTHER READING

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